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*Corso di Laurea Magistrale in GEOSCIENZE*

*Metodi Elettromagnetici in Geofisica (6 CFU)  
- MEMAG -*

**UD-4d: Ground Penetrating Radar  
GPR - inversion**

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# Ground Penetrating Radar: Inversions and analyses

## Outline

1. Multi azimuth acquisitions (co-polarized and cross-polarized). Theory and examples of application for linear targets detection
2. EM amplitude inversion techniques. Theory and examples of application/validation for glaciological targets.
3. GPR borehole data acquisition and inversions (traveltimes and amplitudes)



# Ground Penetrating Radar: multi azimuth acquisition

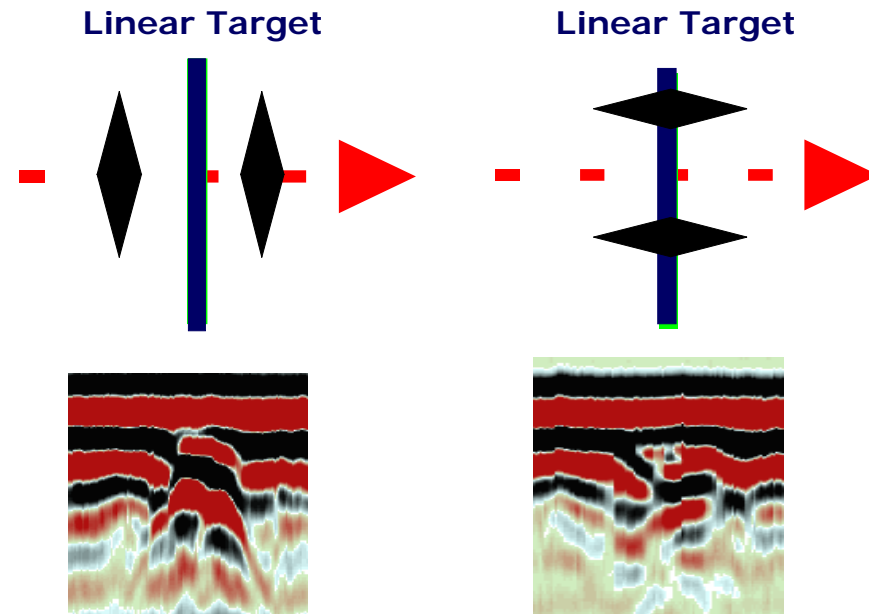


By using 2 separated antennas it is possible to change: Offset and/or Azimuth → AVO  
AVA analyses and polarimetric measurements.

# Ground Penetrating Radar: multi azimuth acquisition

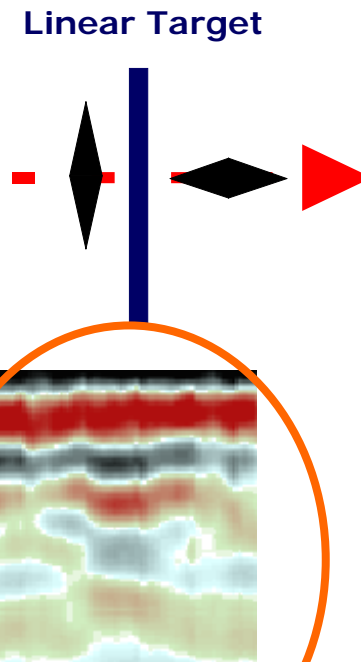
CO-POLARIZATION

TE (Broadside) TM (Broadside)



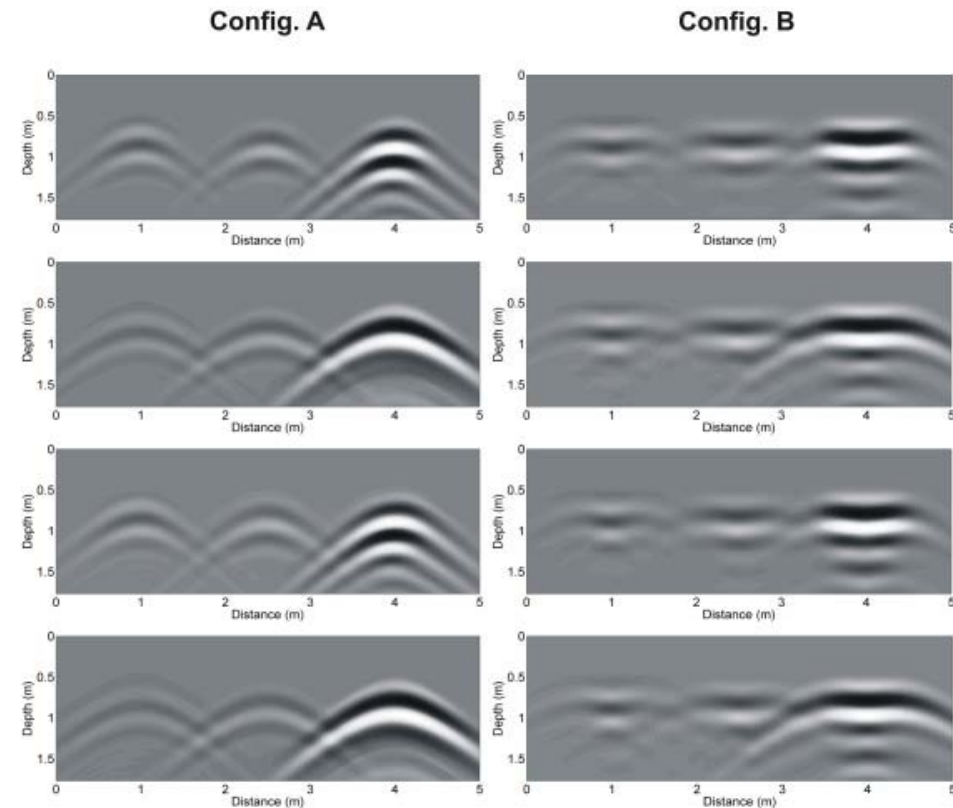
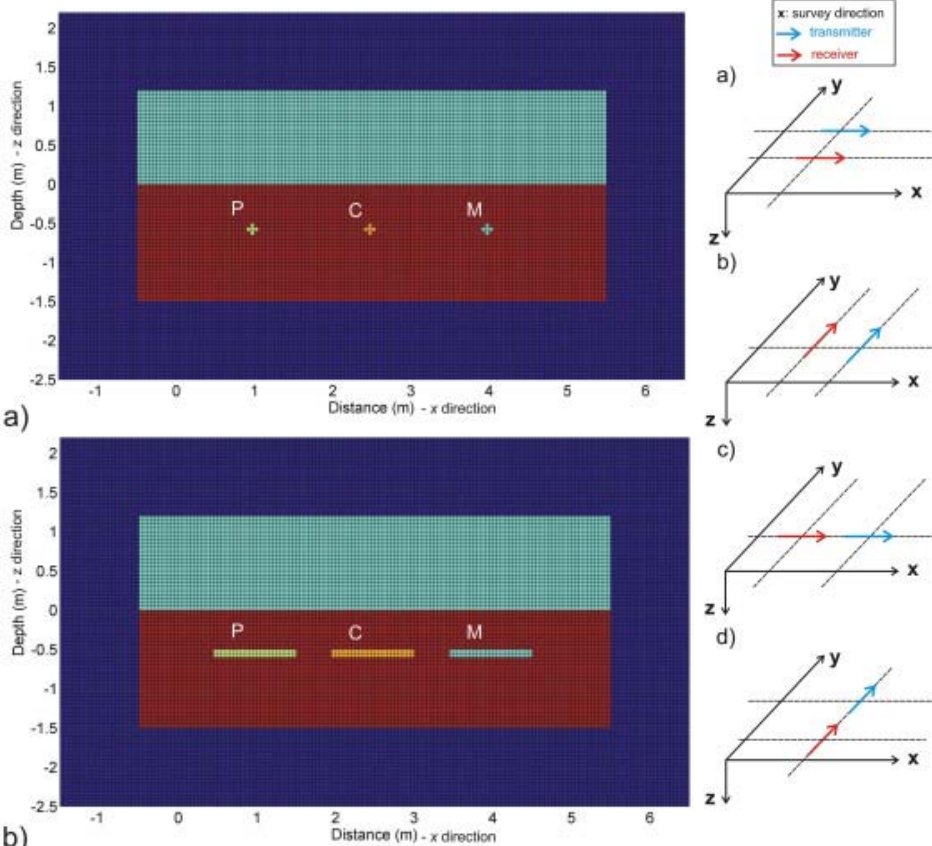
E	H	CO-POLARIZED		CROSS-POLARIZED
		END FIRE	BROADSIDE	
a)	H			
TM	b)		c)	d)
TE	e)	f)	g)	

CROSS-POLARIZATION



**AMPLITUDE  $\ll$  and not negligible only for linear targets due to their DEPOLARIZING EFFECT**

# Ground Penetrating Radar: co- and cross- polarization



Santos and Teixeira, 2017

A: plastic (P), concrete (C) and metallic (M) pipes positioned perpendicular to the GPR survey direction.  
B: plastic (P), concrete (C) and metallic (M) positioned parallel to the GPR survey direction.

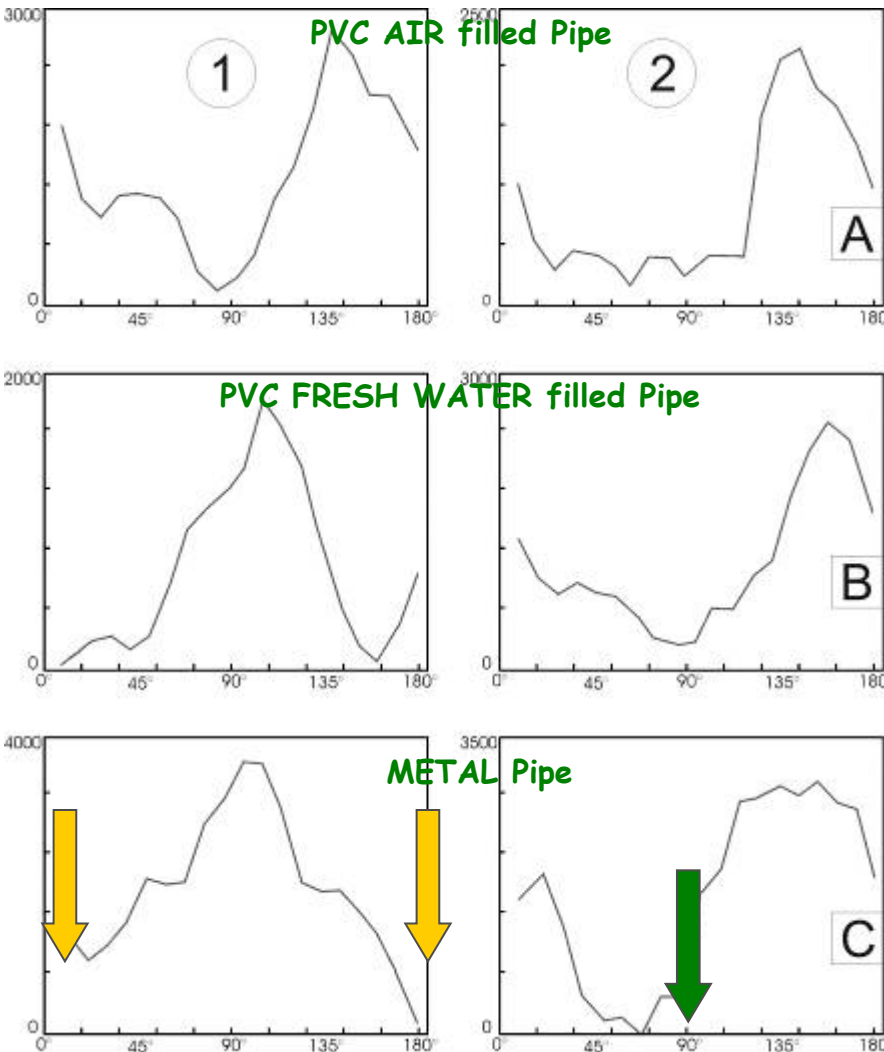
100 MHz GPR results obtained for Configurations A and B, using different **co-polarized** antenna orientations (a-d).

