$$f(x) = \ln\left(\frac{2x+5}{x-4}\right)$$

1) DOMINIO

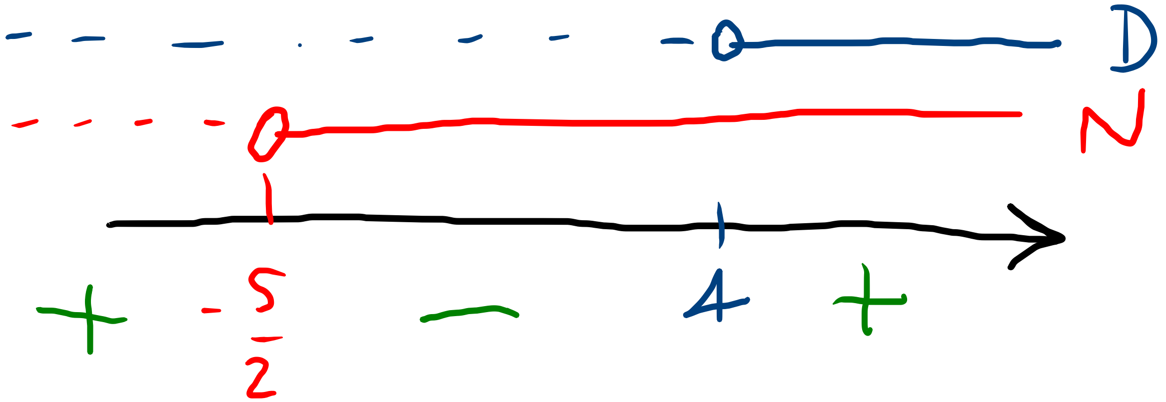
$$\begin{cases} x-4 \neq 0 \\ \dfrac{2x+5}{x-4} > 0 \end{cases} \qquad \begin{cases} x \neq 4 \\ N: \ 2x+5 > 0 \qquad x > -\dfrac{5}{2} \\ D: \ x-4 > 0 \qquad x > 4 \end{cases}$$

DOMINIO

$\dfrac{N}{D} \Rightarrow D \neq 0$

$\sqrt{A} \Rightarrow A \geq 0$

$\ln(B) \Rightarrow B > 0$



$$\Rightarrow D_f = \left]-\infty; -\frac{5}{2}\right[\cup \left]4; +\infty\right[$$

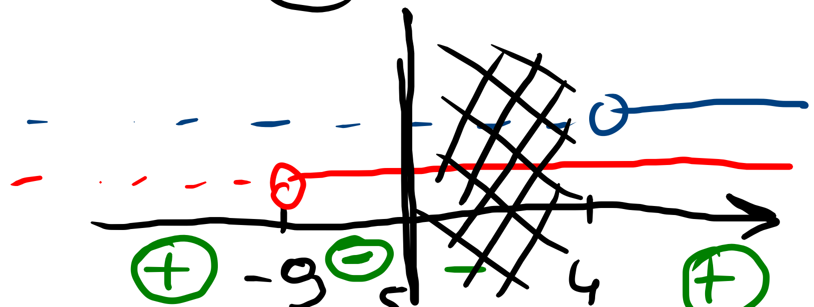
2) SEGNO:

$$\boxed{\ln 1 = 0} \qquad e^{\ln x} = x$$

$$\ln\left(\frac{2x+5}{x-4}\right) > 0$$

$$\ln\left(\frac{2x+5}{x-4}\right) > \ln 1 \Rightarrow e^{\ln\left(\frac{2x+5}{x-4}\right)} > e^{\ln 1}$$

