FOGLIO 7

① Calcolare il rango e determinare una base per il sottospazio generato dalle seguenti matrici a entrate in $\mathbb{C}$, e trovare l'inversa di quelle invertibili.

Ⓐ $\begin{pmatrix} 0 & -1 & 3 \\ 2 & 4 & -1 \\ 1 & -3 & 1 \end{pmatrix}$.

RANGO E BASE?

$$\begin{pmatrix} 0 & -1 & 3 \\ 2 & 4 & -1 \\ 1 & -3 & 1 \end{pmatrix} \xrightarrow[R_3 \leftrightarrow R_1]{} \begin{pmatrix} 1 & -3 & 1 \\ 2 & 4 & -1 \\ 0 & -1 & 3 \end{pmatrix} \xrightarrow[R_2 \to R_2 - 2R_1]{} \begin{pmatrix} 1 & -3 & 1 \\ 0 & 10 & -3 \\ 0 & -1 & 3 \end{pmatrix} \xrightarrow[R_3 \leftrightarrow R_2]{} \begin{pmatrix} 1 & -3 & 1 \\ 0 & -1 & 3 \\ 0 & 10 & -3 \end{pmatrix} \longrightarrow$$

$$\xrightarrow[R_3 \to R_3 + 10R_2]{} \begin{pmatrix} 1 & 3 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 27 \end{pmatrix} \Rightarrow \text{rg} = 3, \text{ una base può essere } \{(1\,3\,-2), (0\,-1\,3), (0\,0\,2\ldots$$

INVERSA?

$$\left(\begin{array}{ccc|ccc} 0 & -1 & 3 & 1 & 0 & 0 \\ 2 & 4 & -1 & 0 & 1 & 0 \\ 1 & -3 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_3 \leftrightarrow R_1]{} \left(\begin{array}{ccc|ccc} 1 & -3 & 1 & 0 & 0 & 1 \\ 2 & 4 & -1 & 0 & 1 & 0 \\ 0 & -1 & 3 & 1 & 0 & 0 \end{array}\right) \xrightarrow[R_2 \to R_2 - 2R_1]{} \left(\begin{array}{ccc|ccc} 1 & -3 & 1 & 0 & 0 & 1 \\ 0 & 10 & -3 & 0 & 1 & -2 \\ 0 & -1 & 3 & 1 & 0 & 0 \end{array}\right) \xrightarrow[R_2 \leftrightarrow R_3]{}$$

$$\xrightarrow[R_2 \leftrightarrow R_3]{} \left(\begin{array}{ccc|ccc} 1 & -3 & 1 & 0 & 0 & 1 \\ 0 & -1 & 3 & 1 & 0 & 0 \\ 0 & 10 & -3 & 0 & 1 & -2 \end{array}\right) \xrightarrow[R_3 \to R_3 + 10R_2]{} \left(\begin{array}{ccc|ccc} 1 & -3 & 1 & 0 & 0 & 1 \\ 0 & -1 & 3 & 1 & 0 & 0 \\ 0 & 0 & 27 & 10 & 1 & -2 \end{array}\right) \xrightarrow[R_3 \to \frac{R_3}{27}]{} \left(\begin{array}{ccc|ccc} 1 & -3 & 1 & 0 & 0 & 1 \\ 0 & -1 & 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{10}{27} & \frac{1}{27} & -\frac{2}{27} \end{array}\right)$$

$$\xrightarrow[\substack{R_2 \to -R_2 + 3R_3 \\ R_1 \to R_1 - R_3}]{} \left(\begin{array}{ccc|ccc} 1 & -3 & 0 & -\frac{10}{27} & -\frac{1}{27} & \frac{29}{27} \\ 0 & +1 & 0 & \frac{1}{9} & \frac{1}{9} & -\frac{2}{9} \\ 0 & 0 & 1 & \frac{10}{27} & \frac{1}{27} & -\frac{2}{27} \end{array}\right) \xrightarrow[R_1 \to R_1 + 3R_2]{} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{27} & \frac{8}{27} & \frac{11}{27} \\ 0 & 1 & 0 & \frac{1}{9} & \frac{1}{9} & -\frac{2}{9} \\ 0 & 0 & 1 & \frac{10}{27} & \frac{1}{27} & -\frac{2}{27} \end{array}\right). \checkmark\checkmark$$

$$A^{-1} = \begin{pmatrix} -\frac{1}{27} & \frac{8}{27} & \frac{11}{27} \\ \frac{1}{9} & \frac{1}{9} & -\frac{2}{9} \\ \frac{10}{27} & \frac{1}{27} & -\frac{2}{27} \end{pmatrix}$$

