$$\frac{N_{i+1} - N_i}{N_i} = \alpha h$$

periodo di tempo

Tasso di (de)crescita del numero N di individui

N_0

$$\frac{N_1 - N_0}{N_0} = \alpha h \Rightarrow N_1 = N_0(1 + \alpha h)$$

$$N_1 - N_0 = \alpha h N_0$$

$$\frac{N_2 - N_1}{N_1} = \alpha h$$

$N_0(1+\alpha h)$

$$N_2 = N_1 + N_1 \alpha h = N_1(1+\alpha h) = \underline{N_0}(1+\alpha h)^2$$

$$\boxed{N_i = (1+\alpha h)^i N_0}$$

$$\boxed{e = \lim_{k \to +\infty} \left(1 + \frac{1}{k}\right)^k}$$

$$(1+\alpha h)^i = (1+\alpha h)^{\frac{t}{h}} = (1+\alpha h)^{\frac{t}{h}} =$$

$$\frac{t}{h} = i \qquad = \left[(1+\alpha h)^{\frac{1}{h}}\right]^t = \left[(1+\alpha h)^{\frac{1}{\alpha h}}\right]^{\alpha t}$$

$$h \to 0 \qquad \frac{1}{h} = k \qquad = \left[\left(1+\frac{\alpha}{k}\right)^{\frac{k}{\alpha}}\right]^{\alpha t} \to e^{\alpha t}$$

$$\to \begin{cases} \dfrac{d}{dt} N(t) = (\overset{\overset{\varepsilon}{\shortparallel}}{\beta - \mu}) N(t) \\ \underline{N(t_0) = \underline{N_0}} \end{cases}$$

$$\boxed{N(t) = N_0 \cdot e^{(\beta-\mu)(t-t_0)}} = N_0 \cdot \frac{e^{\beta(t-t_0)}}{e^{\mu(t-t_0)}}$$

$$\checkmark \quad N'(t) = N_0(\beta-\mu)\, e^{(\beta-\mu)(t-t_0)} = (\beta-\mu) N(t)$$

$$=$$

$$\boxed{\tau = \frac{1}{\mu}}$$

$$\begin{cases} N'(t) = (\overset{\varepsilon}{\beta - \mu}) N(t) = \mu \left(\frac{\beta}{\mu} - 1\right) N(t) = \mu \cdot (R_0 - 1) N(t) \\ N(t_0) = N_0 \end{cases}$$

$$\tau = \frac{1}{\mu} \qquad \beta \tau = \frac{\beta}{\mu} =: R_0$$

$$\begin{cases} N'(t) = \varepsilon \cdot N(t) + m(t) \\ N(t_0) = N_0 \end{cases}$$

∘ $x'(t) = a\,x(t)$

(eq diff. primo ordine in forma normale autonoma a variabili separabili)

$$\frac{d}{dt}x(t) = a\,x(t)$$

$$\int \frac{1}{x}\,dx = \int a\,dt \Rightarrow \quad \ln x(t) = at + C$$

$$\boxed{x(t) = e^{at+C} = x_0 \cdot e^{at}}$$

$$x'(t) = a(t)\,x(t) + m(t)$$

eq diff. primo ordine in forma normale lineare

$N \longrightarrow X$

$a \longrightarrow \varepsilon$

$m = m$

$$\boxed{x'(t) = B(t)\,x(t) + C(t)}$$

$$x(t) = \int$$

$$\begin{cases} \dfrac{dN(t)}{dt} = N'(t) = \underline{a N(t) - b N^2(t)} \\ \quad = \varepsilon N(t) - \dfrac{\varepsilon}{K} N^2(t) = \varepsilon\left(1 - \dfrac{N(t)}{K}\right) N(t) \\ N(t_0) = N_0 \end{cases}$$

$$\begin{cases} x'(t) = a x(t) - b x^2(t) \\ x(t_0) = N_0 \end{cases}$$

eq diff. primo ordine
autonome una variabile
separabile

$$\int \frac{1}{x(a-bx)} dx = \int \frac{dx}{ax - bx^2} = \int dt$$

$$\int \frac{1}{x(a-bx)}\,dx = \int dt$$

$$= t + C$$

$$\frac{A}{x} + \frac{B}{a-bx} = \frac{1}{x(a-bx)}$$

$\forall x$

$a \cdot A - Abx + Bx = 1$

$\Rightarrow a \cdot A = 1$

$(B - Ab) = 0$

$A = 1/a \quad B = +b/a$

$$\int \frac{1}{x(a-bx)}\,dx = \frac{1}{a}\ln x + \frac{b}{a}\ln(a-bx) = \frac{1}{a}\left[\ln x + b\ln(a-bx)\right]$$