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$$\lim_{x \to x_0} f(x) =$$

FORME INDETERMINATE

$\infty - \infty$

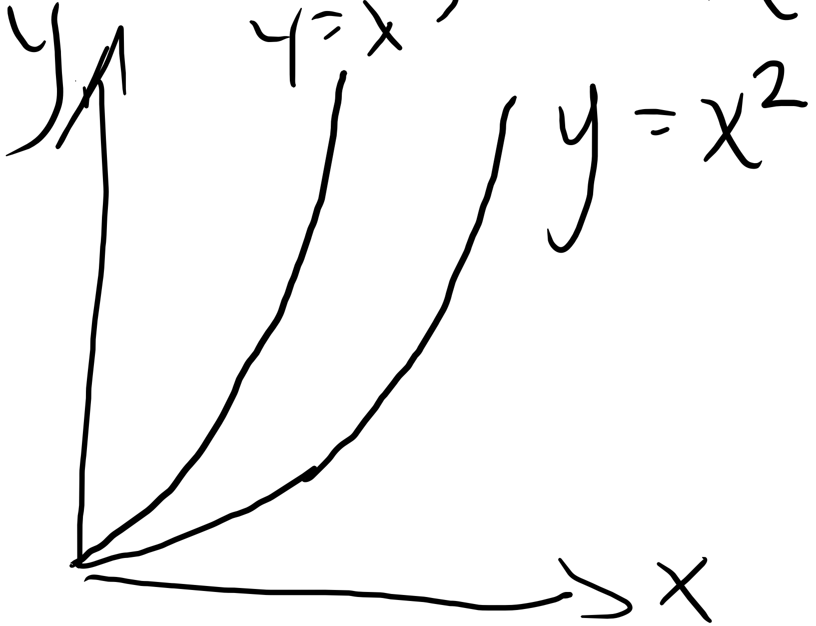
$\dfrac{\infty}{\infty}$ $\quad \dfrac{0}{0}$ $\quad 0 \cdot \infty$

1) $\frac{\infty}{\infty}$ caso dei polinomi

a) $$\lim_{x\to\pm\infty} \frac{x^3-3x}{x^2+1} \underset{x\to\pm\infty}{\overset{\lim}{=}} \frac{x^3\left(1-\frac{3}{x^2}\right)}{x^2\left(1+\frac{1}{x^2}\right)}$$

$\frac{3}{x^2} \to 0$ $\frac{1}{x^2} \to 0$

$$= \lim_{x\to\pm\infty} \frac{x^{\cancel{3}}}{\cancel{x^2}} = \lim_{x\to\pm\infty} x = \pm\infty$$



b) $\lim_{x \to \pm\infty} \frac{x^3 - 3x}{-2x^3 + x^2 + 1} = \lim_{x \to \pm\infty} \frac{x^3\left(1 - \frac{3}{x^2}\right)}{x^3\left(-2 + \frac{1}{x} + \frac{1}{x^3}\right)}$

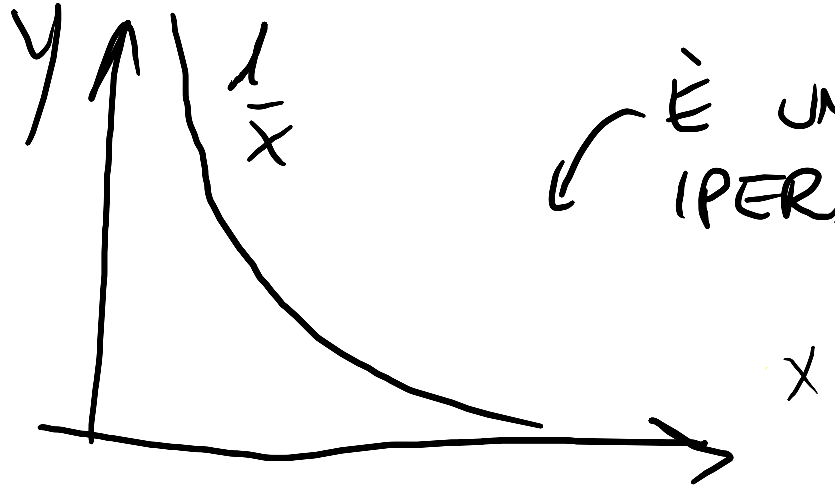
$\frac{3}{x^2} \to 0 \qquad \frac{1}{x} \to 0 \qquad \frac{1}{x^3} \to 0$