It is possible to exploit de-polarization effects to quickly determine the **DIRECTION** of linear targets (pipes, walls,...)

Also the characteristics of materials can be estimated

# Ground Penetrating Radar: Multi-components

## CROSS-POLE CO-POLE



Real DATA  
Pipe direction=0°

Pipan et al., 2003

### Theoretical values

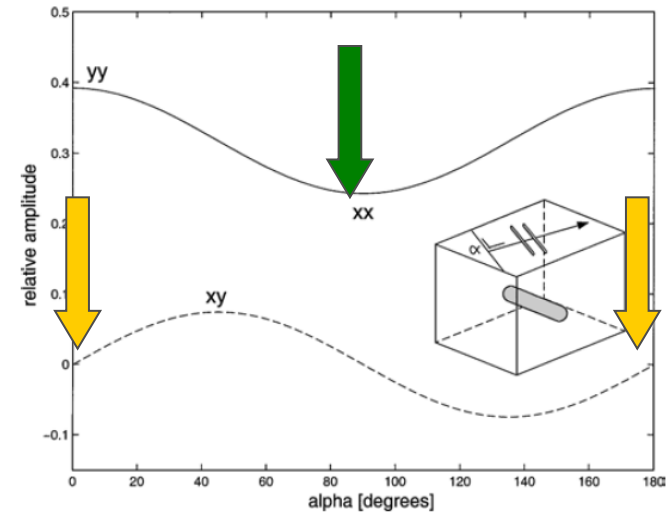
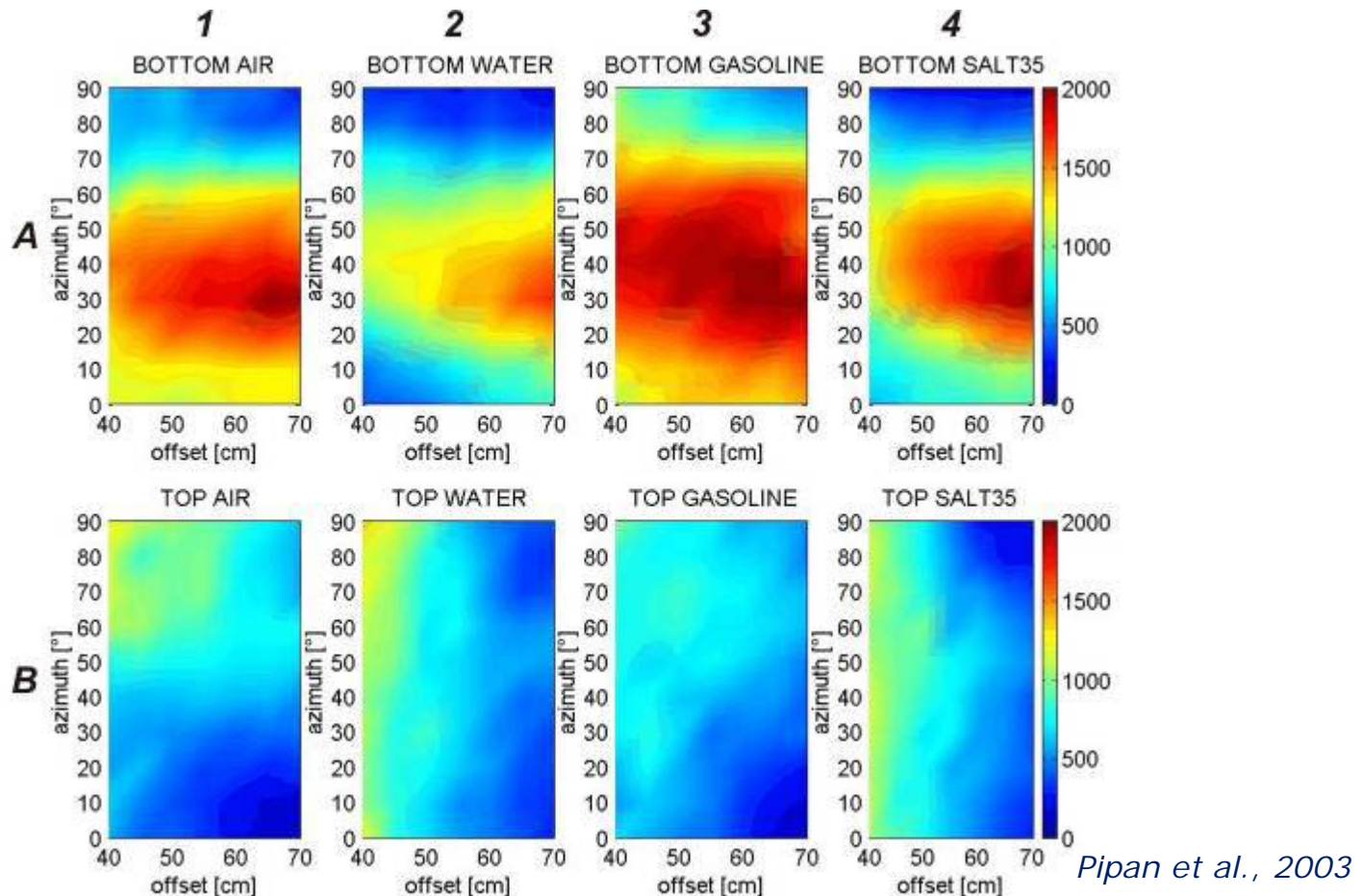


FIG. 3. Theoretical reflection response of a cylinder versus angle  $\alpha$  between the orientation of the antennas and cylinder. Inset shows simplified representation of the setup. The copolarized and crosspolarized configurations are shown with a solid and a dashed line, respectively. For these calculations the perfectly conducting cylinder has a length of 1 m and a diameter of 20 cm and is embedded in sand with  $\epsilon_r = 5$ ,  $\sigma = 0$  mS/m, and  $\mu_r = 1$ .

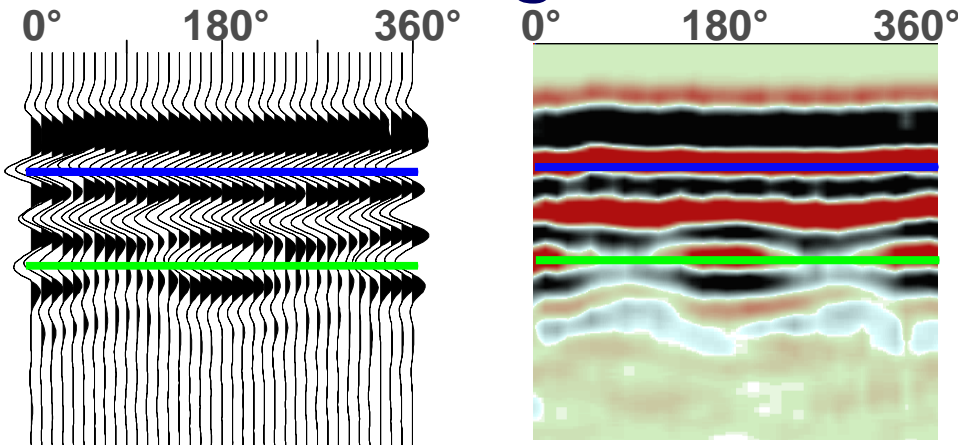
Da Van Gestel e Stoffa., 2002

# Ground Penetrating Radar: Multi-components AVO and AVA

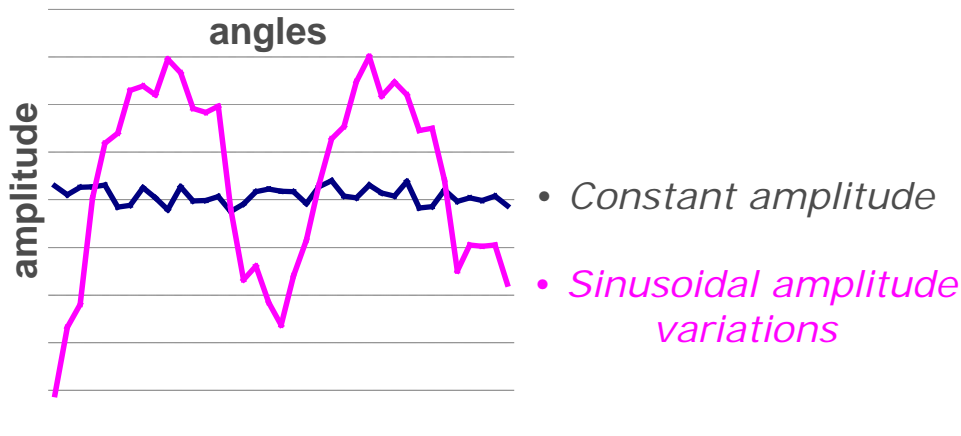


GPR AVO (Amplitude versus Offset) and AVA (Amplitude versus Azimuth) analysis performed on a PVC pipe filled with different fluids. Columns 1 to 4: (1) air, (2) fresh water, (3) gasoline and (4) salt water (salinity about 35 ‰). **Rows A and B: (A) amplitude of reflection from metal base and (B) amplitude of reflection from top of the pipe.**

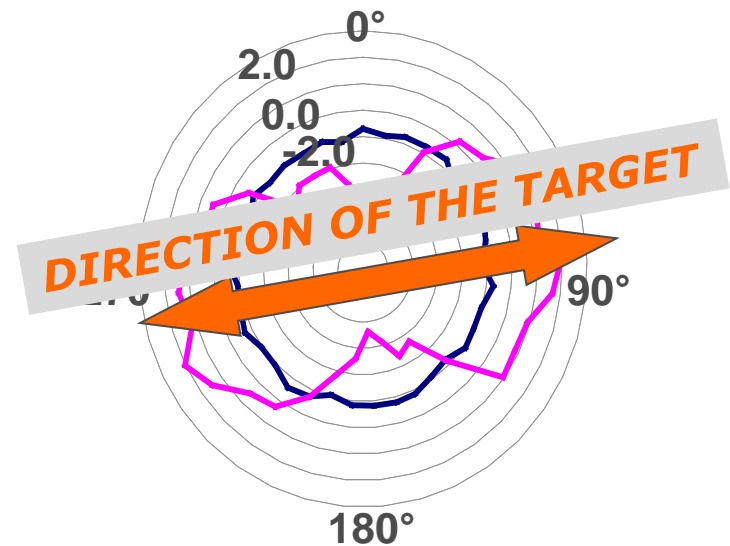
# Ground Penetrating Radar: Multi-components AVO and AVA



When there are AVA variations → there are **linear TARGETS**



*Polar diagram of amplitude variations*



With this approach it is possible to derive the linear target orientation even in zones with logistical constraints (obstacles, limited operative dimensions, surface variations, ...)