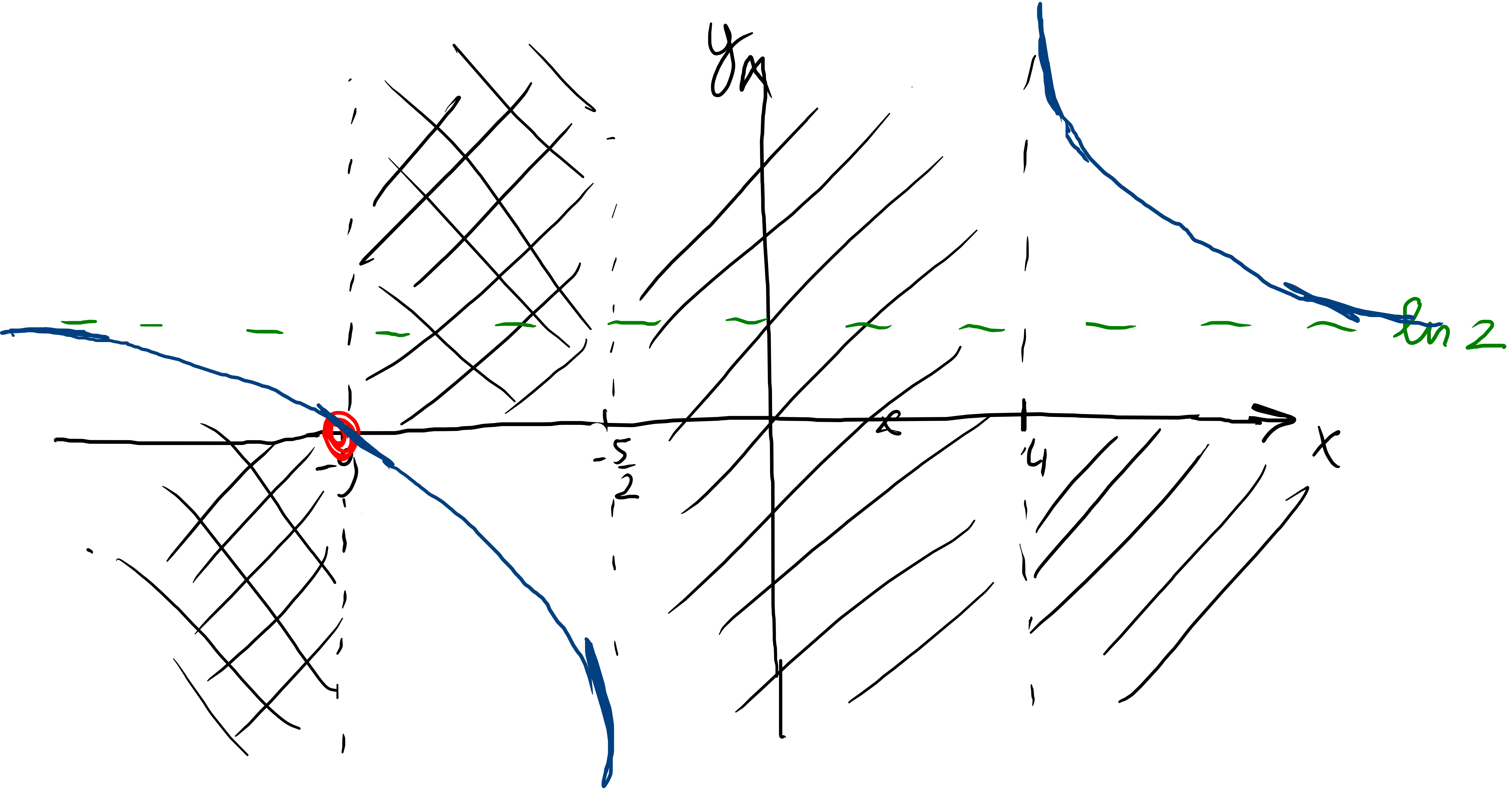
$$\frac{2x+5}{x-4} > 1 \Rightarrow \frac{2x+5}{x-4} - 1 > 0$$



$$\frac{2x+5-x+4}{x-4} > 0 \qquad \frac{x+9}{x-4} > 0$$

N: $x+9>0$ $\quad x>-9$

D: $x-4>0$ $\quad x>4$



y
x
ln 2
-9
$-\frac{5}{2}$
e
4

3) INTERSEZIONI CON GLI ASSI

Non ha senso calcolare l'intersezione con l'asse y

Int. asse x

$$\begin{cases} y = \ln\left(\frac{2x+5}{x-4}\right) \\ y = 0 \end{cases} \Rightarrow \begin{cases} 0 = \ln\left(\frac{2x+5}{x-4}\right) \\ y = 0 \end{cases}$$

$$\begin{cases} 1 = \frac{2x+5}{x-4} \\ y = 0 \end{cases} \quad \begin{cases} \frac{x+9}{x-4} = 0 \\ y = 0 \end{cases} \quad \begin{cases} x = -9 \\ y = 0 \end{cases}$$

