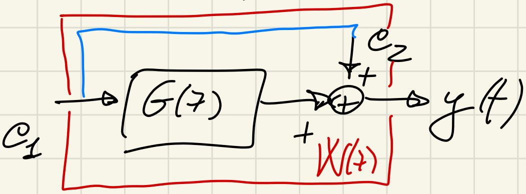
Esercizio del 27/11/2020 caso (c)

$e_1(\cdot) \sim WN(0,1)$



$e_2(t) = e_1(t)$

$$G(z) = \frac{1}{1-\frac{1}{4}z^{-1}} = \frac{z}{z-\frac{1}{4}}$$

$$W(z) = 1 + G(z) = 1 + \frac{z}{z-\frac{1}{4}} = \frac{2z-\frac{1}{4}}{z-\frac{1}{4}} = \frac{2-\frac{1}{4}z^{-1}}{1-\frac{1}{4}z^{-1}}$$

ARMA(1,1)

$$y_c(t) = \frac{1}{4}y_c(t-1) + 2e_1(t) - \frac{1}{4}e_1(t-1)$$

NON è in forma canonica!

$$W(z) = \frac{2-\frac{1}{4}z^{-1}}{1-\frac{1}{4}z^{-1}}$$

Stesso grado N, D

poli, zeri con $|\cdot|<1$

$$\Phi_y(z) = G(z)\cdot G(z^{-1})\cdot\lambda^2 = \left[\frac{1}{\alpha}G(z)\right]\left[\frac{1}{\alpha}G(z^{-1})\right]\alpha^2\lambda^2$$

$$\Phi_y(z) = W(z)\,W(z^{-1})\,\underbrace{\lambda^2}_{1} = H(z)\,H(z^{-1})\cdot \lambda^2 \quad 4$$

$$H(z) = \frac{1}{2}W(z) = \frac{1-\frac{1}{8}z^{-1}}{1-\frac{1}{4}z^{-1}} = \frac{C(z)}{A(z)}$$

$$\eta(t) \longrightarrow \boxed{H(z)} \longrightarrow y(\ )$$

$$H(z) = \frac{\overbrace{1-\frac{1}{8}z^{-1}}^{C(z)}}{\underbrace{1-\frac{1}{4}z^{-1}}_{A(z)}}$$

$$\eta \sim WN(0,4)$$

$$\hat{y}(t|t-1) = \frac{C(z)-A(z)}{C(z)}\,y(t)$$

$$\hat{y}(t+1|t) = \frac{z\left[C(z)-A(z)\right]}{C(z)}\,y(t)$$

$\boxed{\text{Stima bayesiana}}$ 2 v.a scorrelate x, w

$E[x]=0 \qquad E[w]=0$

$\sigma_x^2 = 1 \qquad \sigma_w^2 = 1$

stima valore attuale di x, utilizzando le osservazioni di

$$y = x + w$$

$\hat{x} \rightarrow$ stimatore lineare ottimo

$E[y] = ? \qquad \sigma_y^2 = ? \qquad \sigma_{xy} = ?$

$E(x)=0$

$E(w)=0$

$E[y] = E(x) + E(w) = 0$

x, w scorrelate $\rightarrow \sigma_y^2 = \sigma_x^2 + \sigma_w^2 = 2$

$$\hat{\theta} = \frac{\phi_{\theta d}}{\phi_{dd}} d$$

$\phi_{\theta d} \leftarrow \sigma_{\theta d}$

$d \leftarrow$ dato osservato $\rightarrow y$

$\phi_{dd} \rightarrow \sigma_d^2 \Rightarrow \sigma_y^2$

$\sigma \leftarrow x$

$d \leftarrow y$

$\sigma_{dd} \Rightarrow \sigma_y^2 = 2$

$\sigma_{\sigma d} \rightarrow \sigma_{xy} = E(xy)$

x, w scorr

$y = x + w \longrightarrow E\left[x(x+w)\right] = \underbrace{E\left[x^2\right]}_{\sigma_x^2 = E(x^2) - [E(x)]^2} + \cancel{E\left[xw\right]}_{0}$

$\sigma_{xy} = 1$

$\hat{x} = \frac{\overset{1}{\sigma_{xy}}}{\underset{2}{\sigma_{yy}}} y$ $\qquad \hat{x} = \frac{1}{2} y$

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2 osservazioni

x, w, z v.a.

