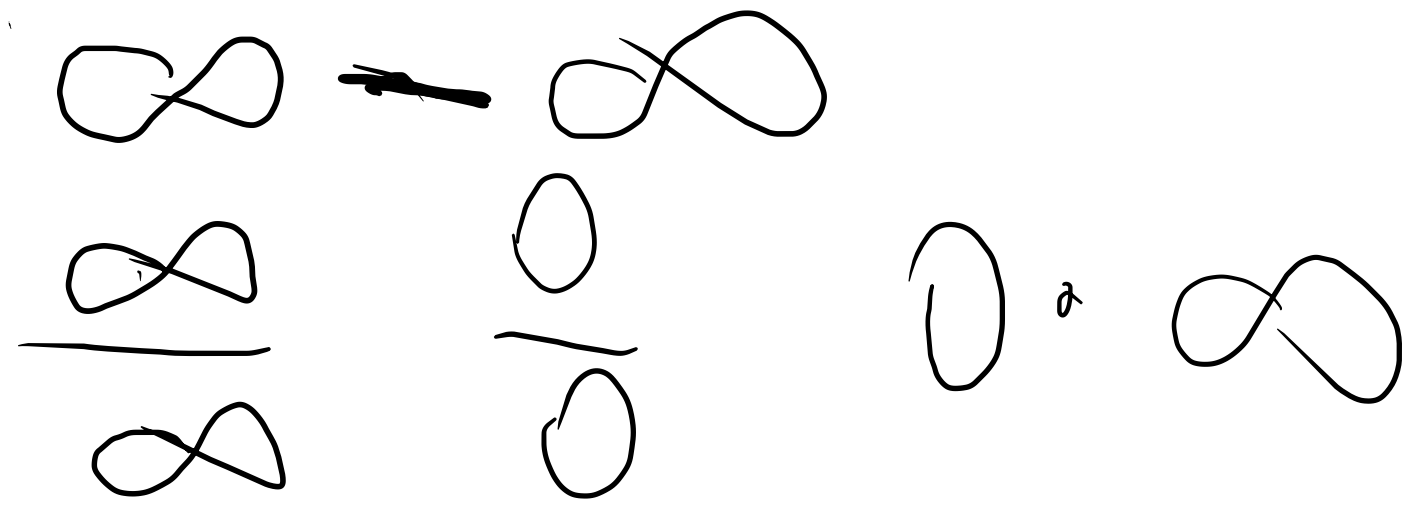


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$$\lim_{x \rightarrow x_0} f(x) =$$

FORME INDETERMINATE



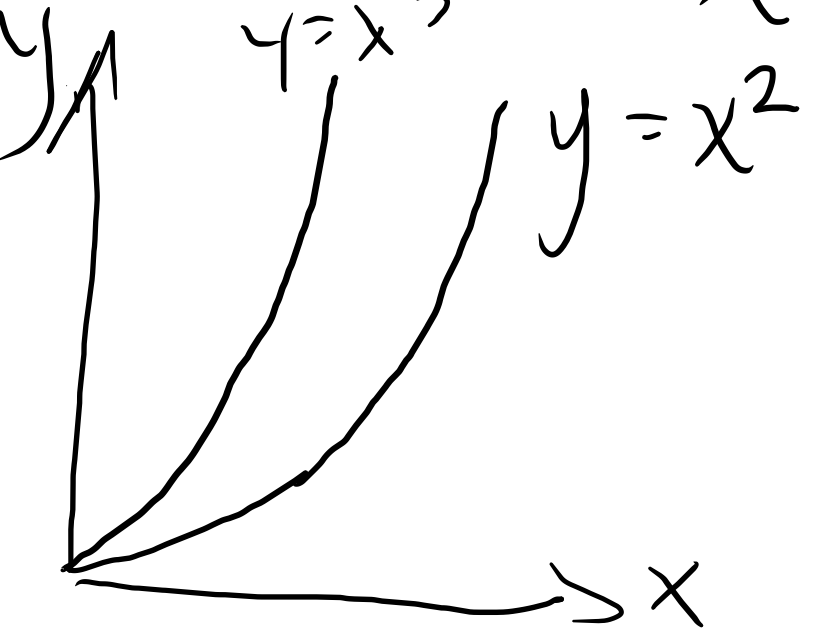
$$1) \frac{\infty}{\infty}$$

Caso dei polinomi

$$\lim_{x \rightarrow \pm\infty} \sqrt{\quad}$$

$$a) \lim_{x \rightarrow \pm\infty} \frac{x^3 - 3x}{x^2 + 1}$$

$$= \frac{x^3 \left(1 - \frac{3}{x^2}\right)}{x^2 \left(1 + \frac{1}{x^2}\right)}$$



$$= \lim_{x \rightarrow \pm\infty} \frac{x^3}{x^2} = \lim_{x \rightarrow \pm\infty} x = \pm\infty$$