Intersezione

$(-9; 0)$

4) STUDIO NEI PUNTI PARTICOLARI

In questo caso $\pm\infty$; $+4$; $-\frac{5}{2}$

$$\lim_{x\to+\infty} \ln\left(\frac{2x+5}{x-4}\right) = \lim_{x\to+\infty} \ln\left[\frac{\cancel{x}\left(2+\frac{5}{x}\right)}{\cancel{x}\left(1-\frac{4}{x}\right)}\right] = \ln(2)$$

$$\lim_{x\to-\infty} \ln\left(\frac{2x+5}{x-4}\right) = \ln(2)$$

ASINTOTO ORIZZONTALE

$y = \ln(2) \simeq 0{,}6$

$$\lim_{x\to 4^+} \ln\left(\frac{2x+5}{x-4}\right) = \ln(+\infty) = +\infty$$

$$\lim_{x\to -\frac{5}{2}^-} \ln\left(\frac{2x+5}{x-4}\right) = \ln(0) = -\infty$$

5) STUDIO DELLA DERIVATA PRIMA

$$f'(x) = \frac{x-4}{2x+5} \cdot \left[\frac{2 \cdot (x-4) - (2x+5) \cdot 1}{(x-4)^2} \right]$$

DERIVATA LOGARITMO

DERIVATA ARGOMENTO

$$f(x) = \ln\left(\frac{2x+5}{x-4}\right)$$

$$= \frac{\cancel{x-4}}{2x+5} \cdot \frac{\cancel{2x}-8-\cancel{2x}-5}{(x-4)^{\cancel{2}}} = \frac{-13}{(2x+5)(x-4)}$$

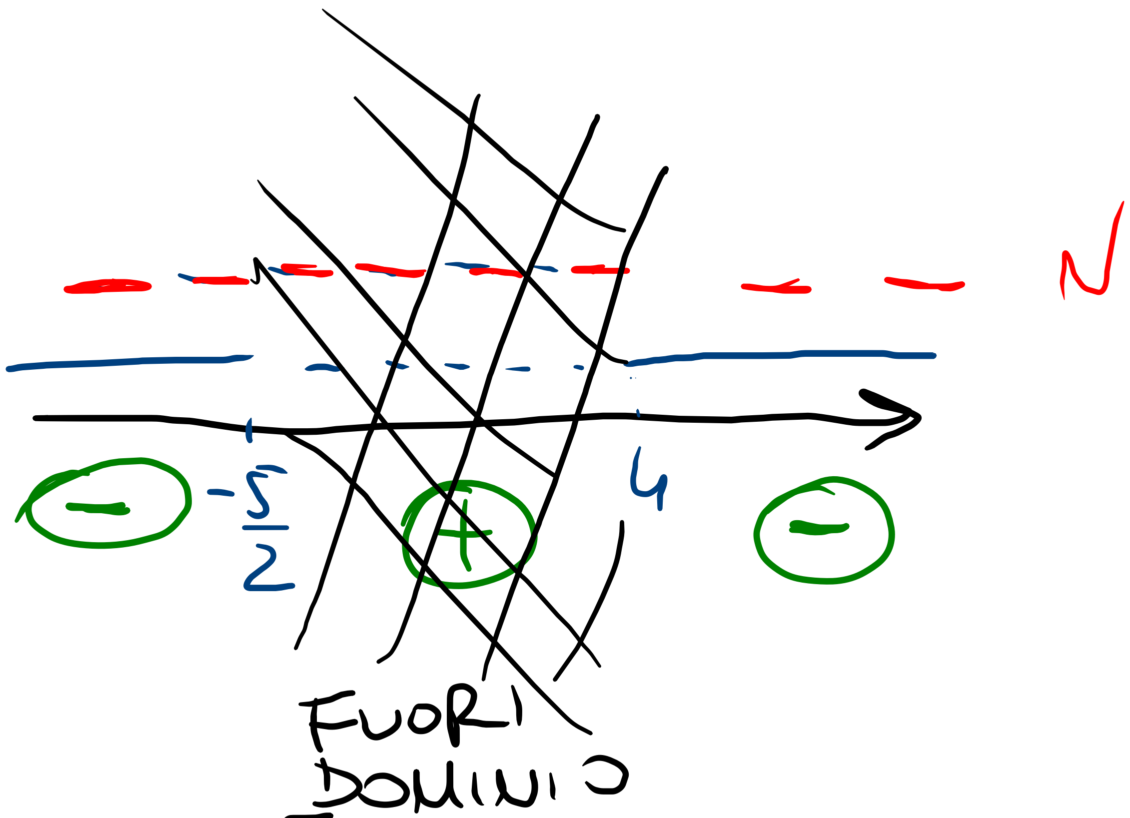
$$f'(x) = 0 \Rightarrow \frac{-13}{(2x+5)(x-4)} = 0 \Rightarrow \text{IMPOSSIBILE} \Rightarrow$$