# Ground Penetrating Radar: Multi-components AVO and AVA

## Pros and Cons of GPR MULTI-AZIMUTH surveys

1. By applying antenna arrays it is possible to collect multi azimuth data during the same survey path. If series of common offset surveys are “simultaneously” collected then ALL the linear targets can be located → very helpful for pipes and technological networks location.
2. Possible experimental problems can make difficult the interpretation (antennas directivity, target within not homogeneous and isotropic materials, ...).
3. The maximum accuracy is compulsory during data acquisition (positioning, combined rotation of the antennas, effective antennas orientation, ...).
4. The approach can be time consuming and can be essential to collect data in several different positions, but it can be preferable respect to 2.5D dense surveys.

Example of “attributes” calculation through  
EM amplitude inversion techniques

*The problem, the implementation, the testing*

# Example of EM amplitude inversion techniques

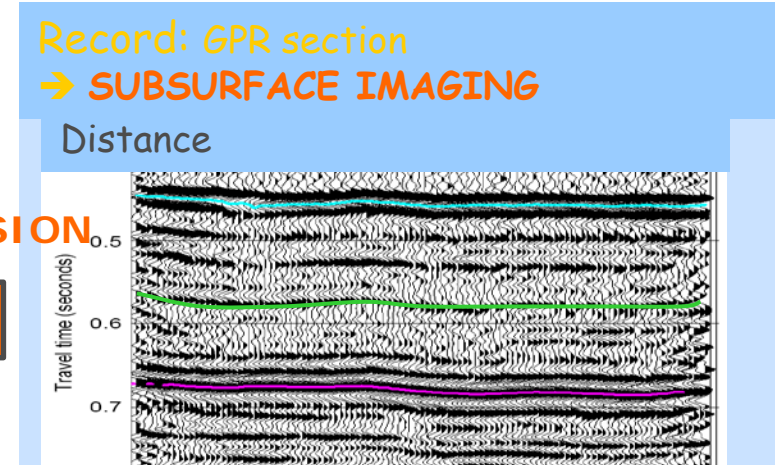
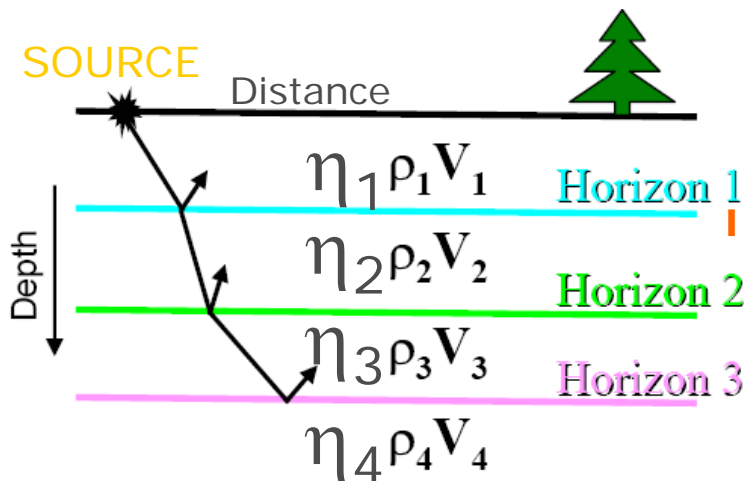
## Statement of the problem/Motivation

**Wave field methods** are based on the propagation of a perturbation (wave) within the earth.

The most commonly used wavefields are **Seismic (elastic waves)** and **Electromagnetic waves**.

The perturbation (or signal) travels into the subsurface, is REFLECTED / REFRACTED / SCATTERED / BACKSCATTERED / CONVERTED and therefore can be recorded at the surface (or into a borehole) by one or more sensors as a function of the time (typically the time zero is the energizing instant).

**No data inversion is required (but it is possible!) → direct IMAGING** of the subsurface.



Statement of the problem/Motivation: is it possible to directly use GPR amplitudes?

We tried to estimate the dielectric permittivity from EM amplitude, considering the reflectivity of the subsurface (i.e. the series of reflection coefficients). This approach is somehow similar to the one normally used for TDR measurements.

In a simple case,  
for just two homogeneous and isotropic media (1 and 2) and vertical incidence:

$$R = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} \Rightarrow \varepsilon_2 = \varepsilon_1 \left( \frac{1 + R}{1 - R} \right)^2 \quad \text{and} \quad R = \frac{A_R}{A_I}$$

QUESTION:

Is it possible to implement a procedure valid for a generalized case?

## Statement of the problem/Motivation

### **WHY The VELOCITY Field (EM or Seismic) is so IMPORTANT?**

- 1) Depth conversion → Reconstruction of the correct depth and geometry of the targets.
- 2) Migration/imaging → Reconstruction of the correct shape of the targets
- 3) Essential parameter for some processing flows (e.g. topographic corrections, divergence corrections,...)
- 4) ADDITIONAL INFORMATION ABOUT THE INVESTIGATED MATERIALS

### ***Estimation of the EM velocity field on REAL DATASET***

*In the general case velocity varies both laterally and vertically!  
Several methods to estimate the EM velocity are available for Multi Offset data, while just a few ones can be adopted for COMMON OFFSET DATA (e.g. diffraction hyperbolas analysis). → About the 95% of GPR surveys!*

#### LIMITATIONS:

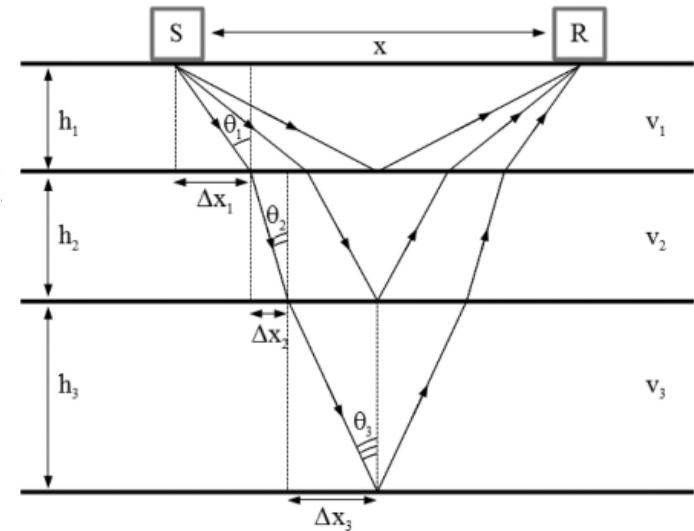
- Effective presence of diffraction hyperbolas
- Low vertical/lateral resolution of the method
- Low overall accuracy (presence of mixed reflected/diffracted events).

## The inversion method

Developed for **COMMON OFFSET TE (broadside) configuration** i.e. the usual one for GPR acquisition.

For each GPR trace the inversion algorithm iteratively calculates for each layer the thickness and the EM velocity by reconstructing from the geometrical data/assumptions and from the picked reflection amplitudes:

- 1) the travel paths of each reflected wave;
- 2) the values of the reflection coefficients.



Each inversion cycle reconstructs the travel path of a reflection.

In the **n-th cycle** we know:

- the first n-1 layer thicknesses
- the first n layer velocities

The **n-th cycle** calculates:

- the n-th layer thickness ( $h_n$ )
- the (n+1)-th layer velocity

$$TWT_n = \sqrt{\frac{x^2}{\bar{v}_n^2} + 4 \left( \sum_{i=1}^n \frac{h_i}{v_i} \right)^2},$$

with

$$\bar{v}_n^2 = \frac{\sum_{i=1}^n v_i h_i}{\sum_{i=1}^n \frac{h_i}{v_i}},$$

$$ah_n^3 + bh_n^2 + ch_n + d = 0,$$

where

$$a = 4/v_n,$$

$$b = \frac{4}{v_n^2} \sum_{i=1}^{n-1} v_i h_i + 8 \sum_{i=1}^{n-1} \frac{h_i}{v_i},$$

$$c = \frac{x^2}{v_n} + \frac{8}{v_n} \left( \sum_{i=1}^{n-1} v_i h_i \right) \left( \sum_{i=1}^{n-1} \frac{h_i}{v_i} \right) + 4v_n \left( \sum_{i=1}^{n-1} \frac{h_i}{v_i} \right)^2 - v_n TWT_n^2,$$

$$d = x^2 \sum_{i=1}^{n-1} \frac{h_i}{v_i} + 4 \left( \sum_{i=1}^{n-1} v_i h_i \right) \left( \sum_{i=1}^{n-1} \frac{h_i}{v_i} \right)^2 - TWT_n^2 \sum_{i=1}^{n-1} v_i h_i.$$

**Forte E.**, Dossi M., Colucci R.R. and Pipan M., 2013, *A new fast methodology to estimate the density of frozen materials by means of common offset GPR data*, JAG, 99, 135-145.

**Forte E.**, Dossi M., Pipan M. and Colucci R.R., 2014, *Velocity analysis from Common Offset GPR data inversion: theory and application to synthetic and real data*, Geophysical Journal International, 197, 3, 1471-1483.

## The inversion method

### INPUTS:

- 1) Offset;
- 2) EM velocity in the shallowest layer;
- 3) Peak amplitude of the wavelet incident on the first interface;
- 4) Peak amplitudes and traveltimes along each reflector.