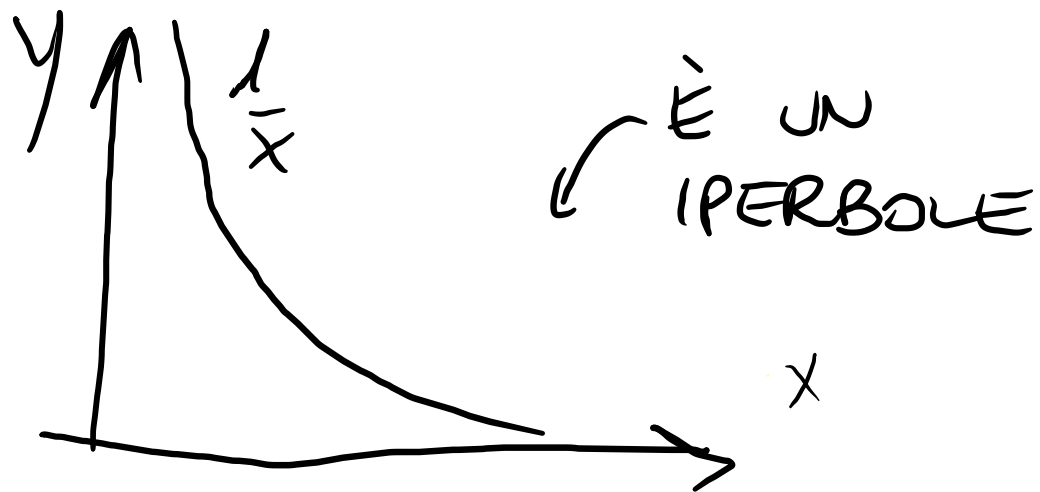
$$b) \lim_{x \rightarrow \pm\infty} \frac{x^3 - 3x}{-2x^3 + x^2 + 1} = \lim_{x \rightarrow \pm\infty} \frac{x^3 \left(1 - \frac{3}{x^2}\right)}{x^3 \left(-2 + \frac{1}{x} + \frac{1}{x^3}\right)}$$

Raccolgo sempre x di grado maggiore

$$\lim_{x \rightarrow \pm\infty} \frac{x^3}{-2x^3} = -\frac{1}{2}$$

$$c) \lim_{x \rightarrow \pm\infty} \frac{x^2 + 5}{x^3 - 2x^2 + 5x} = \frac{x^2 \left(1 + \frac{5}{x^2} \right)}{x^3 \left(1 - \frac{2}{x} + \frac{5}{x^2} \right)}$$

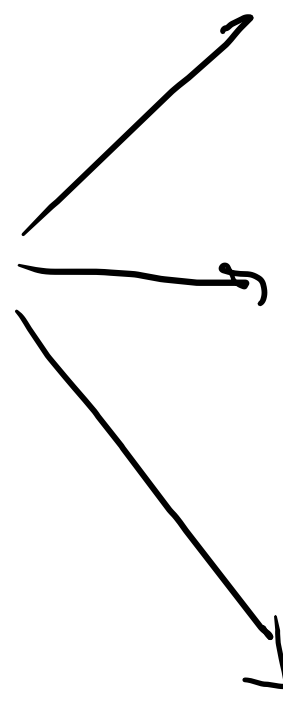
$$\lim_{x \rightarrow \pm\infty} \frac{x^2}{x^3} = \lim_{x \rightarrow \pm\infty} \frac{1}{x} = 0$$



