

Fluid flow through Real pipes



Pump-house

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<http://www.library.ucsb.edu/internal/libwaves/apr04/sempumhouse.html>

Introduction

The flow through most pipes is turbulent. Treatment with classical analytic techniques next to impossible.

Available techniques are basic on experimental data and empirical formulae. The working equations are often derived from dimensional analysis using dimensionless forms.

Often desirable to determine the head loss, h_L so that the energy equation can be used. Pipe systems come with valves, bends, pipe diameter changes, elbows which also contribute to the energy (head) loss.

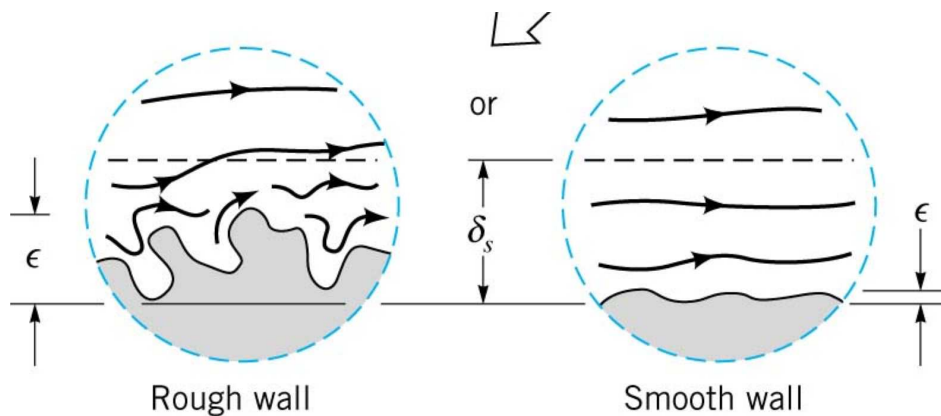
The overall head loss is divided into two parts *major loss* h_{Lmajor} , and *minor loss* h_{Lminor} . The major loss comes from viscosity (in straight pipe) while the minor loss is due to energy loss in the components.

The major loss can actually be smaller than the minor loss for a pipe system containing short pipes and many bends and valves.

Major Losses

The pressure loss in a pipe for turbulent flow depends on the following

- ρ
- μ
- v , l and D
- Surface roughness ϵ . These projections of the wall can and protrude out of the laminar sub-layer.



SO

$$\Delta p = F(v, D, l, \epsilon, \mu, \rho)$$

Dimensional analysis

$$\Delta p = F(v, D, l, \varepsilon, \mu, \rho)$$

Dimensions

$$\dim(l)=L \quad \dim(D)=L \quad \dim(\varepsilon)=L$$

$$\dim(v)=LT^{-1} \quad \dim(\mu)=MT^{-1}L^{-1} \quad \dim(\rho)=ML^{-3}$$

There are 3 basic dimensions, number of Π terms is $k - r = 7 - 3 = 4$.

Will choose ρ , v and D as the repeating variables.
Want Δp to be subject of equation.

$$\Pi_1 = \Delta p D^a v^b \rho^c$$

$$\begin{aligned} \dim(\Pi_1) &= (ML^{-1}T^{-2})L^a(LT^{-1})^b(ML^{-3})^c \\ &= M^{1+c}L^{-1+a+b-3c}T^{-2-b} \end{aligned}$$

Solving equations give $a = 0$, $b = -2$, $c = -1$ so the dimensionless group is

$$\Pi_1 = \frac{\Delta p}{v^2 \rho}$$

Dimensional analysis

The dimensionless group for l is $\Pi_2 = \frac{l}{D}$.

The dimensionless group for ε is $\Pi_3 = \frac{\varepsilon}{D}$. Now do viscosity μ

$$\begin{aligned}\Pi_4 &= \mu D^a v^b \rho^c \\ \dim(\Pi_4) &= (ML^{-1}T^{-1})L^a(LT^{-1})^b(ML^{-3})^c \\ &= M^{1+c}L^{-1+a+b-3c}T^{-1-b}\end{aligned}$$

solving gives $a = -1$, $b = -1$, $c = -1$ and the dimensionless group is

$$\Pi_4 = \frac{\mu}{D\rho v}$$

The representation for the relation is

$$\begin{aligned}\frac{\Delta p}{\frac{1}{2}v^2\rho} &= \phi\left(\frac{\mu}{D\rho v}, \frac{l}{D}, \frac{\varepsilon}{D}\right) \\ &= \phi\left(\text{Re}, \frac{l}{D}, \frac{\varepsilon}{D}\right)\end{aligned}$$

(Note, argument can be **Re** or $1/\text{Re}$)