- 1) *the offset is usually known*
- 2) *the EM velocity in the shallowest layer can be estimated by direct density measurements and assumed constant along each GPR profile;*
- 3) *as reference amplitude we can select the mean peak airwave amplitude recorded by the dedicated measurements;*
- 4) *the reflections are picked along the interpreted horizons, after appropriate data processing.*

### ASSUMPTIONS:

There are some approximations/assumptions necessary for the inversion procedure:

- (A) the propagating radar signal is an EM plane wave;
- (B) each layer is isotropic, homogeneous, lossless and non-dispersive;
- (C) in the neighbourhood of each trace position the reflectors are plane-parallel;
- (D) the amplitudes of the picked reflected waves are only related to the reflection coefficients, while all the other effects are either disregarded or corrected.

**Expected OUTPUT → EM velocity field  
(and so  $\epsilon$  and other physical parameters)**

# The inversion method

From the Snell equation  $v_2 = \frac{\sin(\vartheta_2)}{\sin(\vartheta_1)} v_1$  we obtain:

$$\Delta x_k = \frac{v_k h_k}{v_{k-1} h_{k-1}} \Delta x_{k-1}. \quad \text{With:} \quad \sin(\vartheta_k) \approx \frac{\Delta x_k}{h_k}$$

We calculate the incidence angles along the path of the n-th reflected wave as:

$$\vartheta_k = \text{Arctan} \frac{x v_k}{2 \sum_{i=1}^n v_i h_i},$$

with  $k = 1, 2, \dots, n$ .

With such angles we calculate the n-1 reflection and transmission coefficients using the Fresnel equations for the TE mode.

Using these coefficients

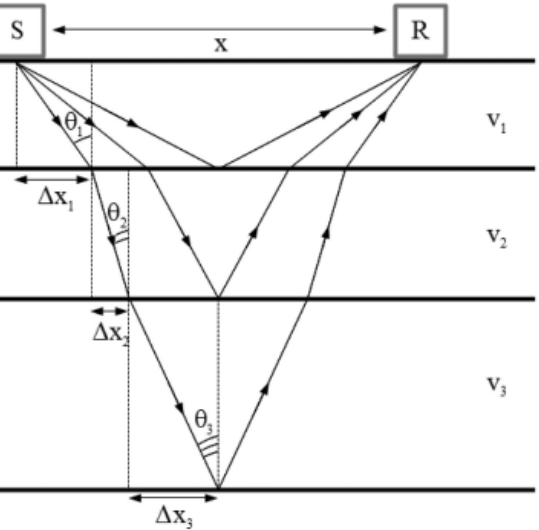
$$A i_n = A i_1 \prod_{i=1}^{n-1} T_i,$$

$$A r_n = \frac{A s_n}{\prod_{i=1}^{n-1} (2 - T_i)}.$$

and so the reflection coefficient of the n-th interface for the n-th reflected wave is

$$R_n = \frac{A r_n}{A i_n}$$

The velocity in the n+1 layer is given by the Snell eq. as:



$$R_k = \frac{\sin(\vartheta_{k+1} - \vartheta_k)}{\sin(\vartheta_{k+1} + \vartheta_k)},$$

$$T_k = 1 + R_k,$$

with  $k = 1, 2, \dots, n - 1$ .

$$v_{n+1} = \frac{\sin(\vartheta_{n+1})}{\sin(\vartheta_n)} v_n,$$

with

$$\vartheta_{n+1} = \text{Arctan} \left( \frac{1 + R_n}{1 - R_n} \tan \vartheta_n \right)$$



## The inversion method

# Data Processing

To make possible the data INVERSION, data must be processed in “true amplitude”

In a GPR experiment, even in case of virtually lossless materials, amplitudes are primarily affected by:

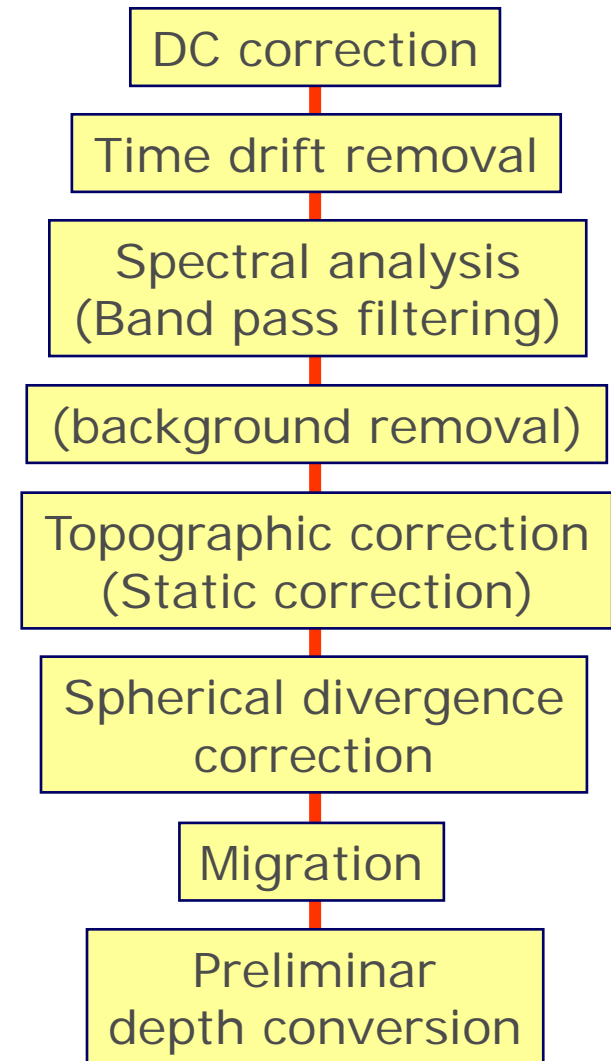
**(A) scattering, (B) geometrical spreading, (C) partial reflections.**

(A) Diffractions can be focused by means of migration algorithms

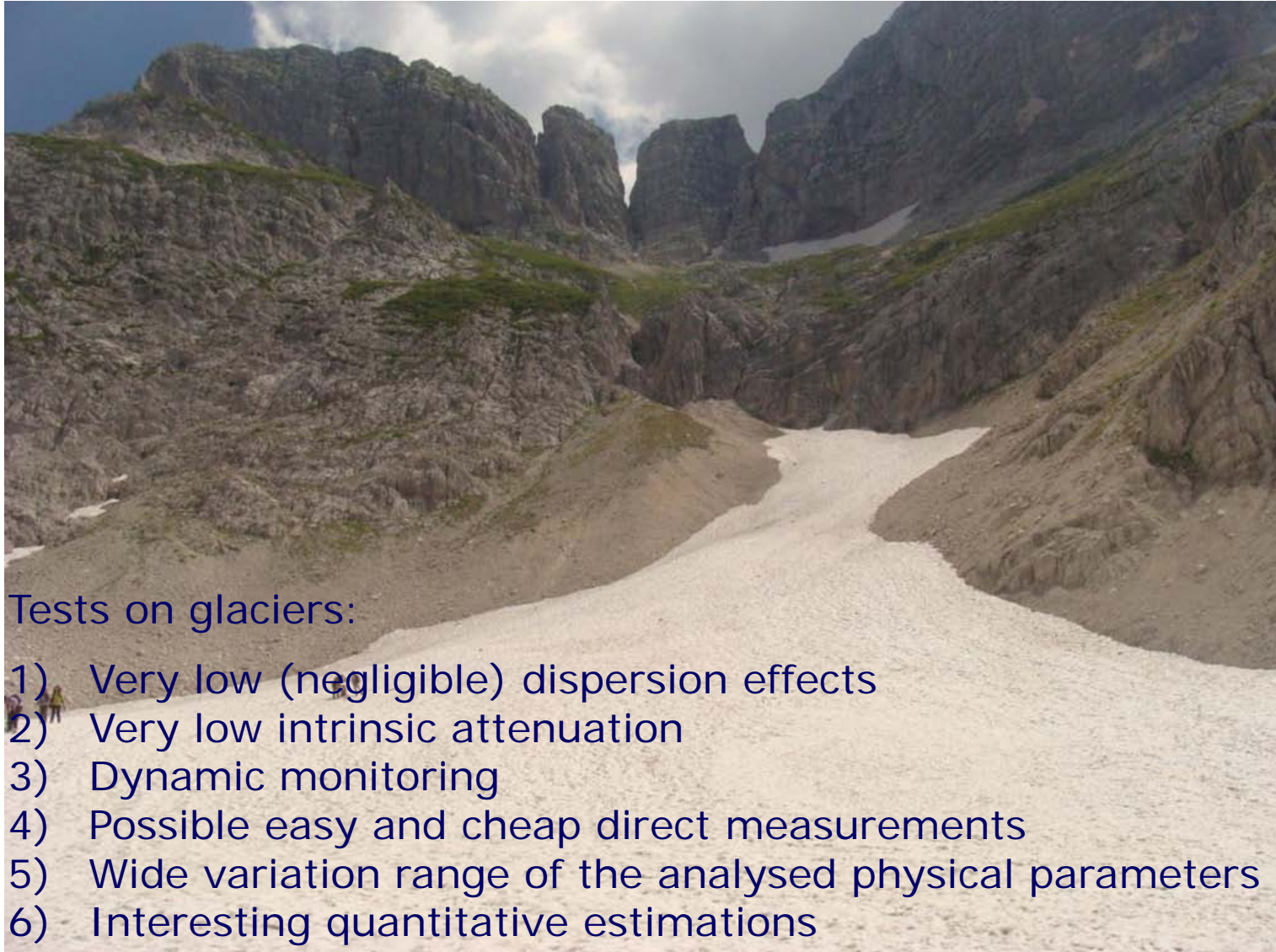
(B) Geometrical spreading can be corrected by using divergence recovery. Due to the antenna directivity, a precise correction can be obtained only if the radiation pattern into the subsurface is known.

Since this can be measured only through complex polarimetric/multicomponent experiments, a spherical divergence correction can be considered a valid first approximation. We apply a spherical divergence correction with a velocity constant for each survey based on combined CMP analysis and direct data validation with glaciological pits.

(C) The effect of partial reflections can be analytically removed starting from the uppermost reflector down to the basal one.