Riepilogo : $f(x)$ $g(x)$ polinomi

$$\lim_{x \rightarrow \pm\infty} \frac{f(x)}{g(x)}$$

=



$$\deg f > \deg g \Rightarrow \infty$$

$$\deg f = \deg g \Rightarrow l$$

$$\deg f < \deg g \Rightarrow 0$$

$$2) \frac{0}{0}$$

$$a^2 - b^2 = (a + b)(a - b)$$

$$\lim_{x \rightarrow 3} \frac{x-3}{\sqrt{x}-\sqrt{3}} = \frac{(\sqrt{x}+\sqrt{3})(\sqrt{x}-\sqrt{3})}{(\sqrt{x}-\sqrt{3})} =$$

$$= \lim_{x \rightarrow 3} (\sqrt{x} + \sqrt{3}) = 2\sqrt{3}$$

$$\lim_{x \rightarrow -1} \frac{x^3 - 3x - 2}{x^4 + 2x^3 - 8x^2 - 18x - 9} =$$

$$\lim_{x \rightarrow -1} \frac{(x+1) \cdot (\dots)}{(x+1) \cdot (\dots)} =$$

$$\lim_{x \rightarrow -1} \frac{\cancel{(x+1)} \cdot (x^2 - x - 2)}{\cancel{(x+1)} \cdot (x^3 + x^2 - 9x - 9)} = \frac{x^3 - 3x - 2}{x^4 + 2x^3 - 8x^2 - 18x - 9}$$

$$= \lim_{x \rightarrow -1} \frac{x^2 - x - 2}{x^3 + x^2 - 9x - 9} = \frac{0}{-1 + 1 + 9 - 9} = \frac{0}{0}$$

$$= \lim_{x \rightarrow -1} \frac{\cancel{(x+1)} (x-2)}{\cancel{(x+1)} (x^2 - 9)} = \lim_{x \rightarrow -1} \frac{x-2}{x^2-9} = \frac{-1-2}{1-9} = \frac{-3}{-8} = \frac{3}{8}$$

$$x^2 \boxed{-x} \boxed{-2} = (x+1)(x-2)$$

$$-2 \cdot 1 = -2 \quad \text{prodotto}$$

$$-2 + 1 = -1 \quad \text{Somma}$$

3) $\infty - \infty$ con radici

$$\lim_{x \rightarrow +\infty} 2x - \sqrt{4x^2 + x} = +\infty - \infty$$

$$= \lim_{x \rightarrow +\infty} \frac{(2x - \sqrt{4x^2 + x})(2x + \sqrt{4x^2 + x})}{(2x + \sqrt{4x^2 + x})}$$

Applico

$$a^2 - b^2 = (a+b)(a-b)$$

$$a = 2x$$
$$b = \sqrt{4x^2 + x}$$

$$\lim_{x \rightarrow +\infty} \frac{\cancel{4x^2} - \cancel{4x^2} - x \rightarrow -(4x^2+x)}{2x + \sqrt{4x^2+x}} = \lim_{x \rightarrow +\infty} \frac{-x}{2x + \sqrt{4x^2+x}}$$

$$= \lim_{x \rightarrow +\infty} \frac{\cancel{-x}}{\cancel{x} \left(2 + \sqrt{4 + \frac{1}{x}} \right)} = \frac{\sqrt{x^2} \left(4 + \frac{1}{x} \right)}{x \sqrt{4 + \frac{1}{x}}}$$

$$= \frac{-1}{2+2} = -\frac{1}{4}$$

$$\lim_{x \rightarrow +\infty} \frac{x^2 - 7x + 1}{4x^3 - 6x^2 + 2} \leftarrow \lim_{x \rightarrow +\infty}$$

$$\frac{x^2 (1 - 7/x + 1/x^2)}{x^3 (4 - 6/x + 2/x^3)}$$

$\begin{matrix} \nearrow 0 & \nearrow 0 \\ \searrow 0 & \searrow 0 \end{matrix}$

$$= \lim_{x \rightarrow +\infty} \frac{x^2}{4x^3} = \lim_{x \rightarrow +\infty} \frac{1}{4x} = 0$$

$$\lim_{x \rightarrow 1} \frac{3x^2 - 8x + 5}{x^2 - 1} = \frac{0}{0}$$

$$= 1x^2 - 8x + 5$$

$$= 3 \left(x^2 - 8x + 5 \right)$$

$$\lim_{x \rightarrow 1} \frac{(x-1)}{(x+1)(x-1)}$$

$$\lim_{x \rightarrow 1} \frac{\cancel{(x-1)} \cdot 3 \left(x - \frac{5}{3} \right)}{\cancel{(x-1)} (x+1)}$$

$$\lim_{x \rightarrow 1} \frac{3 \left(1 - \frac{5}{3} \right)}{(1+1)} = \lim_{x \rightarrow 1} \frac{3 \left(-\frac{2}{3} \right)}{2} = -\frac{2}{2} = -1$$