Ⓑ $$\begin{pmatrix} 2i & -2+\frac{1}{2}i & 1 & 0 \\ -2+2i & -\frac{5}{2}-\frac{3}{2}i & 1+i & 0 \\ 0 & 1 & -1 & i \\ 1 & 1 & 1 & 1 \end{pmatrix} \xrightarrow[\substack{R_1 \leftrightarrow R_4 \\ R_2 \leftrightarrow R_3}]{\text{(COMODITÀ)}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & i \\ -2+2i & -\frac{5}{2}-\frac{3}{2}i & 1+i & 0 \\ 2i & -2+\frac{1}{2}i & 1 & 0 \end{pmatrix} \xrightarrow[\substack{R_3 \to R_3 + (2-2i)R_1 \\ R_4 \to R_4 - 2iR_1}]{} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & i \\ 0 & (-\frac{1}{2}-\frac{7}{2}i) & (3-i) & (2-2i) \\ 0 & (-2-\frac{3}{2}i) & (1-2i) & (-2i) \end{pmatrix}$$

$$\xrightarrow[\substack{R_3 \to 2R_3 \\ R_4 \to 2R_4}]{\text{(COMODITÀ)}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & i \\ 0 & (-1-7i) & (6-2i) & (4-4i) \\ 0 & (-4-3i) & (2-4i) & (-4i) \end{pmatrix} \xrightarrow[\substack{R_3 \to R_3 + (1+7i)R_2 \\ R_4 \to R_4 + (4+3i)R_2}]{} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & i \\ 0 & 0 & (5-9i) & (-3-3i) \\ 0 & 0 & (-2-7i) & (-3) \end{pmatrix} \xrightarrow[R_3 \to R_3 - (1+i)R_4]{} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & (-2-7i) & -3 \end{pmatrix}$$

$\Rightarrow$ rg $= 3$, base $\{(1,1,1,1), (0,1,-1,i), (0,0,-2-7i,3)\}$. NON INVERTIBILE!

Ⓒ $\begin{pmatrix} 1+i & 1-i \\ i & -1 \\ 1-i & 3-i \\ i & i \end{pmatrix} \Rightarrow$ ha rango massimo 2 $\left(= \min\{\text{NUMERO RIGHE}, \text{NUMERO DI COLONNE}\}\right)$;

$(i,-1), (i,i)$ l.i. $\Rightarrow$ rg $= 2$, base $\{(i,-1), (i,i)\}$.

② IN $\mathbb{Z}_2$

$$\begin{pmatrix} 0 & -1 & 3 \\ 2 & 4 & -1 \\ 1 & -3 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \xrightarrow[R_3 \to R_3 - R_2]{R_1 \to R_1 - R_2} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \xrightarrow[R_3 \to R_3 - R_1]{} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \xrightarrow[R_1 \to R_1 + R_3]{} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow rg = 3$. INVERSA?

$$\left(\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_3 \to R_3 - R_1]{} \left(\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 & 1 \end{array}\right) \xrightarrow[R_1 \to R_1 - R_2]{} \left(\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 & 1 \end{array}\right) \rightarrow$$

$$\xrightarrow[\text{(SCAMBIO DI RIGHE)}]{} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right) \Rightarrow \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

$$\begin{pmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ 1 & -3 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}, \quad rg = 3, \text{ INVERSA?}$$

$$\left(\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_3 \to R_3 - R_2]{R_1 \to R_1 - R_2} \left(\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 & 1 \end{array}\right) \xrightarrow[R_1 \leftrightarrow R_3]{} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 & 0 \end{array}\right)$$

INVERSA

IN $\mathbb{Z}_3$

- $$\begin{pmatrix} 0 & -1 & 3 \\ 2 & 4 & -1 \\ 1 & -3 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 0 \\ 2 & 1 & 2 \\ 1 & 0 & 1 \end{pmatrix} \xrightarrow[\substack{R_2 \to R_2 - 2R_1 \\ (= R_2 + R_1)}]{} \begin{pmatrix} 0 & 2 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \xrightarrow[R_1 \to R_1 + R_2]{} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$\Rightarrow rg = 2$, base $\{(0,1,0), (1,0,1)\}$.