## The inversion method: example of application on real data



### Tests on glaciers:

- 1) Very low (negligible) dispersion effects
- 2) Very low intrinsic attenuation
- 3) Dynamic monitoring
- 4) Possible easy and cheap direct measurements
- 5) Wide variation range of the analysed physical parameters
- 6) Interesting quantitative estimations

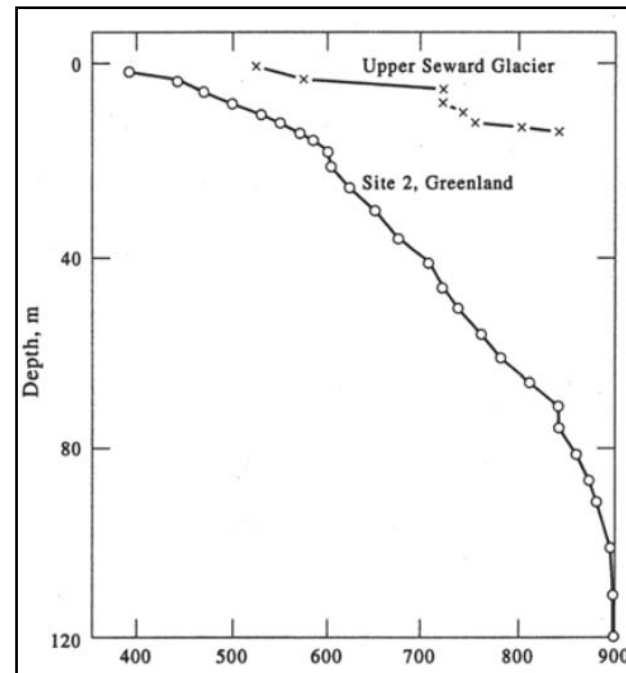
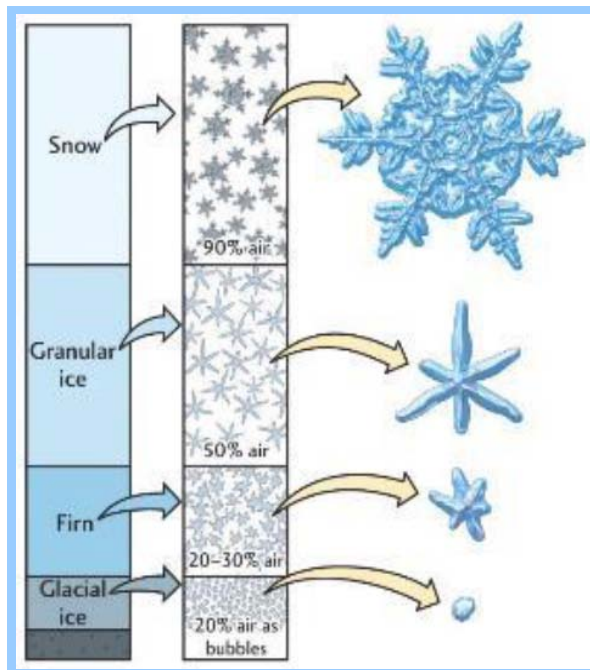
## The inversion method for glaciological surveys: why so?

An accurate EM velocity estimation is essential, since:

- small glaciers and glacierets can show **significant vertical and lateral density variations**.
- **large density changes** correspond to relatively **smaller EM velocity variations**.

Density distribution is commonly assumed to be **constant or slow-varying**, with the only constraints given by **local values** sampled **near the surface**.

GPR surveys allow to probe the **entire volume** of a glacier, with the large number of traces making quantitative analyses statistically sound.



Paterson, 1994

## The inversion method: preliminary inversion tests on real data

$$\varepsilon_{ice} = (1 + 0.845 \cdot \rho)^2 \quad \text{Robin, 1975}$$

$$\varepsilon_{mix} = \left[ \varepsilon_{ice}^{1/2} (1 - \phi) + \varepsilon_{water}^{1/2} \theta + \varepsilon_{air}^{1/2} (\phi - \theta) \right]^2$$

$$\rho_{mix} = [\rho_{ice} (1 - \phi) + \rho_{water} \theta]$$

$$\varepsilon_{mix}^{1/3} - 1 = \frac{\rho_{mix}}{\rho_{ice}} (\varepsilon_{ice}^{1/3} - 1) \quad \text{Looyenga, 1965}$$

*Birchak et al., 1974*

The bulk density of the mixture can be computed according to the contribute of each component.  $\phi$  is the bulk porosity and  $\theta$  is the free water content. This is also known as the **complex refractive index method (CRIM)**

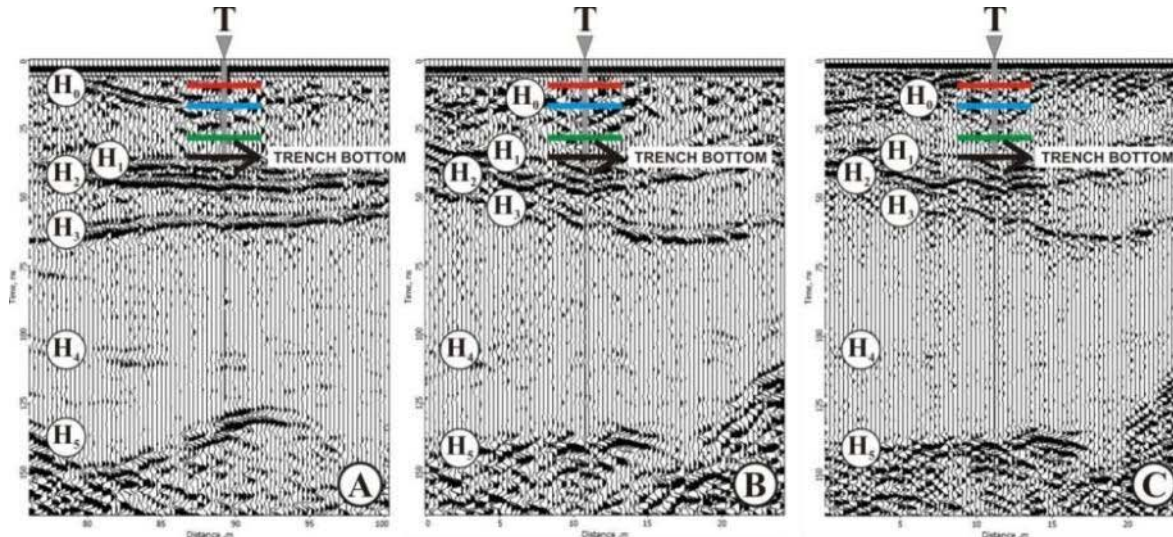
When the effect of free water can be disregarded (i.e. if the ice temperature is considerably lower than 0°C and the pressure small)

	Density [g/cm <sup>3</sup> ]	ROBIN		LOOYENGA	
		( $\varepsilon_r$ )	Velocity [cm/ns]	( $\varepsilon_r$ )	Velocity [cm/ns]
Fresh snow	0.1	1.2	27.7	1.2	27.9
	0.2	1.4	25.7	1.4	25.7
	0.3	1.6	23.9	1.5	24.2
	0.4	1.8	22.4	1.8	22.7
Firn	0.5	2.0	21.1	2.0	21.3
	0.6	2.3	19.9	2.2	20.0
	0.7	2.5	18.9	2.5	18.9
Ice	0.8	2.8	17.9	2.8	17.9
	0.9	3.1	17.0	3.1	17.0

*The main problem is not related to the choice/applicability/accuracy of the empirical relations but to an accurate EM velocity estimation...*

**For frozen materials  $V_{EM}$  from about 0.17 up to 0.28 m/ns !**

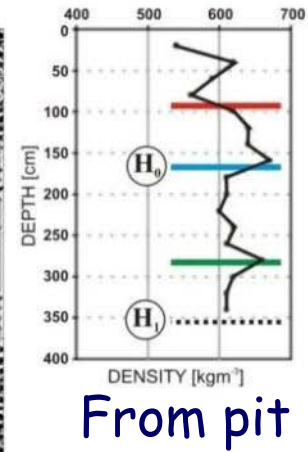
# The inversion method: inversion results vs direct measurements