NESSUN PUNTO STAZION.

$$f'(x) > 0 \qquad \frac{-13}{(2x+5)(x-4)} > 0$$

N: $-13 > 0$ MAI

D: $(2x+5)(x-4) > 0$



$f(x)$ cresce MAI

$f(x)$ decresce $\left]-\infty; -\frac{5}{2}\right[\cup \left]+4; +\infty\right[$

$$f''(x) = \frac{0 \cdot \ldots - (-13) \cdot (4x-3)}{(2x+5)^2\,(x-4)^2}$$

$$f'(x) = \frac{-13}{(2x+5)(x-4)}$$

$$2x^2 - 3x - 20$$

$$= \frac{13(4x-3)}{(2x+5)^2(x-4)^2}$$

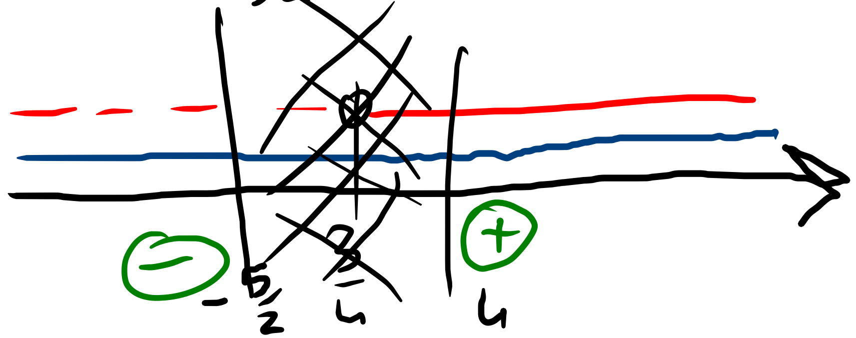
$f''(x) = 0 \Rightarrow 4x - 3 = 0 \qquad x = \frac{3}{4}$ FUORI DOMINIO

