

F06L108

① $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 7 & 2 & 6 & 5 & 1 & 4 \end{pmatrix} \in \Sigma_p$

FATTORIZZAZIONE IN CICLI (SIGNUM)?

$\sigma(1) = 3, \sigma(3) = 7, \sigma(7) = 1 \rightarrow (137)$
 $\sigma(2) = 8, \sigma(8) = 4, \sigma(4) = 2 \rightarrow (248)$
 $\sigma(5) = 6, \sigma(6) = 5 \rightarrow (56)$

$\Rightarrow \sigma = (137)(248)(56)$

σ COME PRODOTTO DI TRASPOSIZIONI?

$(137) = (17)(13), (248) = (28)(24)$

$\Rightarrow \sigma = (17)(13)(28)(24)(56)$

σ^{-1} ?

$\sigma^{-1} = ((17)(13)(28)(24)(56))^{-1} =$
 $= (56)^{-1}(24)^{-1}(28)^{-1}(13)^{-1}(17)^{-1} =$
 $= (56)(24)(28)(13)(17) =$
 $= (56)(284)(173)$

sgn(σ)?

2 METODI:
 - σ È PRODOTTO DI 5 TRASPOSIZIONI $\Rightarrow \text{sgn}(\sigma) = (-1)^5 = -1$ ✓
 - CONTARE LE INVERSIONI: $\sigma(1) > \sigma(2) \dots \sigma(2) > \sigma(3) \dots$ \rightarrow 17 INVERSIONI $\Rightarrow \text{sgn}(\sigma) = (-1)^{17} = -1$ ✓

(68) $\cdot \sigma \cdot (147)$

$= (68) \cdot (137)(284)(56)(147) = \tau$

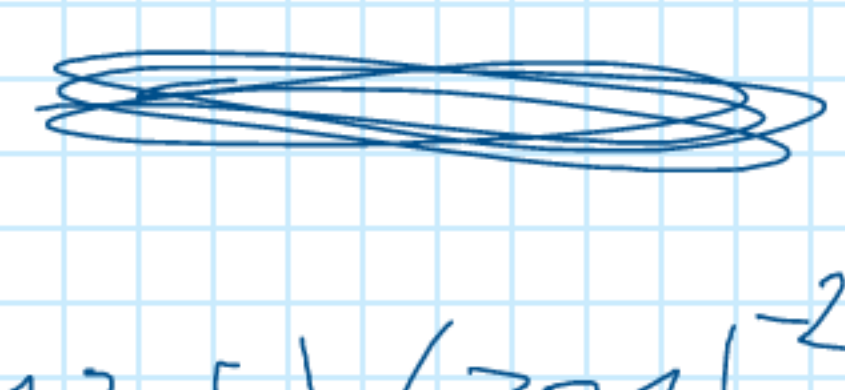
$\tau: 1 \xrightarrow{(147)} 4 \xrightarrow{(56)} 4 \xrightarrow{(284)} 2 \xrightarrow{(17)} 2 \xrightarrow{(68)} 2 \rightsquigarrow \tau(1) = 2$

$\tau: 2 \xrightarrow{(147)} 2 \xrightarrow{(56)} 2 \xrightarrow{(284)} 8 \xrightarrow{(17)} 8 \xrightarrow{(68)} 6 \rightsquigarrow \tau(2) = 6$

$\dots \rightarrow \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 6 & 7 & 1 & 8 & 5 & 3 & 4 \end{pmatrix}$
 $\rightarrow \tau = (126584)(37)$

σ^2

$= (137)(284)(56)(137)(284)(56) =$
 $= (137)(173)(284)(284)(56)(56) =$
 $= (173)(248)$



② $(135)(324)^{-2}(1235)$?

$(324)^{-2} = ((34)(32))^{-2} =$
 $= (32)(24)(342)$

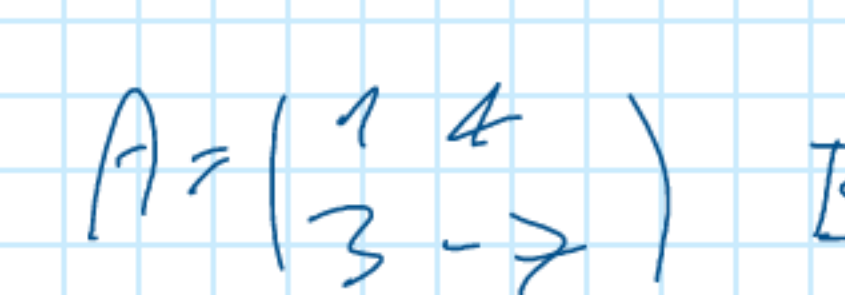
$(135)(342)(1235) =$
 $(153)(24) \checkmark$

③ $\sigma \in \Sigma_n$ K-CICLO $\Rightarrow \sigma^k = \text{id}$?