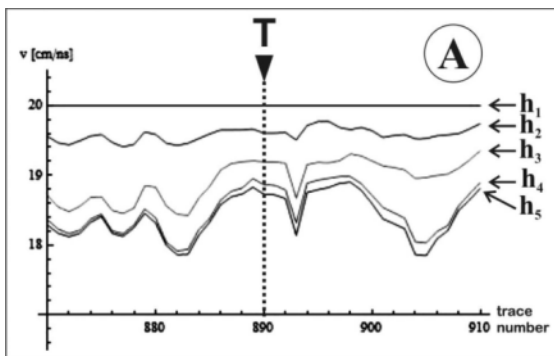
250 MHz

250 MHz perp

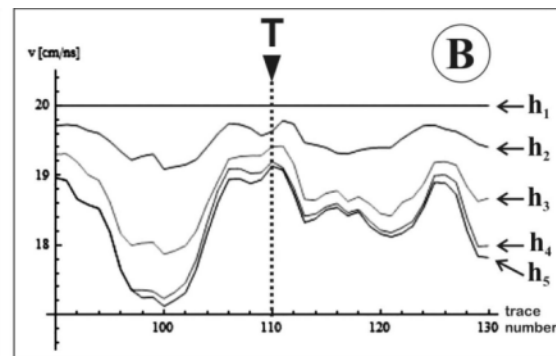
500 MHz perp



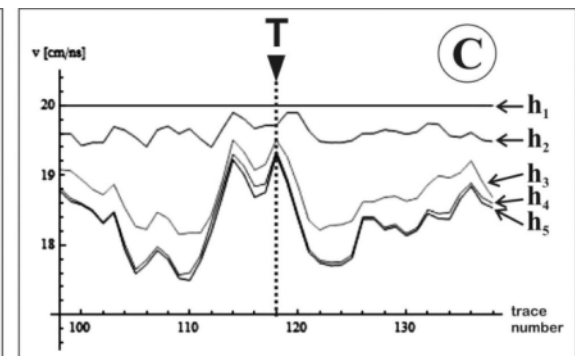
## Inversion ( $V_{EM}$ ) results



250 MHz

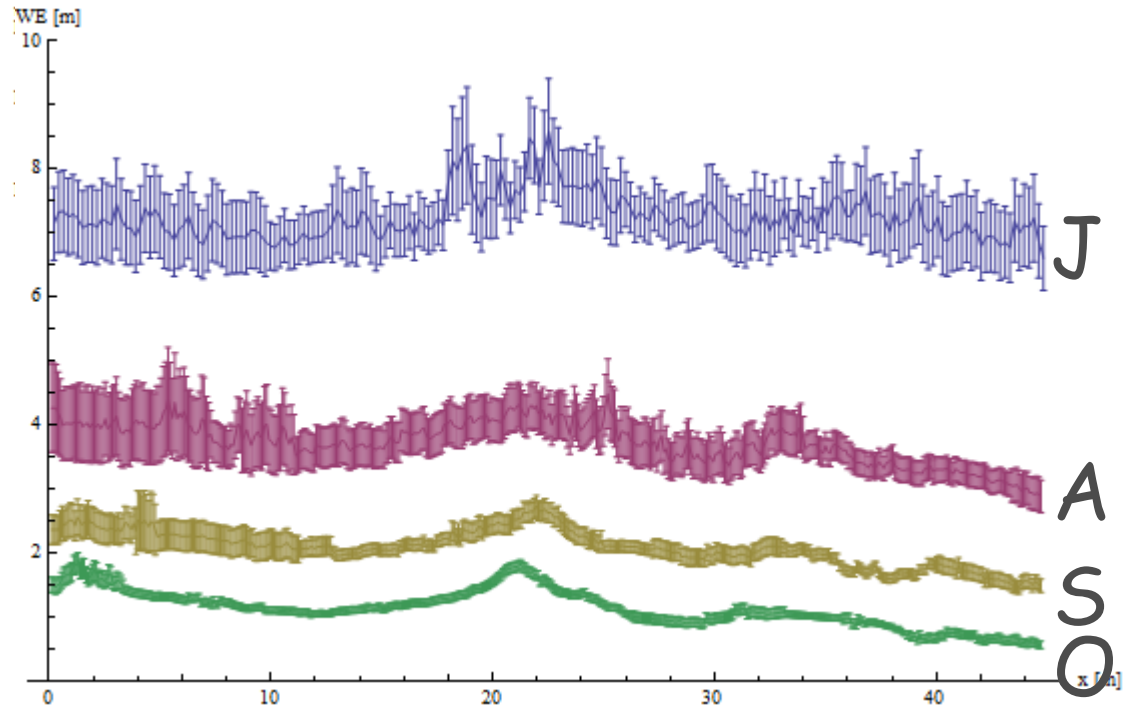


250 MHz perp



500 MHz perp

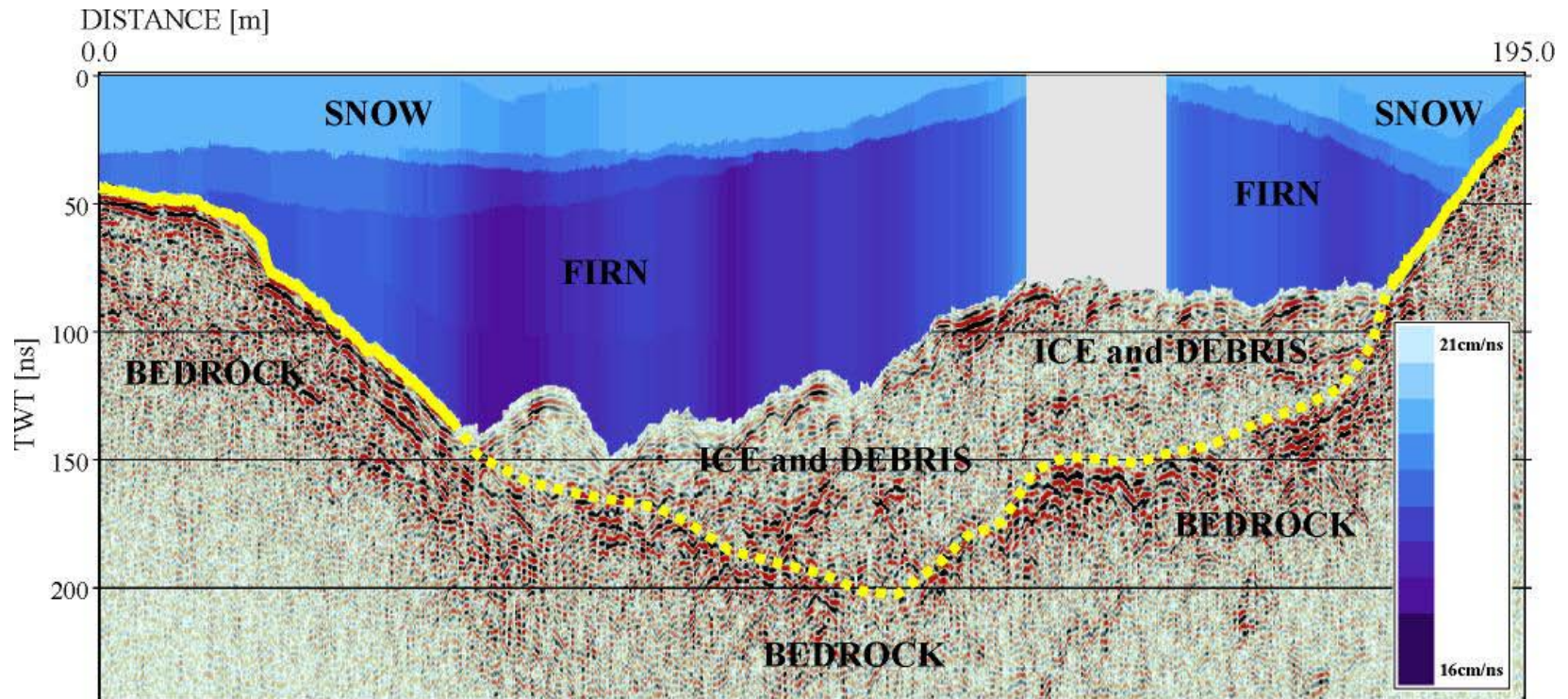
# The inversion method: inversion results vs direct measurements



Comparison of the thicknesses of the different layers obtained by GPR data inversion for using the velocity and densities with the inversion procedure and from direct measures interpolations (dots).

The error bars are obtained applying the propagation of maximum errors on all the inversion equations. The uncertainties of each input parameter are: (1) zero for the offset (i.e. not considered); (2) 0.2 cm/ns for the EM velocity in the shallowest layer; (3) 5% for both reference and reflected amplitudes and (4) half of the sampling interval for the traveltimes (0.119 ns)

## The inversion method: 2D EM velocity field reconstruction

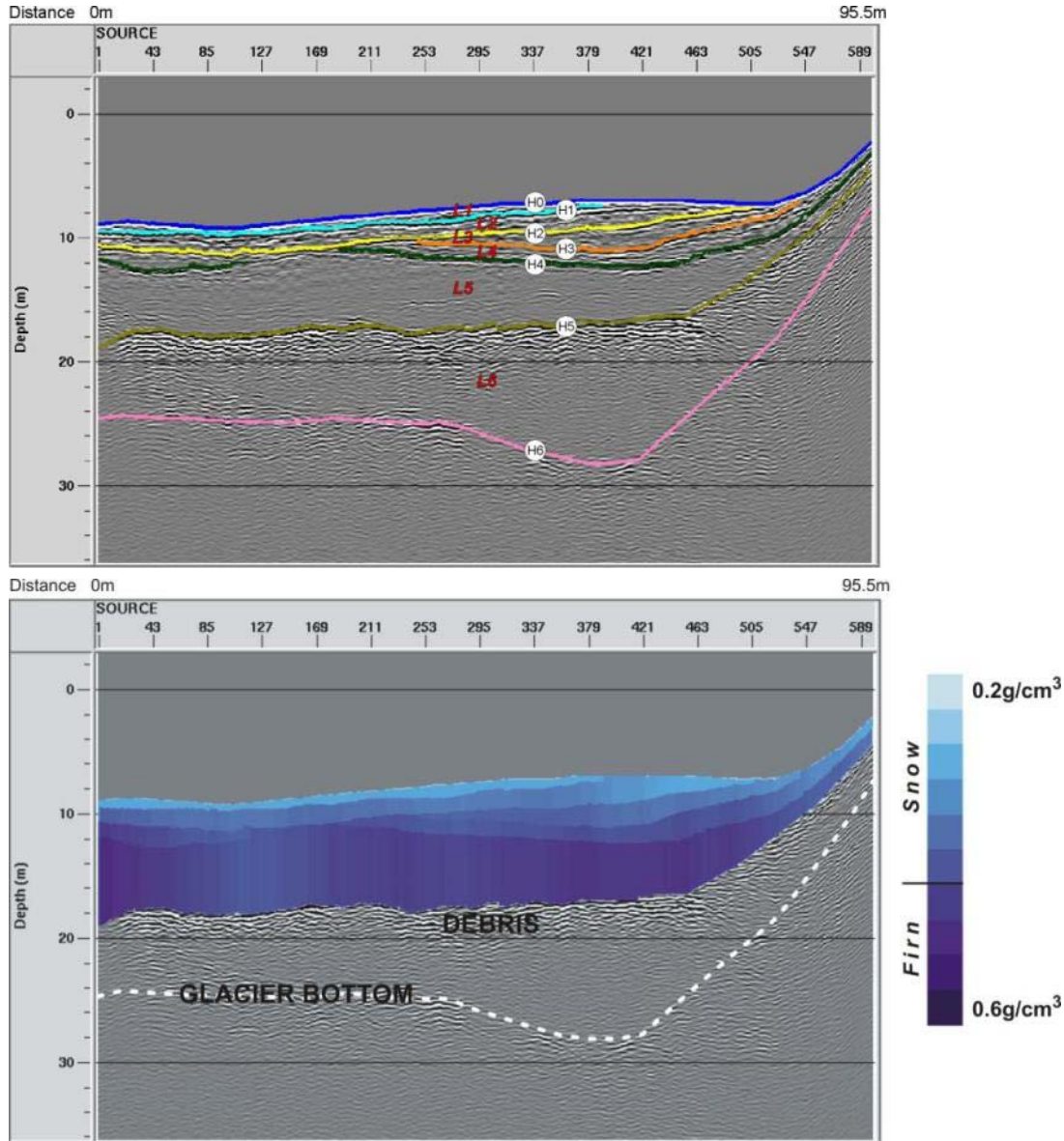


It is possible to highlight both vertical and lateral velocity variations with a (theoretical) resolution equal to the trace interval.

Anyway the validity of the whole procedure is “statistical” → “ZONES” with homogeneous materials.