$f''(x) > 0 \qquad \dfrac{13(4x-3)}{(2x+5)^2(x-4)^2} > 0$

N: $13(4x-3) > 0 \qquad x > \frac{3}{4}$

D: sempre positivo

FUORI DOMINIO



concavità negativa (CONCAVA): $\left]-\infty ; -\frac{5}{2}\right[$

concavità positiva (CONVESSA): $\left]4 ; +\infty\right[$

$$f(x) = x^3 \cdot e^{-x^3}$$

$\Rightarrow$ Dominio

$D_f = \mathbb{R}$ $\quad$ $I: \,]-\infty; +\infty[$

$\Rightarrow$ né PARI né DISPARI

$f(x) = f(-x)$ PARI

$f(x) = -f(-x)$ DISPARI

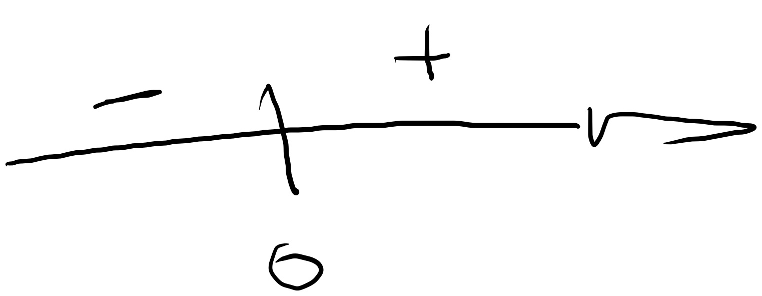
$\Rightarrow$ SEGNO

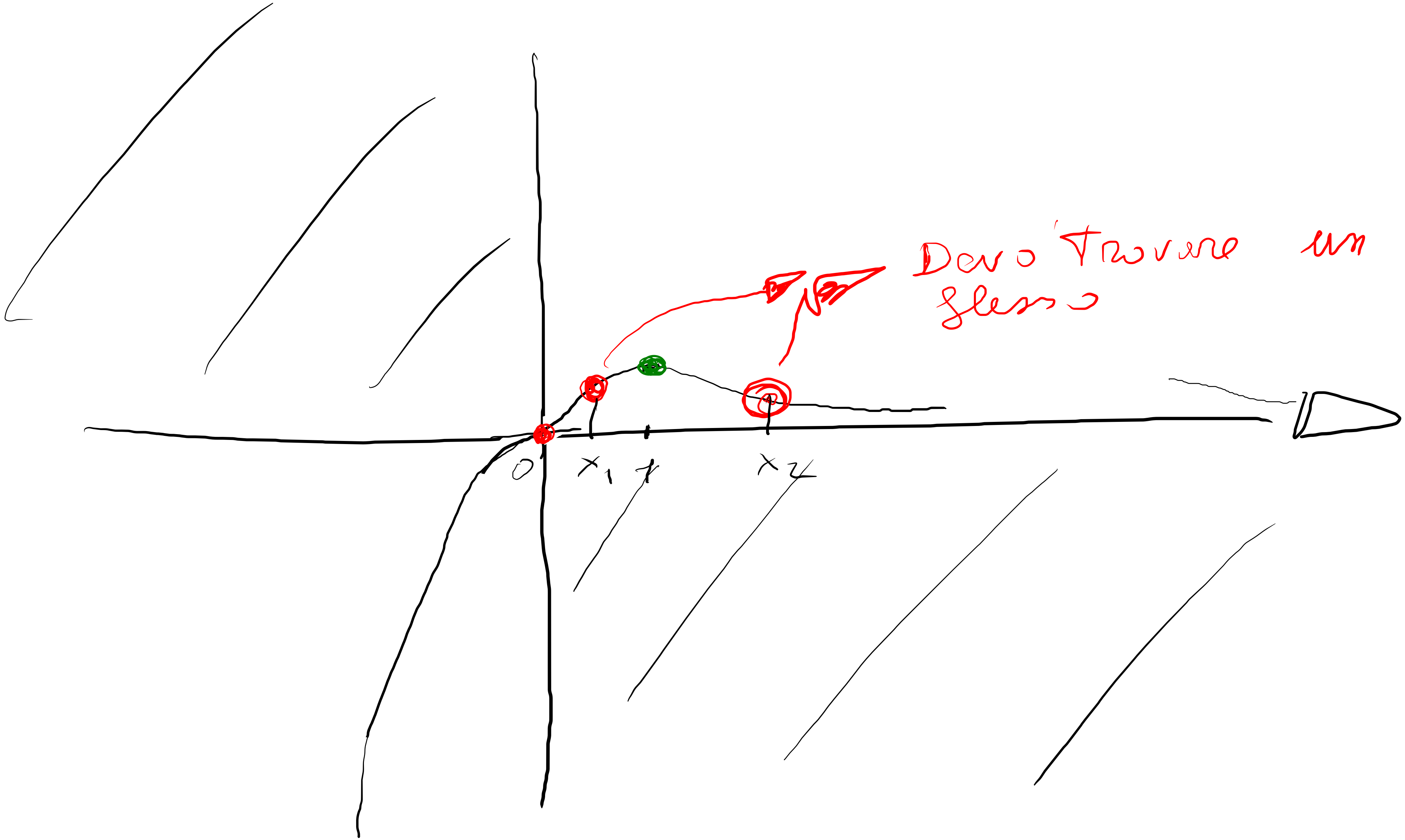
$$x^3 \cdot e^{-x^3} \geq 0$$

$e^{-x^3} > 0 \quad \forall \mathbb{R}$

$x^3 > 0 \quad x > 0$

$x^3 < 0 \quad x < 0$




Devo trovare un flesso
0
x1
1
x2

$\Rightarrow$ INTERSEZIONI

Intersezione in $P(0,0)$

$\Rightarrow$ PUNTI PARTICOLARI $\pm\infty$

$$\lim_{x\to+\infty} x^3 \cdot e^{-x^3} = 0$$

$$\lim_{x\to-\infty} x^3 \cdot e^{-x^3} = (-\infty)\cdot(+\infty) = -\infty$$

$\Rightarrow$ DERIVATA PRIMA

$$f'(x) = 3x^2 \cdot (e^{-x^3}) + x^3 (e^{-x^3}) \cdot (-3x^2) =$$

$$= 3x^2 e^{-x^3}(1-x^3)$$

$$f'(x) = 0$$

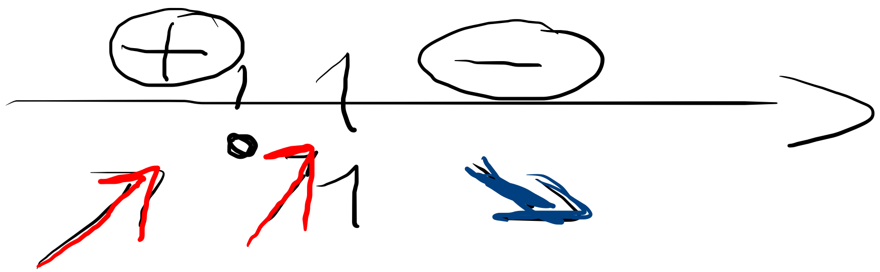
$$3x^2 \cdot e^{-x^3}(1-x^3) = 0 \quad \begin{cases} x = 0 \\ x = 1 \end{cases}$$

