Mathematics Class

December 18, 2020

Remark. Some of the following exercises are extracted from written exams of the past years.

Exercise 1. Solve the following equations or inequalities:

$a)\ 3^x \cdot 3^{2x-1} = \dfrac{3^2}{\sqrt{3}}$ $\quad b)\ \log_2(x+4) - 1 = \dfrac{1}{2}$

$c)\ \dfrac{(|x|-7)\log(-x)}{x^2+1} = 0$ $\quad d)\ \dfrac{e^{4|x|-6} - 2}{\sqrt{x}} = 0$

$e)\ \dfrac{|x|+1}{\log(x-3)} < 0$ $\quad f)\ \log_{\frac{1}{3}}(4 - 9x^2) \geq -1$

$g)\ \log|x-1| < \log x + \log 2$ $\quad h)\ \sqrt{\log_{\frac{1}{2}}(x-1)} \geq 0$

$i)\ \log_2 x + \log_8 x < 1$ $\quad j)\ \log_3|3-2x| \geq -\log_{\frac{1}{3}}(1+x^2)$

$k)\ 2^{|x+2|} \leq 32 \cdot 2^{2x}$ $\quad l)\ 3^{\frac{x^2-1}{|x+2|}} < 9$

Exercise 2. Determine the domain of the following functions:

$a)\ f(x) = \log_4(x^2+1)$ $\quad b)\ f(x) = \log\left(\dfrac{x^2+4}{x}\right)$ $\quad c)\ f(x) = \sqrt{\dfrac{e^{x+2}}{x^3 - x}}$

$d)\ f(x) = \log(1 - \log x)$ $\quad e)\ f(x) = \log_{\frac{1}{2}}\left(\dfrac{x}{\sqrt{x}-2}\right)$ $\quad f)\ f(x) = \dfrac{e^{-x}}{\cos(2x)}$

$g)\ f(x) = e^{\frac{x(x+2)}{(x+1)^3}}$ $\quad h)\ f(x) = \dfrac{1}{2^{x-4} - 7}$ $\quad i)\ f(x) = \dfrac{4 + \cos x}{(3 - e^{2x})^4}$

Exercise 3. The following limits are indeterminate forms. Compute them by applying suitable relevant limits.

$a)\ \lim\limits_{x\to-\infty} \dfrac{x^5 + 2x^3 + 1}{x^2 + 7x + 4}$ $\quad b)\ \lim\limits_{x\to-\infty} \dfrac{x^2 + \sin x}{x^2 + 2x - 5}$ $\quad c)\ \lim\limits_{x\to+\infty} \dfrac{x^2+1}{5^x}$

$d)\ \lim\limits_{x\to+\infty} (x^3 + 3x)2^{-x}$ $\quad e)\ \lim\limits_{x\to+\infty} (x^2 + 3^x - \log_3 x)3^{-x}$ $\quad f)\ \lim\limits_{x\to0} (1+x)^{\frac{1}{2x}}$

$g)\ \lim\limits_{x\to0} \dfrac{\sin(5x)}{7x}$ $\quad h)\ \lim\limits_{x\to0} \dfrac{\sqrt{4 - x + 3x^2} - 2 + \sin(2x)}{3x}$ $\quad i)\ \lim\limits_{x\to0} \dfrac{\sin(x^2+x)}{x}$

$j)\ \lim\limits_{x\to0^+} \dfrac{1 - \cos\sqrt{x}}{x}$ $\quad k)\ \lim\limits_{x\to0^+} \sin x \log_6 x$ $\quad l)\ \lim\limits_{x\to0} \dfrac{\sin(3x)}{\log(1+x)}$

$m)\ \lim\limits_{x\to1} \dfrac{\sin(\sqrt{x}-1)}{x-1}$ $\quad n)\ \lim\limits_{x\to0} \dfrac{x\sin x}{\cos x - 1}$ $\quad o)\ \lim\limits_{x\to0} \dfrac{e^x - 1}{\sqrt{1 - \cos x}}$

$p)\ \lim\limits_{x\to0} \dfrac{e^{x\sin x} - 1}{1 - \cos x}$ $\quad q)\ \lim\limits_{x\to0} \dfrac{e^{\sin^2 x} - 1}{1 - \sqrt{x^2+1}}$ $\quad r)\ \lim\limits_{x\to0} \dfrac{\log(\cos x)}{x^2}$

$s)\ \lim\limits_{x\to-\infty} \log\left(1 - \dfrac{1}{x}\right)^x$ $\quad t)\ \lim\limits_{x\to+\infty} \dfrac{\sqrt{x^2-3} - 4^x}{7x+1}$ $\quad u)\ \lim\limits_{x\to\frac{\pi}{4}} \dfrac{1 - \tan x}{\sin x - \cos x}$

$v)\ \lim\limits_{x\to1} \dfrac{e^{\cos(x-1)} - e}{1 - \cos^2(x-1)}$ $\quad w)\ \lim\limits_{x\to0} \dfrac{e^{e^x} - e}{\log(x+1)}$ $\quad x)\ \lim\limits_{x\to+\infty} \dfrac{e^{\frac{1}{x}} - 1}{\sin\left(\frac{2}{x}\right)}$