$\sigma = (i_2 \dots i_k)$
 $\Rightarrow \sigma(i_2) = i_2, \sigma^2(i_2) = \sigma(i_3) = i_3, \dots$
 $\rightarrow \forall i \in K \quad \sigma(i_2) = i_{2+3}$
 $\Rightarrow \sigma^k(i_2) = \sigma(\sigma^{k-2}(i_2)) = \sigma(i_k) = i_2 \checkmark$

POSSIAMO ANCHE FARE

$\sigma = (i_2 \dots i_k i_1) \rightarrow$ RIASCIAMO IL CICLO IN ORDINE
 $\rightarrow \sigma^k(i_2) = i_2 \rightarrow$ RIPETIAMO E $\sigma^k = \text{id}$ ✓



④ $A = \begin{pmatrix} 1 & 4 \\ 3 & -7 \end{pmatrix} \quad B = \begin{pmatrix} 2i & 1+3i \\ -1 & -1+i \end{pmatrix}$

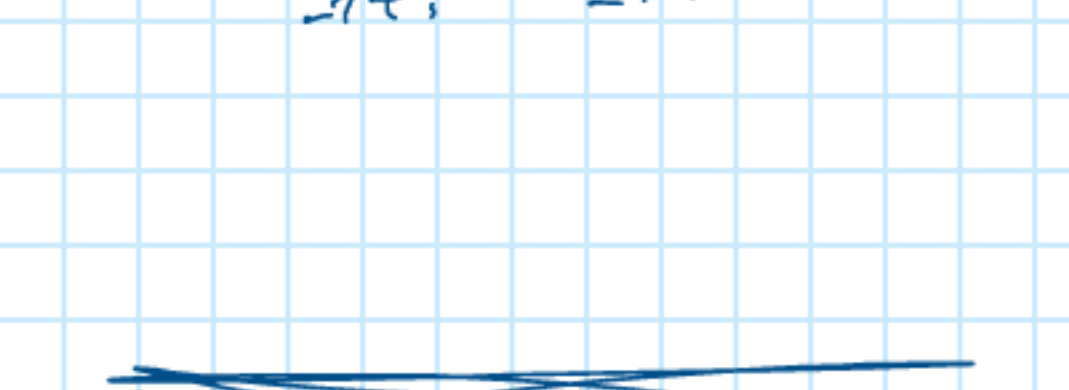
$\det(A) = \begin{vmatrix} 1 & 4 \\ 3 & -7 \end{vmatrix} = 1 \cdot (-7) - 3 \cdot 4 = -19$
 $\det(B) = \begin{vmatrix} 2i & 1+3i \\ -1 & -1+i \end{vmatrix} = (2i)(-1+i) - (-1)(1+i)$
 $= -2i - 2 + 1 + i = -1 - i$

$AB = \begin{pmatrix} -1+2i & -7+2i \\ 7+6i & 10+2i \end{pmatrix} \rightarrow \det(AB) = (-1+2i)(10+2i) - (7+6i)(-7+7i)$
 $= -10 - 2i + 20i - 4 = -14 + 18i$
 $= -19 - 19i = (-19)(-1-i) \checkmark$

$BA = \begin{pmatrix} 3+11i & -7-11i \\ -4+7i & 3-7i \end{pmatrix} \rightarrow \det(BA) = (3+11i)(3-7i) - (-4+7i)(-7-7i)$
 $= 9 - 21i + 33i + 77i^2 - 28 - 28i + 49i + 49i^2 = 19 - 19i = (-19)(-1+i) \checkmark$

$A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix} = \begin{pmatrix} 7/19 & 4/19 \\ 3/19 & -1/19 \end{pmatrix}$
 $\rightarrow \det A^{-1} = \frac{7}{19} \cdot \left(-\frac{1}{19}\right) - \frac{3}{19} \cdot \frac{4}{19} = -\frac{19}{19^2} = -\frac{1}{19} = \frac{1}{\det A} \checkmark$

$B^{-1} = \begin{pmatrix} \frac{-1-i}{-2-i} & \frac{1}{-1-i} \\ \frac{1+3i}{-1-i} & \frac{2i}{-2-i} \end{pmatrix} \rightarrow \det B^{-1} = \frac{(-1-i)(2i)}{(-1-i)^2} - \frac{-1 \cdot (1+3i)}{(-1-i)^2}$
 $= \frac{-2-2i}{-2-4i} - \frac{-1-3i}{-2-4i} = \frac{-2-2i+1+3i}{-2-4i} = \frac{-1+i}{-2-4i} = \frac{1}{-2-i} = \frac{1}{\det B} \checkmark$



⑤ $A(t) = \begin{pmatrix} t & -2 & -1 \\ -2 & 1 & 0 \\ 3 & 0 & t \end{pmatrix}$ $\text{rg } A(t)$? $\det A(t)$?
 $A(t)$ INVARIANTE?

$\det(A(t)) = \begin{vmatrix} t & -2 & -1 \\ -2 & 1 & 0 \\ 3 & 0 & t \end{vmatrix} = 3 \begin{vmatrix} -2 & -1 \\ 1 & 0 \end{vmatrix} + t \begin{vmatrix} t & -2 \\ -2 & 1 \end{vmatrix} =$

$= 3 \cdot (-2 \cdot 0 - 1 \cdot (-2)) + t(t \cdot 1 - (-2) \cdot (-2)) =$
 $= 6 + t(t - 4) = (t-1)(t-3)$

$\rightarrow \forall t \neq 1, 3 \quad \det(A(t)) \neq 0 \rightarrow A(t)$ INVERTIBILE, $\text{rg}(A(t)) = 3$

$\text{rg } A(1), A(3)$?

$\begin{pmatrix} 1 & -2 & -1 \\ -2 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix} \rightarrow$ PARME DUE RIGHE l.i. $\forall t \in \mathbb{R}$

$\Rightarrow \forall t \quad \text{rg } A(t) \geq 2 \Rightarrow \text{rg}(A(1)) = \text{rg}(A(3)) = 2 \checkmark$
 $\text{rg}(A(1)), \text{rg}(A(3)) < 3$

