mercoledì 9 dicembre 2020   13:22

FOGLIO 7

ESERCIZIO 1

$A = \begin{pmatrix} 1 & 2 & -1 & 1 \\ 1 & 1 & 1 & 2 \\ -1 & -2 & 1 & 0 \end{pmatrix}$. Trovare S e T invertibili t.c. $SAT = \left(\begin{array}{c|c} E_r & 0 \\ \hline 0 & 0 \end{array}\right)$.

A è la matrice associata a $L_A : \mathbb{R}^4 \to \mathbb{R}^3$ alle basi canoniche C e C'.

$\Rightarrow$ [Trovare S e T $\Leftrightarrow$ Trovare basi β e β' t.c. $M_\beta^{\beta'}(L_A) = \left(\begin{array}{c|c} E_r & 0 \\ \hline 0 & 0 \end{array}\right)$]

Infatti, $M_\beta^{\beta'}(L_A) = M_{C'}^{\beta'} \cdot M_C^{C'}(L_A) \cdot M_\beta^C$.

· STEP 1: Calcolo rg A.

$\begin{pmatrix} 1 & 2 & -1 & 1 \\ 1 & 1 & 1 & 2 \\ -1 & -2 & 1 & 0 \end{pmatrix} \xrightarrow[R_3 \to R_3 + R_1]{R_2 \to R_2 - R_1} \begin{pmatrix} 1 & 2 & -1 & 1 \\ 0 & -1 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow rg(A) = 3.$

(QUINDI SAT sarà DELLA FORMA $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$.)

· STEP 2: Trovo una base di $\ker L_A$

$Ax = 0 \overset{\text{FORMA RIDOTTA}}{\Longleftrightarrow} \begin{cases} x_1 + 2x_2 - x_3 + x_4 = 0 \\ -x_2 + 2x_3 + x_4 = 0 \\ x_4 = 0 \end{cases} \Longleftrightarrow \begin{cases} x_1 = -3x_3 \\ x_2 = 2x_3 \\ x_4 = 0 \end{cases}$

$\to \ker A = \{(-3a, 2a, a, 0) \mid a \in \mathbb{R}\} = \langle \underbrace{(-3, 2, 1, 0)}_{=: v,\ \text{BASE DI } \ker L_A} \rangle$

· STEP 3: Completo a una base di $\mathbb{R}^4$

$\beta = \{e_1, e_2, e_4, v\}$ base di $\mathbb{R}^4$ ✓.

· STEP 4: TROVO LA BASE DI $\operatorname{Im} L_A$ DATA DA β

$L_A(e_1) = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ $L_A(e_2) = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$, $L_A(e_4) = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$, $L_A(v) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\xrightarrow[\text{GENERATORI di } \operatorname{Im} L_A = 3]{}$ $\left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \right\}$ base di $\operatorname{Im} L_A$

· STEP 5: Completo a una base di $\mathbb{R}^3$.

$\dim \operatorname{Im} L_A = \dim \mathbb{R}^3 \Rightarrow \beta' = \left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \right\}$ base di $\mathbb{R}^3$.

STEP 6: Verifica $M_\beta^{\beta'}(L_A) = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$.

$\beta = \{e_1, e_2, e_4, v\}$.

$L_A(e_1) = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \leftarrow$ PRIMO VETTORE DI β' ✓ $L_A(e_2) = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \leftarrow$ 2° VETT. DI β' ✓

$L_A(e_4) = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \leftarrow$ TERZO VETTORE DI β' ✓ $L_A(v) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \leftarrow$ ULTIMA COLONNA NULLA ✓.

STEP 7: Trovo $T = M_\beta^C$, $S = M_{C'}^{\beta'}$.

$T = M_\beta^C = M_\beta^C(id_{\mathbb{R}^4}) = \begin{pmatrix} \text{COLONNE} = \\ \text{VETTORI DI} \\ \beta \text{ IN COMPONENTI} \\ \text{RISPETTO A } C \end{pmatrix} \overset{C \text{ BASE CANONICA}}{=} \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$.

$S = M_{C'}^{\beta'} = \left(M_{\beta'}^{C'}\right)^{-1} = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ -1 & -2 & 0 \end{pmatrix}^{-1} \overset{(*)}{=} \begin{pmatrix} -4 & 2 & -3 \\ 2 & -1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ ✓.

(*): $\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ -1 & -2 & 0 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right)$

$\to \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -4 & 2 & -1 \\ 0 & 1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right)$

ESERCIZIO 2

(a) $A = \begin{pmatrix} 1 & 1 & 2 & 4 \\ 1 & 3 & 4 & -2 \\ 0 & 1 & 3 & 6 \\ 1 & 3 & 5 & 3 \end{pmatrix}$. A invertibile in $\mathbb{R}$ e $\mathbb{Z}_7$? Trovare l'inversa di A.

