Monotone functions

Given A, $B \subset \mathbb{R}$, a function $f : A \to B$ is

- nondecreasing if, for all $x_1, x_2 \in A$, with $x_1 < x_2$, it holds

$$f(x_1) \leq f(x_2)$$

- nonincreasing if, for all $x_1, x_2 \in A$, with $x_1 < x_2$, it holds

$$f(x_1) \geq f(x_2)$$

- increasing if, for all $x_1, x_2 \in A$, with $x_1 < x_2$, it holds

$$f(x_1) < f(x_2)$$

- decreasing if, for all $x_1, x_2 \in A$, with $x_1 < x_2$, it holds

$$f(x_1) > f(x_2)$$

Monotonicity vs injectivity

$f : A \to B$ is strictly monotone $\to$ f is injective

Monotonicity vs injectivity

$f : A \to B$ is strictly monotone $\to$ f is injective

$f : A \to B$ is injective $\not\to$ f is strictly monotone

Definitions about bounded sets

A subset A of $\mathbb{R}$ is said to be

- lower bounded if there exists $m \in \mathbb{R}$ such that, for all $x \in A$, it holds $m \leq x$. m is a lower bound of A;
- upper bounded if there exists $M \in \mathbb{R}$ such that, for all $x \in A$, it holds $x \leq M$. M is an upper bound of A;
- bounded if A is both lower bounded and upper bounded.

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- If m is a lower bound of A and $m \in A$, then m is the minimum of A, $\min(A)$ $\longrightarrow \min(A) \in \mathbb{R}$.
- If M is an upper bound of A and $M \in A$, then M is the maximum of A, $\max(A)$ $\longrightarrow \max(A) \in \mathbb{R}$.

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- If A is lower bounded, the infimum of A, $\inf(A)$, is the largest lower bound of A $\longrightarrow \inf(A) \in \mathbb{R}$.
- If A is upper bounded, the supremum of A, $\sup(A)$, is the smallest upper bound of A $\longrightarrow \sup(A) \in \mathbb{R}$.

Remark

- If A is lower bounded, $\inf(A)$ always exists, whereas $\min(A)$ may not exist.
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If A is lower unbounded, then conventionally $\inf(A) = -\infty$.

If A is upper unbounded, then conventionally $\sup(A) = +\infty$.