- $$\begin{pmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ 1 & -3 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix} \xrightarrow[R_2 \to R_2 + R_1]{} \begin{pmatrix} 2 & 1 & 0 \\ 3 & 3 & 0 \\ 1 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 2 \end{pmatrix}$$

$\Rightarrow rg = 2$, base $\{(2,1,0), (1,0,2)\}$.

③ determinare $r, s \in \mathbb{Z}$ t.c. $ra + sb = MCD(a,b)$, con:

ⓐ $(a,b) = (14, 12)$

ALGORITMO DI BEZOUT: $\underbrace{14}_{a} = \underbrace{1}_{q_1} \cdot \underbrace{12}_{b} + \underbrace{2}_{r_1}$, $\underbrace{12}_{b} = \underbrace{6}_{q_2} \cdot \underbrace{2}_{r_1} + \underbrace{0}_{r_2}$

$r_2 = 0 \Rightarrow r_1 = MCD(14,12) \Rightarrow MCD(14,12) = 2$;

(DA PRIMA) $14 = 1 \cdot 12 + 2 \Rightarrow 1 \cdot 14 + (-1) \cdot 12 = 2 \Rightarrow r = 1, s = -1$ ✓.

(B) $(a,b) = (-20,7)$

Calcolando per $(a,b) = (20,7)$. Basterà cambiare il segno al coefficiente di 20.

Alg. MCD: $\underbrace{20}_{a} = \underbrace{2}_{q_1} \cdot \underbrace{7}_{b} + \underbrace{6}_{r_1}$, $\quad 7 = \underbrace{1}_{q_2} \cdot \underbrace{6}_{r_1} + \underbrace{1}_{r_2}$ $\quad \underbrace{6}_{r_1} = \underbrace{6}_{q_3} \cdot \underbrace{1}_{r_2} + \underbrace{0}_{r_3} \Rightarrow r_3 = 0 \Rightarrow r_2 = 1 = MCD(7,20)$.

$20 = 2\cdot 7 + 6 \Rightarrow 6 = 20 - 2\cdot 7 \Rightarrow 7 = 1\cdot 6 + 1 = 1\cdot(20 - 2\cdot 7) + 1$

$\Rightarrow 1 = 7 - (20 - 2\cdot 7) = -20 + 3\cdot 7 \Rightarrow r = 1, s = 3$ ✓.

(per $(a,b) = (-20,7)$)

(C) $(a,b) = (42,11)$.

$42 = 3\cdot 11 + 9$, $\quad 11 = 1\cdot 9 + 2$, $\quad 9 = 4\cdot 2 + 1 \Rightarrow MCD(42,11)$.

$9 = 42 - 3(11)$ $\quad 2 = 11 - 9$

$2 = 11 - 42 + 3\cdot 11$

$= -1(42) + 4(11) \longrightarrow 42 - 3(11) = 4(-42 + 4\cdot 11) + 1$

$\Rightarrow 1 = 42 - 3\cdot 11 + 4\cdot 42 - 16\cdot 11 = 5\cdot 42 - 19\cdot 11 \Rightarrow r = 5, s = -19$

(4) 4^{-1} in $\mathbb{Z}_3, \mathbb{Z}_{11}, \mathbb{Z}_{13}$.

- $\mathbb{Z}_3$

$4 \equiv 1 \Rightarrow 1^{-1} = 1$ ✓.

$\mathbb{Z}_{11}$

$4\cdot 3 = 12 = 11 + 1 \equiv 1 \Rightarrow 3 = 4^{-1}$ ✓.

- $\mathbb{Z}_{13}$

$MCD(4,13) = 1 \Rightarrow \exists\, r,s$ t.c. $r4 + s\cdot 13 = 1 \Rightarrow r\cdot 4 \equiv 1 \pmod{13} \Rightarrow r = 4^{-1}$ in $\mathbb{Z}_{13}$.