$\det(A) = \begin{vmatrix} 1 & 1 & 2 & 4 \\ 1 & 3 & 4 & -2 \\ 0 & 1 & 3 & 6 \\ 1 & 3 & 5 & 3 \end{vmatrix} \underset{\substack{R_4 \to R_4 - R_2 \\ R_2 \to R_2 - R_1}}{=} \begin{vmatrix} 1 & 1 & 2 & 4 \\ 0 & 2 & 2 & -6 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 1 & 5 \end{vmatrix} = \begin{vmatrix} 2 & 2 & -6 \\ 1 & 3 & 6 \\ 0 & 1 & 5 \end{vmatrix} =$ (SVILUPPO)

$= 2\begin{vmatrix} 3 & 6 \\ 1 & 5 \end{vmatrix} - \begin{vmatrix} 2 & -6 \\ 1 & 5 \end{vmatrix} = 2(3 \cdot 5 - 1 \cdot 6) - (2 \cdot 5 - 1 \cdot (-6)) = 2 \neq 0$ in $\mathbb{R}$ e $\mathbb{Z}_7$

$\Rightarrow$ A INVERTIBILE IN $\mathbb{R}$ E $\mathbb{Z}_7$.

INVERSA DI A?

TROVIAMO L'INVERSA IN $\mathbb{R}$, CON I GIUSTI ACCORGIMENTI TROVEREMO L'INVERSA IN $\mathbb{Z}_7$ A PARTIRE DA QUELLA.

$\left(\begin{array}{cccc|cccc} 1 & 1 & 2 & 4 & 1 & 0 & 0 & 0 \\ 1 & 3 & 4 & -2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & 6 & 0 & 0 & 1 & 0 \\ 1 & 3 & 5 & 3 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_4 \to R_4 - R_1]{R_2 \to R_2 - R_1} \left(\begin{array}{cccc|cccc} 1 & 1 & 2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & 2 & -6 & -1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 6 & 0 & 0 & 1 & 0 \\ 0 & 2 & 3 & -1 & -1 & 0 & 0 & 1 \end{array}\right)$

$\xrightarrow{R_2 \leftrightarrow R_3} \left(\begin{array}{cccc|cccc} 1 & 1 & 2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & 6 & 0 & 0 & 1 & 0 \\ 0 & 2 & 2 & -6 & -1 & 1 & 0 & 0 \\ 0 & 2 & 3 & -1 & -1 & 0 & 0 & 1 \end{array}\right) \xrightarrow[\substack{R_3 \to R_3 - 2R_2 \\ R_4 \to R_4 - 2R_2}]{R_1 \to R_1 - R_2} \left(\begin{array}{cccc|cccc} 1 & 0 & -1 & -2 & 1 & 0 & -1 & 0 \\ 0 & 1 & 3 & 6 & 0 & 0 & 1 & 0 \\ 0 & 0 & -4 & -18 & -1 & 1 & -2 & 0 \\ 0 & 0 & -3 & -13 & -1 & 0 & -2 & 1 \end{array}\right)$

$\xrightarrow[R_3 \to -\frac{1}{4}R_3]{(*)} \left(\begin{array}{cccc|cccc} 1 & 0 & -1 & -2 & 1 & 0 & -1 & 0 \\ 0 & 1 & 3 & 6 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{9}{2} & \frac{1}{4} & -\frac{1}{4} & \frac{1}{2} & 0 \\ 0 & 0 & -3 & -13 & -1 & 0 & -2 & 1 \end{array}\right) \xrightarrow[R_4 \to R_4 + 3R_3]{\substack{R_1 \to R_1 + R_3 \\ R_2 \to R_2 - 3R_3}} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & \frac{5}{2} & \frac{5}{4} & -\frac{1}{4} & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & -\frac{15}{2} & -\frac{3}{4} & \frac{3}{4} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & \frac{9}{2} & \frac{1}{4} & -\frac{1}{4} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{2} & 0 \end{array}\right)$

$\xrightarrow[\substack{R_2 \to R_2 + 15R_4 \\ R_3 \to R_3 - 9R_4 \\ R_4 \to 2R_4}]{R_1 \to R_1 - 5R_4} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{5}{2} & \frac{7}{2} & 2 & -5 \\ 0 & 1 & 0 & 0 & -\frac{9}{2} & -\frac{21}{2} & -8 & 15 \\ 0 & 0 & 1 & 0 & \frac{5}{2} & \frac{13}{2} & 5 & -5 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & -\frac{3}{2} & -1 & 2 \end{array}\right)$ (INVERSA IN $\mathbb{R}$) ✓.