Raccolgo sempre x di grado maggiore

$\lim_{x \to \pm\infty} \frac{\cancel{x^3}}{-2\cancel{x^3}} = -\frac{1}{2}$

c) $\displaystyle \lim_{x\to\pm\infty} \frac{x^2+5}{x^3-2x^2+5x} = \frac{x^2\left(1+\underbrace{\frac{5}{x^2}}_{\to 0}\right)}{x^3\left(1-\underbrace{\frac{2}{x}}_{\to 0}+\underbrace{\frac{5}{x^2}}_{\to 0}\right)}$

$$\lim_{x\to\pm\infty} \frac{\cancel{x^2}}{\cancel{x^3}} = \lim_{x\to\pm\infty} \frac{1}{x} = 0$$



È UN IPERBOLE

Riepilogo: $f(x)$ $g(x)$ polinomi

$$\lim_{x\to\pm\infty} \frac{f(x)}{g(x)} = \begin{cases} \deg f > \deg g \Rightarrow \infty \\ \deg f = \deg g \Rightarrow l \\ \deg f < \deg g \Rightarrow 0 \end{cases}$$

$$a^2-b^2=(a+b)(a-b)$$

2) $\frac{0}{0}$

$$\lim_{x\to 3}\frac{x-3}{\sqrt{x}-\sqrt{3}}=\frac{(\sqrt{x}+\sqrt{3})\cancel{(\sqrt{x}-\sqrt{3})}}{\cancel{(\sqrt{x}-\sqrt{3})}}=$$

$$=\lim_{x\to 3}(\sqrt{x}+\sqrt{3})=2\sqrt{3}$$

$$\lim_{x \to -1} \frac{x^3 - 3x - 2}{x^4 + 2x^3 - 8x^2 - 18x - 9} =$$

$$\lim_{x \to -1} \frac{(x+1) \cdot (\cdot \ \cdot \ \cdot \ \cdot \ )}{(x+1) \cdot (\cdot \ \cdot \ \cdot \ \cdot \ )} =$$

$$\lim_{x \to -1} \frac{\cancel{(x+1)} \cdot (x^2 - x - 2)}{\cancel{(x+1)} \cdot (x^3 + x^2 - 9x - 9)} = \frac{x^3 - 3x - 2}{x^4 + 2x^3 - 8x^2 - 18x - 9}$$

$$= \lim_{x \to -1} \frac{x^2 - x - 2}{x^3 + x^2 - 9x - 9} = \frac{0}{-1+1+9-9} = \frac{0}{0}$$

$$= \lim_{x \to -1} \frac{\cancel{(x+1)}(x-2)}{\cancel{(x+1)}(x^2-9)} = \lim_{x \to -1} \frac{x-2}{x^2-9} = \frac{-1-2}{1-9} = \frac{-3}{-8}$$

$$= \frac{3}{8}$$

$$x^2 \boxed{-x} \boxed{-2} = (x+1)(x-2)$$

$$-2 \cdot 1 = -2 \quad \text{prodotto}$$

$$-2+1 = -1 \quad \text{Somma}$$

3) $\infty - \infty$ con radici

$$\lim_{x \to +\infty} 2x - \sqrt{4x^2+x} = +\infty - \infty$$

$$= \lim_{x \to +\infty} \left(2x - \sqrt{4x^2+x}\right) \frac{\left(2x + \sqrt{4x^2+x}\right)}{\left(2x + \sqrt{4x^2+x}\right)}$$

Applico $a^2 - b^2 = (a+b)(a-b)$

$a = 2x$

$b = \sqrt{4x^2+x}$

$$\lim_{x\to+\infty}\frac{\cancel{4x^2}-\cancel{4x^2}-x}{2x+\sqrt{4x^2+x}} = \lim_{x\to+\infty}\frac{-x}{2x+\sqrt{4x^2+x}}$$

$\to -(4x^2+x)$

$$= \lim_{x\to+\infty}\frac{-\cancel{x}}{\cancel{x}\left(2+\sqrt{4+\frac{1}{x}}\right)}$$