# The inversion method: from EM velocity... to ice density

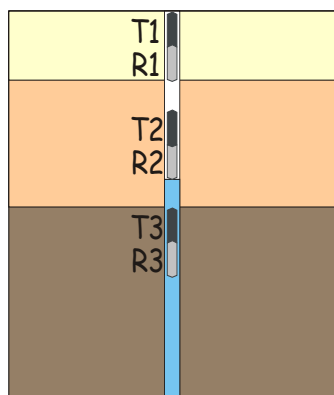




## Borehole GPR: motivations

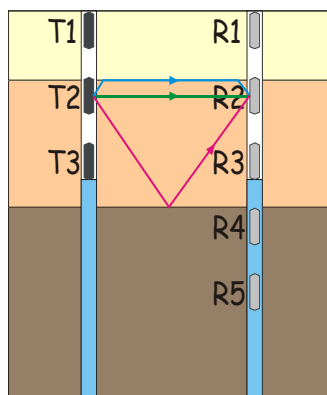
1. Extend the information available from boreholes stratigraphy.
2. Overcome penetration depth limitations of surface GPR surveys.
3. Possible data inversion (both traveltimes → velocity and amplitude → attenuation) to recover “global” EM characteristics of the materials in addition to subsurface imaging.
4. Strategies already developed and exploited for reflection seismics can be adapted to EM waves

### Reflection



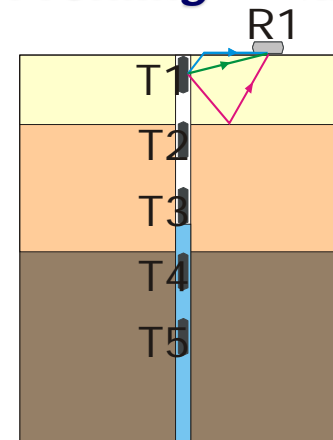
*T and R within the same borehole*

### Tomography



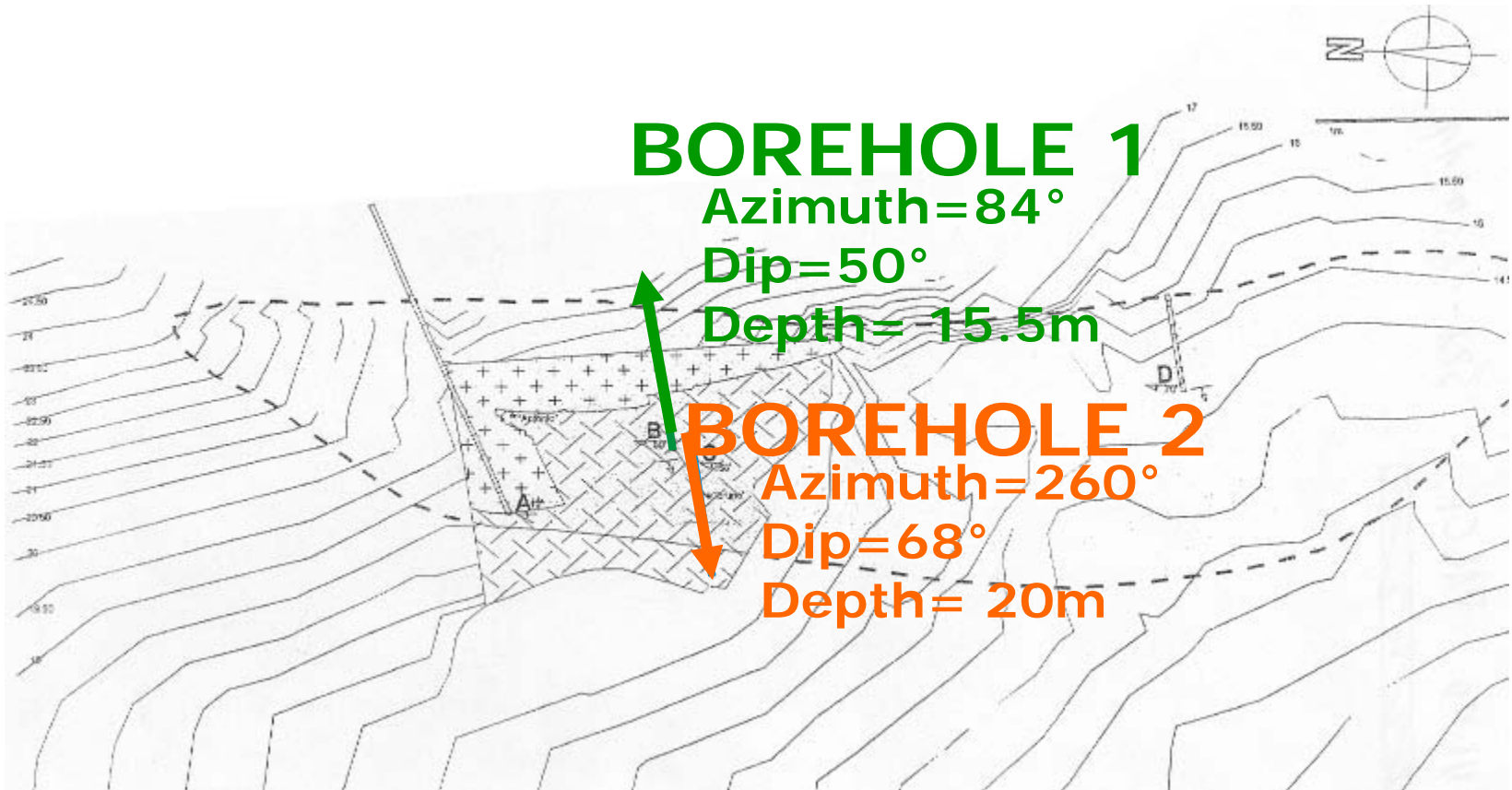
*T and R within 2 different boreholes*

### Vertical Radar Profiling - VRP



*T within a borehole  
R on the surface  
(or viceversa)*

## A Borehole GPR Tomography example



*Geological settings:*

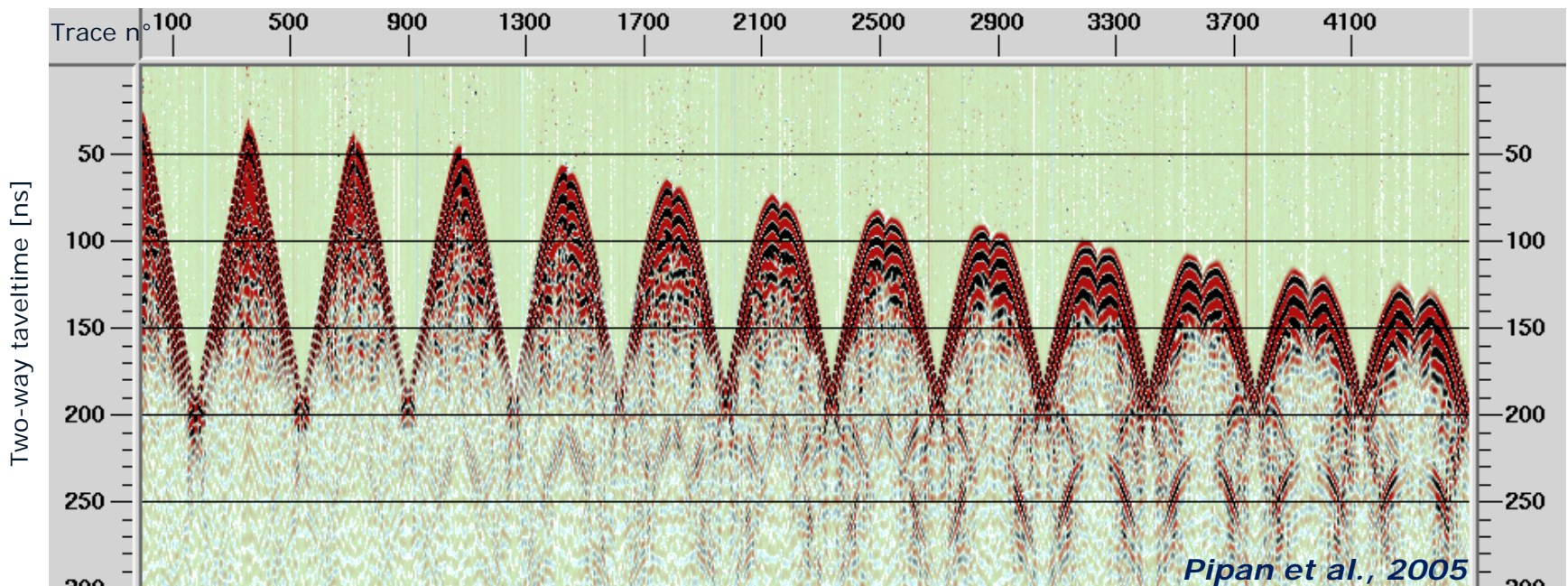
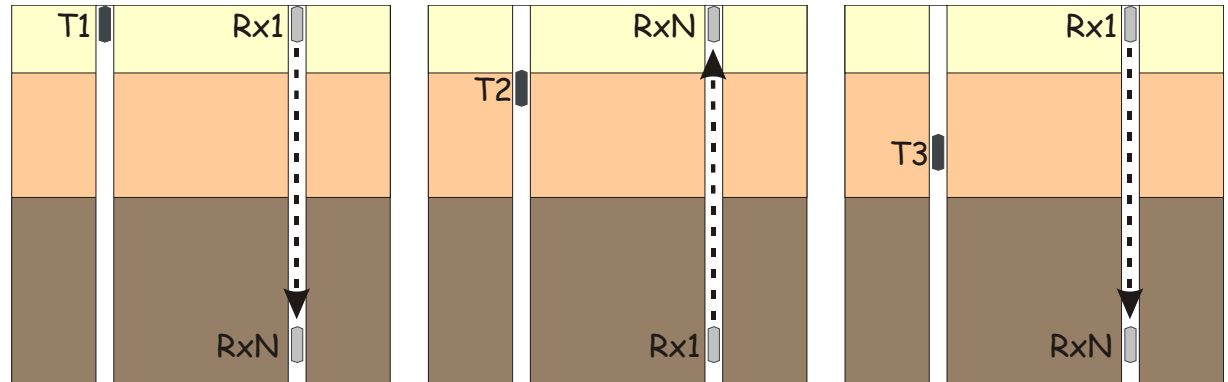
*Grey or blackish limestone, with laminithic levels characterised by different organic material content.*