(*): DA QUESTO PASSAGGIO IN POI BISOGNA DIFFERENZIARE $\mathbb{Z}_7$ DA $\mathbb{R}$.

Tutti i passaggi, in un certo senso, sono validi in $\mathbb{Z}_7$: bisogna però ricordare che dividere per 4 (o per qualsiasi altro numero) significa moltiplicare per 4^{-1}.

Dunque $A^{-1} \underset{\text{in } \mathbb{Z}_7}{=} \begin{pmatrix} 2^{-1} \cdot 5 & 2^{-1} \cdot 7 & -2 & -5 \\ -2^{-1} \cdot 9 & -2^{-1} \cdot 21 & -8 & 15 \\ -2^{-1} \cdot 5 & 2^{-1} \cdot 13 & 5 & -9 \\ -2^{-1} & -2^{-1} \cdot 3 & -1 & 2 \end{pmatrix} \underset{\substack{\text{SOSTITUIAMO } 2^{-1} = 4 \text{ E SCRIVIAMO} \\ \text{LE CLASSI DI RESTO}}}{=} \begin{pmatrix} 6 & 0 & 5 & 2 \\ 6 & 0 & 6 & 1 \\ 1 & 3 & 5 & 5 \\ 3 & 2 & 6 & 2 \end{pmatrix}$.

(b) $A(x) = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & x \end{pmatrix}$, $x \in \mathbb{R}$. $A(x)$ INVERTIBILE? $(A(x))^{-1}$?

$\det(A(x)) = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & x \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 1 & x \end{vmatrix} - \begin{vmatrix} 1 & 0 \\ 1 & x \end{vmatrix} = 2x - 1 - x = x - 1.$

Quindi $A(x)$ invertibile $\Leftrightarrow x - 1 \neq 0 \Leftrightarrow x \neq 1$.

IN TAL CASO:

Calcoliamo l'inversa di A tramite la sua trasposta.

$A'_{11} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & x \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 1 & x \end{vmatrix} = 2x - 1$ $A'_{12} = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & x \end{vmatrix} = -x$ $A'_{13} = \begin{vmatrix} 0 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 0 \end{vmatrix} = 1$.

$A'_{21} = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & x \end{vmatrix} = -x$ $A'_{22} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & x \end{vmatrix} = x$ $A'_{23} = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = -1$

$A'_{31} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 1$ $A'_{32} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = -1$ $A'_{33} = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$.

$\Rightarrow \tilde{A} = \begin{pmatrix} 2x-1 & -x & 1 \\ -x & x & -1 \\ 1 & -1 & 1 \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{x-1} \tilde{A}$ ✓✓✓

! Visto che A è simmetrica, anche $\tilde{A}$ lo è! Avremmo potuto calcolare solo gli A'_{ij} con $i \geq j$.

(3) $V = \mathbb{R}[t]_{\leq 2}$. $A = \{ \underbrace{t^2 - 2t}_{p_1(t)}, \underbrace{1 + 2t}_{p_2(t)}, \underbrace{2 - t^2}_{p_3(t)} \}$

$B = \{ \underbrace{-1 + t}_{q_1(t)}, \underbrace{-1 + t + t^2}_{q_2(t)}, \underbrace{2t + 2t^2}_{q_3(t)} \}$.

A, B basi? M_A^B?

A base $\Leftrightarrow rg(p_1(t) | p_2(t) | p_3(t)) = 3 \overset{\substack{\text{SCRIVIAMO RISPETTO} \\ \text{A } C = \{1, t, t^2\}}}{\Longleftrightarrow} rg \underbrace{\begin{pmatrix} 0 & 1 & 2 \\ -2 & 2 & 0 \\ 1 & 0 & -1 \end{pmatrix}}_{= M_A^C!} = 3$

$\begin{vmatrix} 0 & 1 & 2 \\ -2 & 2 & 0 \\ 1 & 0 & -1 \end{vmatrix} \underset{\text{SVILUPPO 3ª RIGA}}{=} \begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 1 \\ -2 & 2 \end{vmatrix} = -4 - 2 = -6 \neq 0$ ✓.

Analogamente, B base $\Leftrightarrow rg \underbrace{\begin{pmatrix} -1 & -1 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix}}_{= M_B^C!} = 3$.