$\frac{1}{x} \to 0$

$$= \sqrt{x^2\left(4+\frac{1}{x}\right)} = x\sqrt{\left(4+\frac{1}{x}\right)}$$

$$= \frac{-1}{2+2} = -\frac{1}{4}$$

$$\lim_{x\to+\infty} \frac{x^2-7x+1}{4x^3-6x^2+2} = \lim_{x\to+\infty} \frac{x^2(1-\overset{\to 0}{7/x}+\overset{\to 0}{1/x^2})}{x^3(4-\underset{\to 0}{6/x}+\underset{\to 0}{2/x^3})}$$

$$= \lim_{x\to+\infty} \frac{\cancel{x^2}}{4x^3} = \lim_{x\to+\infty} \frac{1}{4x} = 0$$

$$\lim_{x \to 1} \frac{3x^2 - 8x + 5}{x^2 - 1} = \frac{0}{0}$$

$$\underline{\underline{1}}x^2 - Sx + P$$

$$\underbrace{3\left(x^2 - \frac{8}{3}x + \frac{5}{3}\right)}_{\text{Regola somma prodotto}}$$

$$\lim_{x \to 1} \frac{(x -}{(x+1)(x-1)}$$

$$\lim_{x \to 1} \frac{\cancel{(x-1)} \cdot 3\left(x - \frac{5}{3}\right)}{\cancel{(x-1)}(x+1)}$$

$$\lim_{x \to 1} \frac{3\left(1 - \frac{5}{3}\right)}{(1+1)} = \lim_{x \to 1} \frac{3\left(-\frac{2}{3}\right)}{2} = -\frac{2}{2} = -1$$

h) $\lim_{x \to +\infty} \left(\sqrt[3]{2+x^3} - \sqrt[3]{1+2x^2+x^3} \right)$

$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

$a = \sqrt[3]{2+x^3}$

$b = \sqrt[3]{1+2x^2+x^3}$

$a^{\frac{1}{n}} = \sqrt[n]{a}$

$$\lim_{x \to +\infty} \left(\sqrt[3]{2+x^3} - \sqrt[3]{1+2x^2+x^3}\right) \cdot \frac{\overbrace{(2+x^3)^{\frac{2}{3}}}^{a^2} + \overbrace{\sqrt[3]{(2+x^3)(1+2x^2+x^3)}}^{ab} + \overbrace{(1+2x^2+x^3)^{\frac{2}{3}}}^{b^2}}{}$$

$$\|$$

$$\lim_{x \to +\infty} \frac{2+\cancel{x^3} - (1+2x^2+\cancel{x^3})}{(2+x^3)^{\frac{2}{3}} + \sqrt[3]{(2+x^3)(1+2x^2+x^3)} + (1+2x^2+x^3)^{\frac{2}{3}}}$$

$$\lim_{x\to\infty} \frac{-2x^2+1}{(2+x^3)^{\frac{2}{3}} + \sqrt[3]{(2+x^3)(1+2x^2+x^3)} + (1+2x^2+x^3)^{\frac{2}{3}}}$$

$$\simeq (x^3)^{\frac{2}{3}} = x^2 \qquad \simeq \sqrt[3]{x^3 \cdot x^3} = \sqrt[3]{x^6} = x^2$$

Mi aspetto un limite finito $\neq 0$

$$\lim_{x\to\infty} \frac{x^2\left(-2+\frac{1}{x^2}\right)}{x^2\left(\frac{2}{x^3}+1\right)^{\frac{2}{3}} + x^2\left(1+\frac{2}{x}+\frac{1}{x^3}\right)^{\frac{2}{3}} + \otimes}$$

$$x^2\left(\frac{2}{x^3}+1\right)^{\frac{2}{3}} = \left(2+x^3\right)^{\frac{2}{3}}$$

$$\otimes \quad x^2\left(\sqrt[3]{\left(\frac{2}{x^3}+1\right)\left(1+\frac{2}{x}+\frac{1}{x^3}\right)}\right)$$

$$= \lim_{x\to\infty} \frac{-2\cancel{x^2}}{\cancel{x^2}(1+1+1)} = -\frac{2}{3}$$

LIMITI NOTEVOLI

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0} \frac{\tan x}{x} = 1$$

$$\lim_{x \to 0} \frac{\ln(x+1)}{x} = 1$$

$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{x\to 0} \frac{\sin x}{\tan x} =$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{\tan x}{x} = 1$$

$$= \lim_{x\to 0} \frac{\sin x}{\tan x} \cdot \frac{x}{x} = 1 \cdot 1 = 1$$

$\to 1$

$\to 1$

i) $\lim_{x\to 0} \frac{\sin(\sin x)}{x} =$

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$= \lim_{x\to 0} \frac{\sin(\sin x)}{x} \cdot \frac{x}{x}$

NO son
divers.

$= \lim_{x\to 0} \frac{\sin(\sin x)}{x} \cdot \frac{\sin x}{\sin x} = 1$

$\to 1$

$\to 1$

$x = \sin x$