The pressure loss equation

$$\frac{\Delta p}{\frac{1}{2}v^2\rho} = \phi \left(\text{Re}, \frac{l}{D}, \frac{\varepsilon}{D} \right)$$

Impose the condition that $\Delta p \propto l$ (this is very reasonable and supported by experiments), so

$$\frac{\Delta p}{\frac{1}{2}v^2\rho} = \frac{l}{D} \phi \left(\text{Re}, \frac{\varepsilon}{D} \right)$$

The left hand side of this equation is just the Darcy friction factor so

$$\begin{aligned} \Delta p &= f \frac{1}{2}v^2\rho \frac{l}{D} \\ f &= \phi \left(\text{Re}, \frac{\varepsilon}{D} \right) \end{aligned}$$

This is the parametrization applied to analysis of turbulent pipe flows. The friction factor depends on the Reynolds number and the surface roughness in a complicated way and practical usage relies the synthesis of exhaustive experiments as charts or empirical formulae (for laminar flow $f = 64/\text{Re}$).

The head loss

$$\frac{p_1}{\gamma} + \alpha_1 \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2 \frac{v_2^2}{2g} + z_2 + h_L$$

Suppose the pipe-diameter is constant, then $v_1 = v_2$, and the pipe is horizontal so $z_1 = z_2$, and the flow is fully developed so $\alpha_1 = \alpha_2$, then

$$\frac{p_1}{\gamma} = \frac{p_2}{\gamma} + h_L \Rightarrow \Delta p = \gamma h_L$$

Then

$$h_L = \frac{\Delta p}{\gamma} = f \frac{1}{2} v^2 \frac{\rho}{\gamma} \frac{l}{D} = f \frac{v^2 l}{2gD}$$

This is called the Darcy-Weisbach equation.

Although derived for $z_1 = z_2$, it is valid for pipes with vertical drops since the working equation is for a change in head. Consider pressure change

$$p_1 - p_2 = \gamma(z_2 - z_1) + \gamma h_L = \gamma(z_2 - z_1) + f \frac{\rho v^2 l}{2D}$$

Part of the pressure change is due to elevation changes while part is due to the friction factor.

The friction factor

$$h_{Lmajor} = f \frac{v^2 l}{2gD}$$

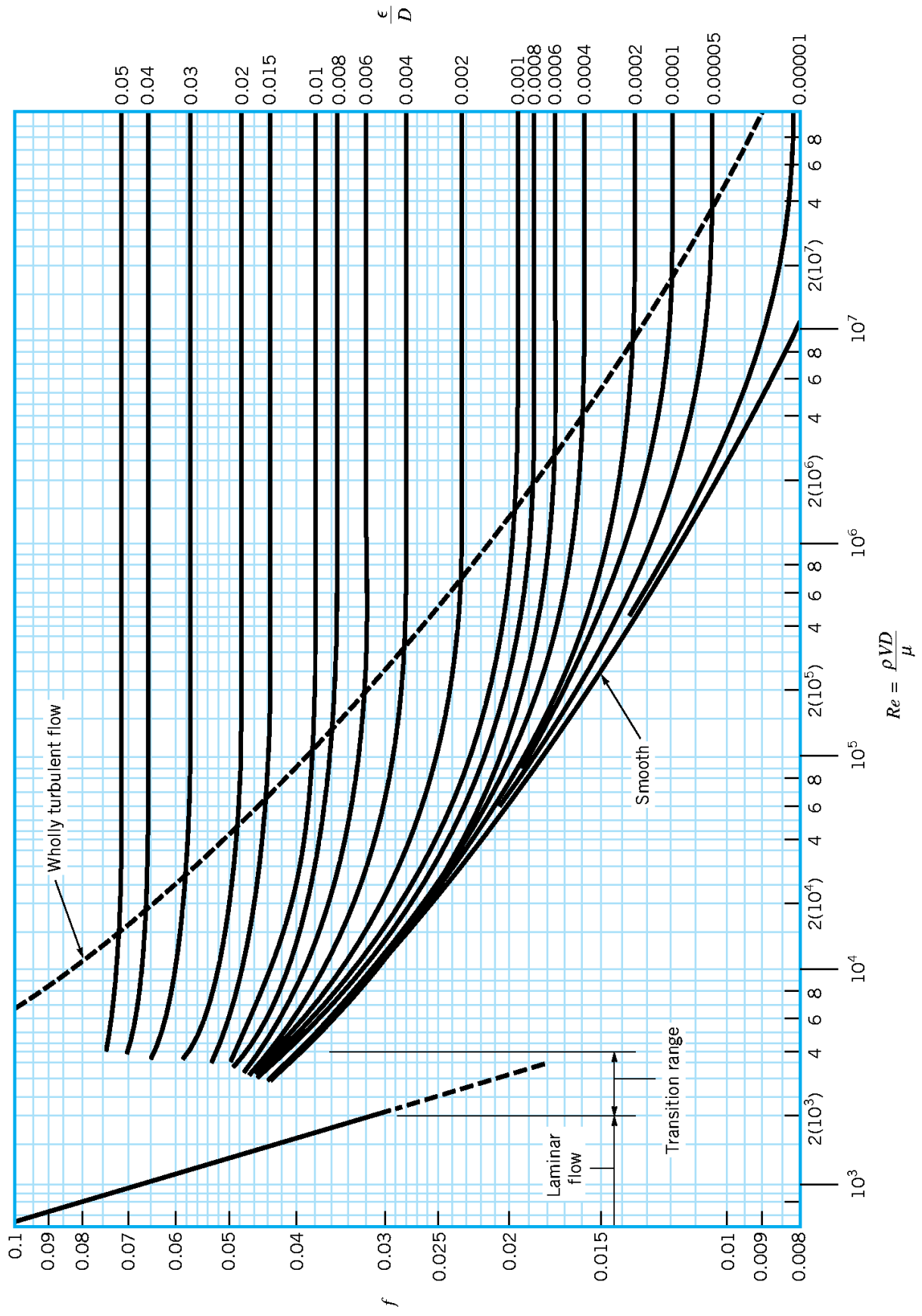
The friction factor was determined by comprehensive experiments on firstly tubes with sand grain glued inside and experiments on typical commercial pipes.