Rechnerische
Produkte

$$h) \lim_{x \rightarrow +\infty} \left(\sqrt[3]{2+x^3} - \sqrt[3]{1+2x^2+x^3} \right)$$

$$a \pm b = (a \pm b) (a^2 \mp ab + b^2)$$

$$a = \sqrt[3]{2+x^3}$$

$$b = \sqrt[3]{1+2x^2+x^3}$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$\lim_{x \rightarrow +\infty} \left(\sqrt[3]{2+x^3} - \sqrt[3]{1+2x^2+x^3} \right) \cdot \begin{matrix} ab \\ b^2 \end{matrix}$$

$$\frac{\left((2+x^3)^{\frac{2}{3}} + \sqrt[3]{(2+x^3)(1+2x^2+x^3)} + (1+2x^2+x^3)^{\frac{2}{3}} \right)}{\dots}$$

$$\lim_{x \rightarrow +\infty} \frac{2 + \cancel{x^3} - (1 + 2x^2 + \cancel{x^3})}{(2+x^3)^{\frac{2}{3}} + \sqrt[3]{(2+x^3)(1+2x^2+x^3)} + (1+2x^2+x^3)^{\frac{2}{3}}}$$

$$\lim_{x \rightarrow \infty} \frac{-2x^2 + 1}{(2 + x^3)^{\frac{2}{3}} + \sqrt[3]{(2 + x^3)(1 + 2x^2 + x^3)} + (1 + 2x^2 + x^3)^{\frac{2}{3}}}$$

$$\approx (x^3)^{\frac{2}{3}} = x^2 \quad \approx \sqrt[3]{x^3 \cdot x^3} = \sqrt[3]{x^6} = x^2$$

Mi aspetto un limite finito $\neq 0$

$$\begin{aligned}
 & \lim_{x \rightarrow \infty} \frac{x^2(-2 + \sqrt{x^2})}{x^2 \left(\frac{2}{x^3} + 1 \right)^{\frac{2}{3}} + x^2 \left(1 + \frac{2}{x} + \frac{1}{x^3} \right)^{\frac{2}{3}} + \textcircled{X}}{x^2 \sqrt[3]{ \left(\frac{2}{x^3} + 1 \right) \left(1 + \frac{2}{x} + \frac{1}{x^3} \right) }} \\
 & \stackrel{\textcircled{X}}{=} \lim_{x \rightarrow \infty} \frac{\cancel{x^2}}{x^2 (1 + 1 + 1)} = \frac{1}{3}
 \end{aligned}$$

Handwritten notes and corrections:

- Red annotations: $\frac{2}{x^3} \rightarrow 0$, $1 \rightarrow 0$, $\frac{2}{x} \rightarrow 0$, $\frac{1}{x^3} \rightarrow 0$.
- Blue annotations: 1 (circled), 1 (circled), 1 (circled).
- Red equation: $x^2 \left(\frac{2}{x^3} + 1 \right)^{\frac{2}{3}} = (2 + x^3)^{\frac{2}{3}}$
- Red annotations on the denominator: $\frac{2}{x^3} \rightarrow 0$, $1 \rightarrow 0$, $\frac{2}{x} \rightarrow 0$, $\frac{1}{x^3} \rightarrow 0$.
- Red annotations on the denominator's inner terms: $\frac{2}{x^3} \rightarrow 0$, $1 \rightarrow 0$, $\frac{2}{x} \rightarrow 0$, $\frac{1}{x^3} \rightarrow 0$.

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = \underline{1} \quad \text{LIMIT}$$

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = \underline{1} \quad \text{NOTEVAL}$$

$$\lim_{x \rightarrow 0} \frac{\ln(x+1)}{x} = \underline{1}$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \underline{1}$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{\tan x} =$$

$$= \lim_{x \rightarrow 0} \frac{\sin x}{\tan x} \cdot \frac{x}{x}$$

The expression $\frac{\sin x}{\tan x} \cdot \frac{x}{x}$ is shown with annotations. A green circle highlights $\frac{\sin x}{\tan x}$ and a red circle highlights $\frac{x}{x}$. A green arrow points from the green circle to a green '1', and a red arrow points from the red circle to a red '1'.

$$= 1 \cdot 1 = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$
$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

$$2) \lim_{x \rightarrow 0} \frac{\sin(\sin x)}{x} =$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

~~$$= \lim_{x \rightarrow 0} \frac{\sin(\sin x)}{x} \cdot \frac{x}{x}$$~~

~~$$= \lim_{x \rightarrow 0} \frac{\sin(\sin x)}{x} \cdot \frac{\sin x}{\sin x} = 1 \cdot 1 = 1 \quad x = \sin x$$~~

NO some divers.