*Presence of fractures locally with karstic phenomena and vertebrate fossils.*

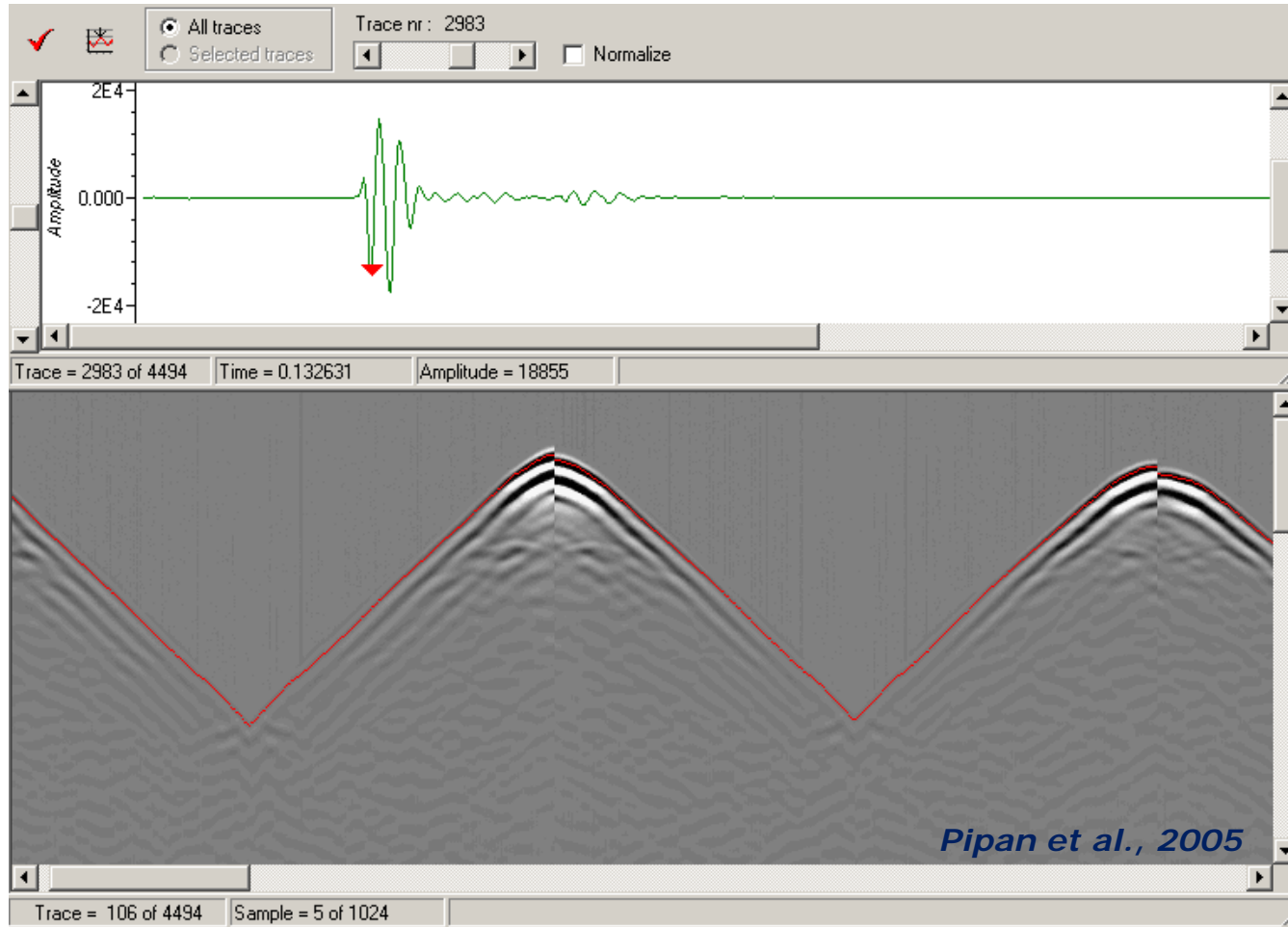
*Pipan et al., 2005*

# A Borehole GPR Tomography example

Example of **borehole GPR tomography** acquisition scheme:  
Tx increment = 50cm  
Rx increment = 10cm



# A Borehole GPR Tomography example

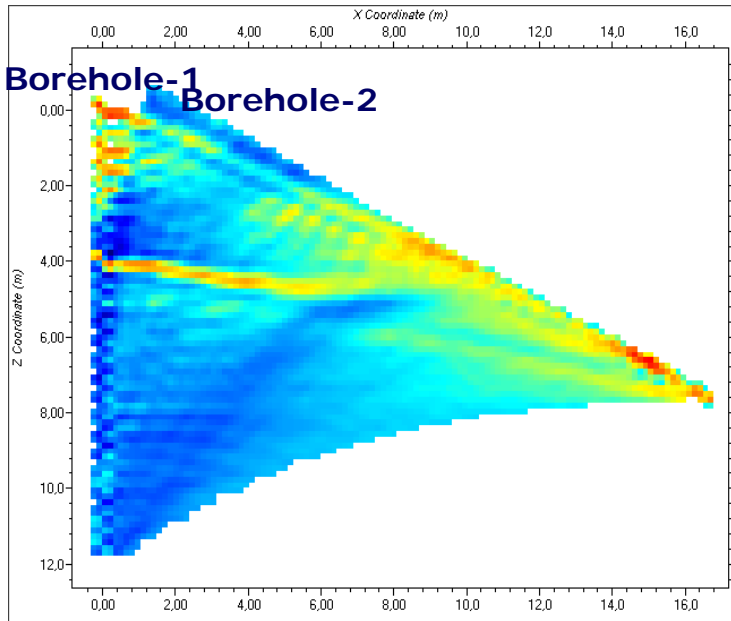


**First break picking**  *Traveltime inversion* → velocity  
*Amplitude inversion* → attenuation

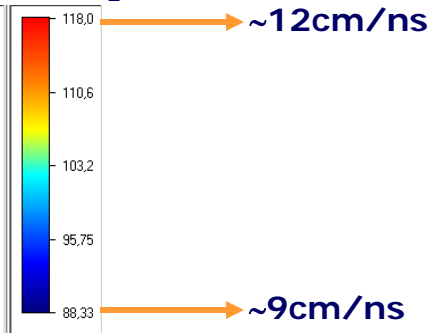


# A Borehole GPR Tomography example

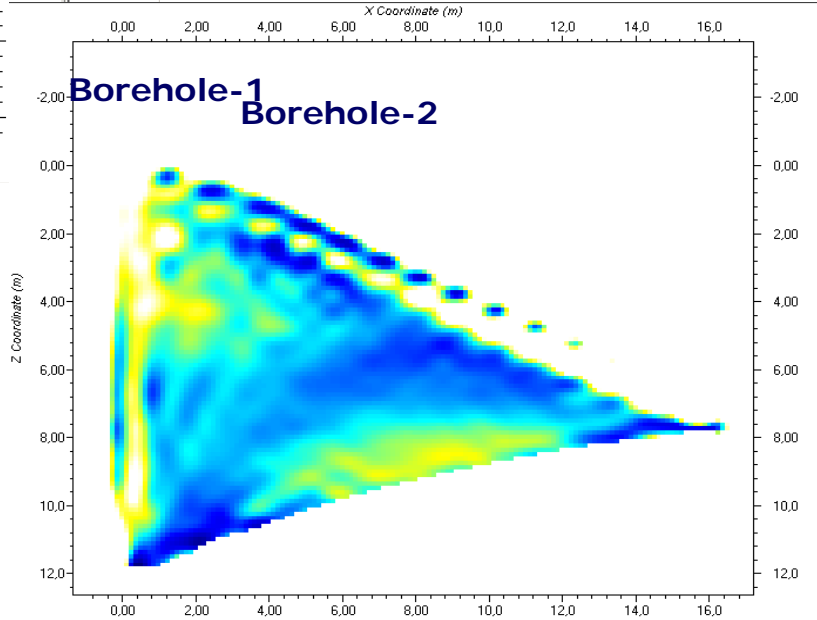
## Velocity Field



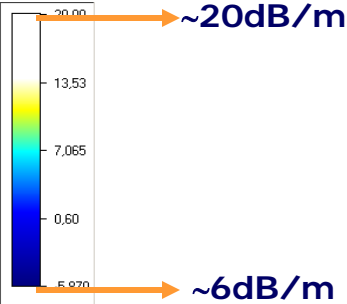
Velocity  
[mm/ns]



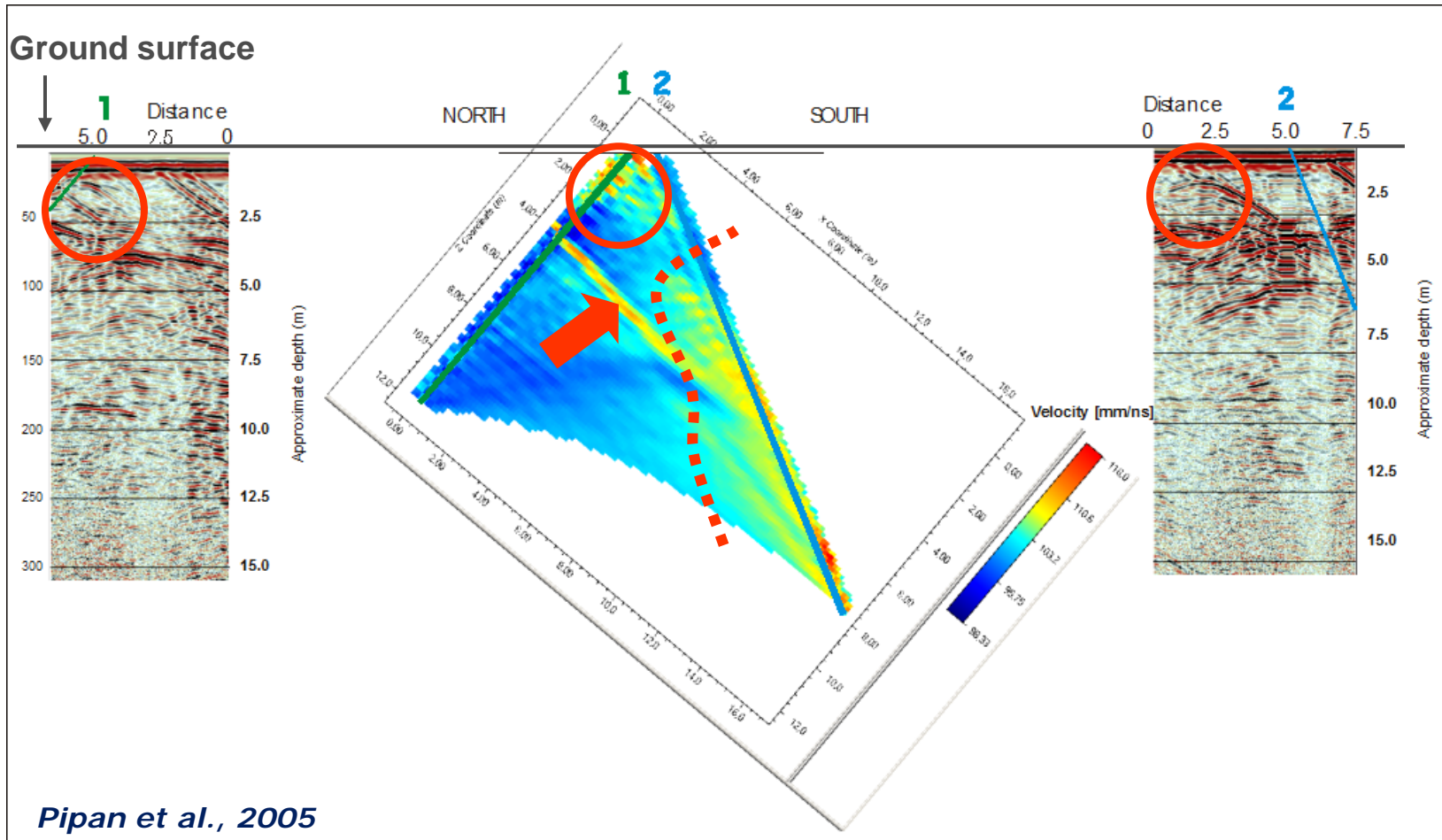
## Attenuation field



Attenuation  
dB/m



# A Borehole GPR Tomography example





***Questions?***