$$f'(x) > 0 \qquad \underbrace{3x^2 \cdot e^{-x^3}}_{>0\ \forall \mathbb{R}}(1-x^3) > 0$$

$$1 - x^3 > 0$$

$$1 > x^3$$

$$x < 1$$



$x < 1 \vee x \neq 0$ f crescente

$x > 1$ f decrescente

$x = 0 \Rightarrow$ flesso a tangente orizzontale

$x = 1 \Rightarrow$ punto di MASSIMO ASSOLUTO

$\Rightarrow$ DERIVATA SECONDA

$$f'(x) = 3x^2 \cdot e^{-x^3}(1-x^3) = 3e^{-x^3}(x^2-x^5)$$

$$f''(x) = 3e^{-x^3}(-3x^2)(x^2-x^5) +$$
$$+ 3e^{-x^3}(2x-5x^4) =$$

$$= 3e^{-x^3}x(-3x^3+3x^6+2-5x^3)$$
$$= 3e^{-x^3}\cdot x(3x^6-8x^3+2)$$

$$f''(x) = 0 \begin{cases} \Rightarrow x = 0 \\ \Rightarrow 3t^2-8t+2=0 \end{cases}$$

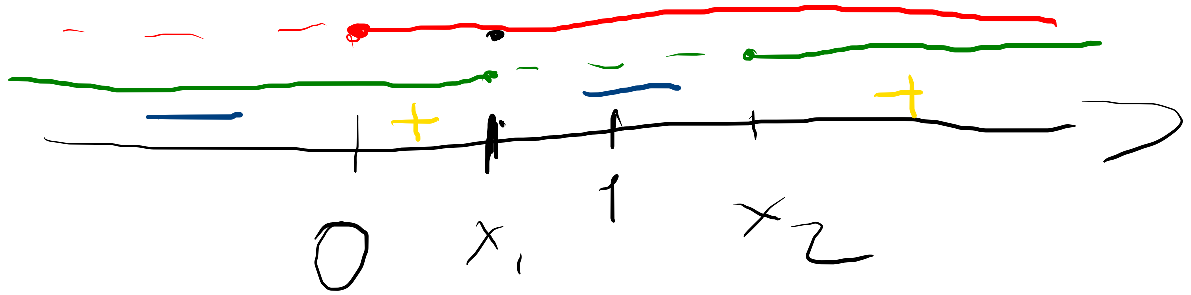
$$\frac{\Delta}{4} = 16-6 = \sqrt{10}$$

$$t_{1,2} = \frac{4 \pm \sqrt{10}}{3} \begin{cases} t_1 \\ \end{cases}$$

$$t = x^3 \begin{cases} x_1 = \sqrt[3]{\dfrac{4-\sqrt{10}}{3}} \cong 0,65 \\ x_2 = \sqrt[3]{\dfrac{4+\sqrt{10}}{3}} \cong 1,34 \end{cases}$$

$$f''(x) > 0$$

$$3e^{-x^3}x\left(3x^6-8x^3+2\right) > 0$$



$]-\infty, 0[$ concavità neg.

$]0, x_1[$ concavità pos.

$]x_1, x_2[$ concavità neg

$]x_2, +\infty[$ concavità pos.

$x = 0 \Rightarrow$ flesso ascendente

$x = x_1 \Rightarrow$ flesso disc.

$x = x_2 \Rightarrow$ flesso asc.