The information is summarized by

- a The Moody chart
- b Tables of surface roughness for pipes of various materials (note the tabulated roughness ε is not exactly equal to roughness determined by geometric inspection).

Note, tables of surface roughness pertain to new clean pipes. After usage, corrosion or a buildup of scale can make the roughness larger (by a factor of 10!). Very old pipes may have their diameter changed.

The Moody chart



Surface roughness table

Material	ε mm
Riveted Steel	0.90-9.0
Concrete	0.30-3.0
Wood stave	0.18-0.90
Cast iron	0.26
Galvanized iron	0.15
Commercial steel	0.045
Drawn tubing	0.0015
Plastic, glass	0.0

There are tabulations in the Handbook of fluid dynamics

The Moody chart

- Laminar flow $f = 64/\text{Re}$ independent of ε .
- The friction factor is non-zero even when $\varepsilon = 0$.
- For large Re , one finds f independent of Re . This is called completely turbulent flow. The Laminar sub-layer is so thin it has no impact.
- No data on diagram $2000 < \text{Re} < 4000$. Can find an unsteady mix of both flows.
- Pipe manufacturers will sometimes give formulae covering kinematic ranges relevant for their pipes. They are just segments of Moody chart.
- There are uncertainties in tabulations of ε and in the Moody chart itself. Accuracy of about 10% is expected.

Some equations

The information in the Moody diagram also exists as a number of formulae.

The Colebrook formulae is

$$\frac{1}{\sqrt{f}} = -2.0 \log_{10} \left(\frac{\varepsilon}{3.7 \times D} + \frac{2.51}{\text{Re}\sqrt{f}} \right)$$

is actually a transcendental non-linear equation that has to be solved! (Note, the Moody chart is just the graphical representation of this formulae).

The Moody formulae is

$$f = 0.0055 \left[1 + \left(20000\varepsilon + \frac{10^6}{\text{Re}} \right)^{1/3} \right]$$

The von Karman formulae for fully turbulent flows is

$$\frac{1}{\sqrt{f}} = -2.0 \log_{10} \left(\frac{\varepsilon}{3.7 \times D} \right)$$

The Blasius equation for smooth pipes ($\text{Re} < 10^5$) is

$$f = \frac{0.316}{\text{Re}^{1/4}}$$

Application of Moody chart

Air (standard conditions) flows through 4.0 mm diameter drawn tubing with $v = 50 \text{ m/s}$. Under these conditions flow is expected to be turbulent, but for dust free air, a very smooth entrance to the tube, and no tube vibration, it may be possible to maintain laminar flow. Determine the pressure drop in a 0.100 m length of tube assuming

- (a) Laminar flow
- (b) Turbulent flow

First determine standard conditions (see Table 2.1 for standard atmosphere). Use $\rho = 1.23 \text{ kgm}^{-3}$, $\mu = 1.79 \times 10^{-5} \text{ kg m}^{-1}\text{s}^{-1}$.