$\begin{vmatrix} -1 & -1 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{vmatrix} = -\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} -1 & 0 \\ 1 & 2 \end{vmatrix} = -2 \neq 0$ ✓✓✓.

$M_A^B = M_C^B M_A^C = \left(M_B^C\right)^{-1} \cdot M_A^C$

$\left(\begin{array}{ccc|ccc} -1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_2 \to R_2 + R_1} \left(\begin{array}{ccc|ccc} -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_3 \to R_3 - R_2} \left(\begin{array}{ccc|ccc} -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 & 1 \end{array}\right)$

$\xrightarrow[R_2 \to R_2/2]{R_1 \to -R_1 - R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 & -1 & -1 & 1 \end{array}\right) \xrightarrow{R_2 \leftrightarrow R_3} \left(\begin{array}{ccc|ccc} 1 & & & 0 & 1 & -1 \\ & 1 & & -1 & -1 & 1 \\ & & 1 & \frac{1}{2} & \frac{1}{2} & 0 \end{array}\right)$, $\left(M_B^C\right)^{-1} = M_C^B$.

$\Rightarrow M_A^B = M_C^B M_A^C = \begin{pmatrix} 0 & 1 & -1 \\ -1 & -1 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 2 \\ -2 & 2 & 0 \\ 1 & 0 & -1 \end{pmatrix} = \begin{pmatrix} -3 & 2 & 1 \\ 3 & -3 & -3 \\ -1 & \frac{3}{2} & 1 \end{pmatrix}$ ✓✓.

(4) $\beta = \{ \underbrace{(1, 0, 1)}_{v_1}, \underbrace{(0, 1, -1)}_{v_2}, \underbrace{(0, 0, 2)}_{v_3} \} \subseteq \mathbb{Q}^3$

$\beta' = \{ \underbrace{(3, 1, 0)}_{w_1}, \underbrace{(-1, 0, 2)}_{w_2}, \underbrace{(0, 2, 0)}_{w_3} \} \subseteq \mathbb{Q}^3$.

$\exists! T : \mathbb{Q}^3 \to \mathbb{Q}^3$ t.c. $T(v_i) = w_i$ $\forall i = 1, 2, 3$? $M_\beta^\beta(T)$? $M_C^C(T)$? (C BASE CANONICA)

$\exists! T \Leftrightarrow B$ base di $\mathbb{Q}^3 \Leftrightarrow rg\, M_\beta^C = 3$.

$M_\beta^C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & 2 \end{pmatrix} \Rightarrow rg = 3$ ✓.

! ALLO STESSO MODO, NOTIAMO β' BASE DI $\mathbb{Q}^3$.

$M_\beta^{\beta'}(T) = I \Rightarrow M_\beta^\beta(T) = M_{\beta'}^\beta M_\beta^{\beta'}(T) = M_{\beta'}^\beta \cdot I = M_{\beta'}^\beta$

$\Rightarrow M_\beta^\beta(T) = M_{\beta'}^\beta = M_C^\beta M_{\beta'}^C = \left(M_\beta^C\right)^{-1} \cdot M_{\beta'}^C$

$M_C^C(T) = M_{\beta'}^C \underbrace{M_\beta^{\beta'}(T)}_{= I} M_C^\beta = M_{\beta'}^C M_C^\beta = M_{\beta'}^C \cdot \left(M_\beta^C\right)^{-1}$.

$M_{\beta'}^C = \begin{pmatrix} 3 & -1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix}$ $\left(M_\beta^C\right)^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & 2 \end{pmatrix}^{-1} \overset{(*)}{=} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

(*): $\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & -1 & 2 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_3 - R_1 + R_2 \over 2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{array}\right)$

$\Rightarrow M_\beta^\beta(T) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 3 & -1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 3 & -1 & 0 \\ 1 & 0 & 2 \\ 1 & \frac{3}{2} & 1 \end{pmatrix}$

$M_C^C(T) = \begin{pmatrix} 3 & -1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 3 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 2 & 0 \end{pmatrix}$

(5) $f : \mathbb{R}^3 \to \mathbb{R}^2$, $M_C^{C'}(f) = A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \end{pmatrix}$. (BASI CANONICHE)

$\beta = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$, $\beta' = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$. $M_\beta^{\beta'}(f)$?

$M_\beta^{\beta'}(f) = \underbrace{M_{C'}^{\beta'}}_{\left(M_{\beta'}^{C'}\right)^{-1}} M_C^{C'}(f) \cdot M_\beta^C = \underbrace{\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^{-1}}_{= \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

$\Rightarrow M_\beta^{\beta'}(f) = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 3 & 3 \end{pmatrix}$. ✓✓✓