mercoledì 16 dicembre 2020 14:14

FOGLIO 8

① $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 7 & 2 & 6 & 5 & 1 & 4 \end{pmatrix} \in \Sigma_8.$

FATTORIZZAZIONE IN CICLI DISGIUNTI?

$\sigma(1)=3,\ \sigma(3)=7,\ \sigma(7)=1 \to (137)$

$\sigma(2)=8,\ \sigma(8)=4,\ \sigma(4)=2 \to (248)$

$\sigma(5)=6,\ \sigma(6)=5 \longrightarrow (56)$

$\Rightarrow \sigma = (137)(248)(56).$

σ COME PRODOTTO DI TRASPOSIZIONI?

$(137) = (17)(13),\ (248) = (28)(24).$

$\Longrightarrow \sigma = (17)(13)(28)(24)(56).$

σ^{-1}?

$\sigma^{-1} = ((17)(13)(28)(24)(56))^{-1} =$

$= (56)^{-1}(24)^{-1}(28)^{-1}(13)^{-1}(17)^{-1} =$

$= (56)(24)(28)(13)(17) =$

$= (56)(284)(173).$

sgn(σ)?

2 METODI:
- σ È PRODOTTO DI 5 TRASPOSIZIONI $\Rightarrow sgn(\sigma) = (-1)^5 = -1.$ ✓
- CONTIAMO LE INVERSIONI: $\#\{(i,j) \mid i<j,\ \sigma(i)>\sigma(j)\}$. $\sigma(1)>\sigma(4)$, $\sigma(2)>\sigma(3)$, $\sigma(1)>\sigma(7)$, $\sigma(2)>\sigma(3)$ … $\to$ 17 INVERSIONI $\Rightarrow sgn(\sigma) = (-1)^{17} = -1$ ✓

$(68)\cdot\sigma\cdot(147)$

$= (68)\cdot(137)(284)(56)(147) =: \tau.$

$\tau: 1 \xmapsto{(147)} 4 \xmapsto{(56)} 4 \xmapsto{(284)} 2 \xmapsto{(137)} 2 \xmapsto{(68)} 2 \rightsquigarrow \tau(1)=2$

$\tau: 2 \xmapsto{(147)} 2 \xmapsto{(56)} 2 \xmapsto{(284)} 8 \xmapsto{(137)} 8 \xmapsto{(68)} 6 \rightsquigarrow \tau(2)=6$

$\cdots \longrightarrow \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 6 & 7 & 1 & 8 & 5 & 3 & 4 \end{pmatrix}$

$\longrightarrow \tau = (12658 4)(37).$

σ^2

$= (137)(284)(56)(137)(284)(56) \overset{\text{CICLI DISGIUNTI}}{=}$

$= (137)(137)(284)(284)(56)(56)$

$= (173)(248).$

② $(135)(324)^{-2}(1235)$?

$(324)^{-2} = ((34)(32))^{-2} =$

$= (32)(24) = (342).$

$(135)(342)(1235) =$

$(153)(24)$ ✓.

③ $\sigma \in \Sigma_n$ K-CICLO $\Rightarrow \sigma^K = id$?

$\sigma = (i_1 \text{———} i_K).$

$\Rightarrow \sigma(i_1) = i_2,\ \sigma^2(i_1) = \sigma(i_2) = i_3 \ldots$

$\longrightarrow \forall j<K \quad \sigma^j(i_1) = i_{1+j}.$

$\Rightarrow \sigma^K(i_1) = \sigma(\sigma^{K-1}(i_1)) = \sigma(i_K) = i_1$ ✓.

POSSIAMO ANCHE SCRIVERE

$\sigma = (i_2 \text{———} i_K\ i_1) \longrightarrow$ RIPETIAMO IL RAGIONAMENTO

$\longrightarrow \sigma^K(i_2) = i_2 \rightsquigarrow$ RIPETIAMO E $\sigma^K = id$ ✓.