$E(x) = 0 \quad E(w) = 0 \quad E(z) = 0$

$\sigma_x^2 = 1 \quad \sigma_w^2 = 1 \quad \sigma_z^2 = 1$

$\sigma_{xw} = 0 \quad \sigma_{xz} = 0$

$\sigma_{wz} = 0$

$\begin{cases} y_1 = x + w \\ y_2 = z \end{cases}$

$\hat{x} = ?$

$$\hat{\theta} = h(d) \qquad d = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \qquad \theta = x$$

$$\Lambda_{dd} = \begin{bmatrix} \sigma_{y_1}^2 & \sigma_{y_1 y_2} \\ \sigma_{y_2 y_1} & \sigma_{y_2}^2 \end{bmatrix} \qquad \Lambda_{d\theta} = E[d\theta] = \begin{bmatrix} E(y_1 x) \\ E(y_2 x) \end{bmatrix}$$

$$\Lambda_{\theta d} = \Lambda_{d\theta}^{T}$$

$$\sigma_{y_1}^2 = 2 \qquad \sigma_{y_2}^2 = \sigma_z^2 = 1 \qquad \sigma_{y_1 y_2} = E(y_1 y_2) = 0 = \sigma_{y_2 y_1}$$

z, w, x scorelate!

$$\Lambda_{dd} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \qquad \Lambda_{dd}^{-1} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Lambda_{d\theta} = \begin{bmatrix} E(y_1 x) \\ E(y_2 x) \end{bmatrix} \qquad E(y_1 x) = E[(x+w)x] = 1$$

$$E(y_2 x) = E[zx] = 0$$

$$\Lambda_{d\theta} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \Lambda_{\theta d} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$\hat{x} = \hat{\theta} = \Lambda_{\theta d} \cdot \Lambda_{dd}^{-1} \cdot d = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$= \frac{1}{2} y_1$$

È se fosse $y_2 = x + z$?

$E(z) = 0$ $\quad z, w, x$ v.a. indip

$\sigma_z^2 = d^2 \quad d \in \mathbb{R}^+$

$$\Lambda_{dd} = \begin{bmatrix} \sigma_{y_1}^2 & \sigma_{y_1 y_2} \\ \sigma_{y_2 y_1} & \sigma_{y_2}^2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & (1+d^2) \end{bmatrix}$$

$$\sigma_{y_2}^2 = \sigma_x^2 + \sigma_z^2 = 1 + d^2$$

$$\sigma_{y_1 y_2} = E\left[y_1 \cdot y_2\right] = E\left[(x+w)(x+z)\right] =$$

$$= E\left[x^2 + \underbrace{wx + zx + wz}_{0}\right] = 1$$

$$\Lambda_{yy}^{-1} = \begin{bmatrix} \dfrac{\alpha^2+1}{2\alpha^2+1} & -\dfrac{1}{2\alpha^2+1} \\ -\dfrac{1}{2\alpha^2+1} & \dfrac{2}{2\alpha^2+1} \end{bmatrix}$$

$$\Lambda_{yx} = \begin{bmatrix} E(y_1 x) \\ E(y_2 x) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \Lambda_{xy} = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$\hat{x} = \Lambda_{xy} \, \Lambda_{yy}^{-1} \cdot y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \Lambda_{yy}^{-1} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} =$$

$$= \frac{1}{2\alpha^2+1} \left[y_2 + \alpha^2 y_1 \right]$$

se $\alpha^2 \gg 1$ $\Longrightarrow$ $\hat{x} \xrightarrow[\alpha^2 \to +\infty]{} \frac{1}{2} y_1$

$\sigma_z^2 \quad \sigma_w^2$

se $\alpha^2 \ll 1$ $\Longrightarrow$ $\hat{x} \xrightarrow[\alpha^2 \to 0^+]{} y_2$

$\sigma_z^2 \quad \sigma_w^2$