$13 = 4\cdot 3 + 1 \Rightarrow 1 = 13 + (-3)\cdot 4 \Rightarrow 4^{-1} = -3 = 10 \pmod{13}$ ✓✓✓.

⑤ $\begin{cases} f: \mathbb{R}^3 \to \mathbb{R}^2 \\ (x_1,x_2,x_3) \longmapsto (-2x_1+x_3,\ x_1+2x_2-x_3). \end{cases}$

Le componenti di f sono polinomi di primo grado in $x_1, x_2, x_3 \Rightarrow f$ lineare ✓.

BASE DI Ker, Im f?

- $\operatorname{Ker} f = \{(x_1,x_2,x_3) \in \mathbb{R}^3 \mid -2x_1+x_3 = x_1+2x_2-x_3 = 0\}$

$\to$ risolviamo $\begin{cases} -2x_1 + x_3 = 0 \\ x_1+2x_2-x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_3 = -2x_1 \\ x_2 = \frac{x_3 - x_1}{2} = -\frac{3}{2}x_1 \end{cases}$

$\Rightarrow \operatorname{Ker} f = \{(a, -2a, -\frac{3}{2}a) \mid a \in \mathbb{R}\} = \langle (1,-2,-\frac{3}{2}) \rangle \to$ base $\{(1,-2,-\frac{3}{2})\}$

- $\operatorname{Im} f = \{(-2a+c,\ a+2b+c) \in \mathbb{R}^2 \mid a,b,c \in \mathbb{R}\} = \{a(-2,1)+b(0,2)+c(1,1) \mid a,b,c \in \mathbb{R}\}$

$= \langle (-2,1), (0,2), (1,1) \rangle = \mathbb{R}^2. \Rightarrow$ base $\{(0,2),(1,1)\}$.

$\Rightarrow$ base di $\mathbb{R}^3$ $B\{\underbrace{(1,-2,-\frac{3}{2})}_{u_1}, \underbrace{(1,0,0)}_{u_2}, \underbrace{(0,1,0)}_{u_3}\}$, base di $\mathbb{R}^2$ $C = \{(0,2),(1,1)\}$.

MATRICE DI f?

$\Rightarrow M^C_B(f) = (f(u_1) \mid f(u_2) \mid f(u_3))$ in componenti rispetto a C.

$f(u_1) = (0,0) = 0(0,2) + 0(1,1) \to$ PRIMA COLONNA $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$f(u_2) = (-2,1) = (0,3) - 2(1,1) = \frac{3}{2}(0,2) - 2(1,1) \to$ SECONDA COLONNA $\begin{pmatrix} \frac{3}{2} \\ -2 \end{pmatrix}$

$f(u_3) = (0,2) = 1 \cdot (0,2) + 0(1,1) \to$ TERZA COLONNA $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\Rightarrow M^C_B(f) = \begin{pmatrix} 0 & 3/2 & 1 \\ 0 & -2 & 0 \end{pmatrix}$.

$B' = \{\underbrace{(1,1,1)}_{v_1}, \underbrace{(1,-2,0)}_{v_2}, \underbrace{(0,1,1)}_{v_3}\}$ base di $\mathbb{R}^3$? $C' = \{\underbrace{(1,-1)}_{w_1}, \underbrace{(2,3)}_{w_2}\}$ base di $\mathbb{R}^2$?

B' base $\Leftrightarrow \operatorname{rg}\begin{pmatrix} 1 & 1 & 1 \\ 1 & -2 & 0 \\ 0 & 1 & 1 \end{pmatrix} = 3$. $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -2 & 0 \\ 0 & 1 & 1 \end{pmatrix} \xrightarrow[R_2 \to R_2 - R_1]{} \begin{pmatrix} 1 & 1 & 1 \\ 0 & -3 & -1 \\ 0 & 1 & 1 \end{pmatrix} \xrightarrow[R_2 \to R_2 + R_3]{} \begin{pmatrix} 1 & 1 & 1 \\ 0 & -2 & 0 \\ 0 & 1 & 1 \end{pmatrix} \Rightarrow \operatorname{rg} = 3$ ✓.

C' base $\Leftrightarrow \operatorname{rg}\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} = 2$ $\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \xrightarrow[R_2 \to R_2 - 2R_1]{} \begin{pmatrix} 1 & -2 \\ 0 & 5 \end{pmatrix}$

MATRICE $M^{C'}_{B'}(f)$?