④ $A = \begin{pmatrix} 1 & 4 \\ 3 & -7 \end{pmatrix} \quad B = \begin{pmatrix} 2i & 1+3i \\ -1 & -1+i \end{pmatrix}.$

$\det(A) = \begin{vmatrix} 1 & 4 \\ 3 & -7 \end{vmatrix} = -7 - 12 = -19$

$\det(B) = \begin{vmatrix} 2i & 1+3i \\ -1 & -1+i \end{vmatrix} = (2i)(-1+i) - (-1)\cdot(1+3i)$

$= -2i - 2 + 1 + 3i = -1 + i.$

$AB = \begin{pmatrix} -4+2i & -3+7i \\ 7+6i & 10+2i \end{pmatrix} \to \det(AB) = (-4+2i)(10+2i) - (7+6i)(-3+7i)$

$= -40 - 8i + 20i - 4 + 21i - 49i + 18i + 42$

$= 19 - 19i = (-19)(-1+i)$ ✓.

$BA = \begin{pmatrix} 3+11i & -7-13i \\ -4+3i & 3-7i \end{pmatrix} \to \det(BA) = (3+11i)(3-7i) - (-4+3i)(-7-13i)$

$= 9 - 21i + 33i + 77i - 28 - 52i + 21i - 39$

$= 19 - 19i = (-19)(-1+i)$ ✓

$A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix} = \begin{pmatrix} 7/19 & 4/19 \\ 3/19 & -1/19 \end{pmatrix}$

$\longrightarrow \det A^{-1} = \frac{7}{19}\cdot\left(-\frac{1}{19}\right) - \frac{3}{19}\cdot\frac{4}{19} = -\frac{19}{19^2} = -\frac{1}{19} = \frac{1}{\det(A)}$ ✓.

$B^{-1} = \begin{pmatrix} \frac{-1+i}{-1+i} & \frac{-1}{-1+i} \\ \frac{1+3i}{-1+i} & \frac{2i}{-1+i} \end{pmatrix} \to \det B^{-1} = \frac{(-1+i)(2i)}{(-1+i)^2} - \frac{-1\cdot(1+3i)}{(-1+i)^2} =$

$= \ldots = \frac{1}{-1+i} = \frac{1}{\det B}$ ✓.

⑤ $A(t) = \begin{pmatrix} t & -2 & -1 \\ -2 & 1 & 0 \\ 3 & 0 & t \end{pmatrix}.$ rg $A(t)$? det $A(t)$? $A(t)$ INVERTIBILE?

$\det(A(t)) = \begin{vmatrix} t & -2 & -1 \\ -2 & 1 & 0 \\ 3 & 0 & t \end{vmatrix} \underset{\text{SVILUPPO 3ª RIGA}}{=} 3\begin{vmatrix} -2 & -1 \\ 1 & 0 \end{vmatrix} + t\begin{vmatrix} t & -2 \\ -2 & 1 \end{vmatrix} =$

$= 3\cdot(-2\cdot 0 - 1\cdot(-1)) + t(t\cdot 1 - (-2)\cdot(-2)) =$

$= t^2 - 4t + 3 = (t-1)(t-3).$

$\longrightarrow \forall t \neq 1, 3 \quad \det(A(t)) \neq 0 \rightsquigarrow A(t)$ INVERTIBILE, $rg(A(t)) = 3.$

rg $A(1)$, $A(3)$?

$\begin{pmatrix} t & -2 & -1 \\ -2 & 1 & 0 \\ 3 & 0 & t \end{pmatrix} \to$ PRIME DUE RIGHE l.i. $\forall t \in \mathbb{R}$

$\Longrightarrow \forall t$ rg $A(t) \geq 2 \Longrightarrow rg(A(1)) = rg(A(3)) = 2.$ ✓✓

($rg(A(1)), rg(A(3)) < 3.$)