$f(1,1,1) = (-2,2) = -2(1,-1) + 0(2,3)$, $f(1,-2,0) = (-2,-3) = 0(1,-1) - 1 \cdot (2,3)$

$f(0,1,1) = (1,1)$. $a(1,-1) + b(2,3) = (1,1) \Leftrightarrow \begin{cases} a+2b = 1 \\ -a+3b = 1 \end{cases} \Leftrightarrow \begin{cases} a = \frac{1}{5} \\ b = \frac{2}{5} \end{cases} \Rightarrow M^{C'}_{B'}(f) = \begin{pmatrix} -2 & 0 & 1/5 \\ 0 & -1 & 2/5 \end{pmatrix}$.

(6) $v_1 = (1,1,-3,1)$, $v_2 = (-2,0,1,0)$ l.i.?

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & -3 & 1 \\ -2 & 0 & 1 & 0 \end{pmatrix} \xrightarrow{R_2 \to R_2 + 2R_1} \begin{pmatrix} 1 & 1 & -3 & 1 \\ 0 & 2 & -5 & 2 \end{pmatrix} \Rightarrow rk = 2 \Rightarrow v_1, v_2 \text{ l.i. } //.$$

COMPLETARE A UNA BASE DI $\mathbb{R}^4$?

Consideriamo
$$\begin{pmatrix} v_1 \\ v_2 \\ \hline e_1 \\ \cdots \\ e_4 \end{pmatrix} = \begin{pmatrix} 1 & 1 & -3 & 1 \\ -2 & 0 & 1 & 0 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Vogliamo estrarre 4 righe l.i.
$\Rightarrow$ riduciamo $\frac{v_1}{v_2}$ per righe e prendiamo gli e_i corrispondenti alle colonne senza pivot.

$$\begin{pmatrix} 1 & 1 & -3 & 1 \\ -2 & 0 & 1 & 0 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_2 \to R_2 + 2R_1} \begin{array}{c} \phantom{\boxed{1}} \qquad \overbrace{}^{\text{SENZA PIVOT!}} \\ \begin{pmatrix} \boxed{1} & 1 & -3 & 1 \\ 0 & \boxed{2} & -5 & 2 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array} \Rightarrow (v_1, v_2, e_3, e_4) \text{ base di } \mathbb{R}^4 \; //.$$

(7) TABELLA DI ADDIZIONE IN $\mathbb{Z}_3$

	0	1	2
0	0	1	2
1	1	2	3
2	2	3	4

$3 \equiv 0 \mod 3$

$4 \equiv 1 \mod 3$

$\Rightarrow$

	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

(8) l'unico omomorfismo $f: \mathbb{Z}_3 \to \mathbb{Z}_4$ di gruppi additivi è quello nullo.

Basta stabilire $f([1])$: infatti, $f([0]) = [0]$, e $f([2]) = f([1]+[1]) = f([1]) + f([1])$.

Sappiamo $[1]+[1]+[1] = [3] = [0] \Rightarrow f([1]) + f([1]) + f([1]) = [0]$ in $\mathbb{Z}_4$.

Osserviamo che l'unico $[a] \in \mathbb{Z}_4$ t.c. $[a]+[a]+[a] = [0]$ è $[0] \Rightarrow f([1]) = [0]$

$$\Rightarrow \begin{cases} f([0]) = [0] \\ f([1]) = [0] \\ f([2]) = f([1]) + f([1]) = [0] \end{cases} \Rightarrow f \equiv 0 \text{ omomorfismo nullo } ///.$$

(9) l'unico omomorfismo $f: \mathbb{Z}_n \to \mathbb{C}$ è quello nullo $\forall n \geq 2$.

Stesso procedimento dell'esercizio (8):

- STEP 1: $\underbrace{f([1]) + \ldots + f([1])}_{n \text{ VOLTE}} = f([n]) = f([0]) = 0$

- STEP 2: $\forall [j] \in \mathbb{Z}_n$ $\quad [j] = \underbrace{[1] + \ldots + [1]}_{j \text{ VOLTE}} \Rightarrow f([j]) = f([1]) + \ldots + f([1]) = 0 + \ldots + 0 = 0$

$///$.