Application of Moody chart

Determine Re first

$$Re = \frac{\rho D v}{\mu}$$

$$Re = \frac{1.23 \times 0.0040 \times 50}{1.79 \times 10^{-5}} = 1.37 \times 10^4$$

(a) Laminar flow, now $f = 64/13700 = 0.00467$

$$\Delta p = \frac{f l \rho v^2}{2D}$$

$$= \frac{0.00467 \times 0.100 \times 1.23 \times 50.0^2}{2 \times 0.0040}$$

$$= 179.0 \text{ Pa}$$

can also obtain from

$$\Delta p = \frac{32 \mu l v}{D^2}$$

$$= \frac{32 \times 1.79 \times 10^{-5} \times 0.100 \times 50.0}{0.0040^2}$$

$$= 179.0 \text{ Pa}$$

Application of Moody chart: continued

(b) Turbulent flow from Table 8.1 $\varepsilon = 0.00150$ mm .
So $\varepsilon/D = 0.0015/4.0 = 0.000375$. On the Moody chart, $f = 0.028$.

$$\begin{aligned}\Delta p &= \frac{fl\rho v^2}{2D} \\ &= \frac{0.028 \times 0.100 \times 1.23 \times 50.0^2}{2 \times 0.0040} \\ &= 1076 \text{ Pa}\end{aligned}$$

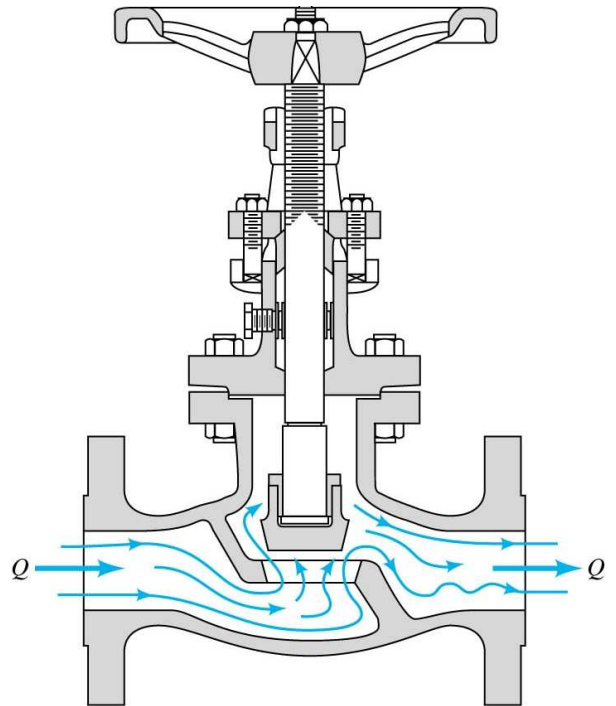
The pressure drop is **6** times as large for the laminar flow situation since the friction factor is **6** times larger.

Minor losses

There is more to pipe systems than pipes. Valves (e.g. a tap head) provide a means to regulate the flow. Bends exist to redirect the flow.

Trying to determine the pressure drop due to valve would be complicated.

Also the pressure drop will on the extent to which the valve is open.



Pressure drops are given in terms of dimensionless variables with the functional form or correction factors obtained from experiment.

The loss coefficient

The loss coefficient, K_m is the most commonly used method to describe pressure loss. It is defined as

$$K_L = \frac{h_{Lminor}}{v^2/(2g)} = \frac{\Delta p}{\frac{1}{2}\rho v^2}$$

or

$$\Delta p = K_L \frac{1}{2}\rho v^2$$

$$h_{Lminor} = K_L \frac{v^2}{2g}$$

The minor losses increase as v^2 . The minor losses increase by 20% if the flow rate increases by 10% .

The equivalent length

The pressure loss is sometimes given in terms of an equivalent length of the same pipe that would give the same pressure loss.

$$h_{Lminor} = K_L \frac{v^2}{2g} = f \frac{l_{eq}}{D} \frac{v^2}{2g}$$

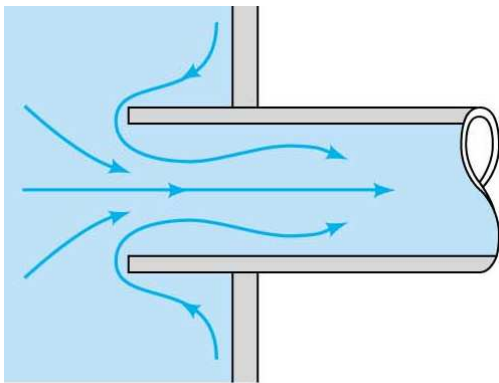
giving

$$l_{eq} = \frac{K_L D}{f}$$

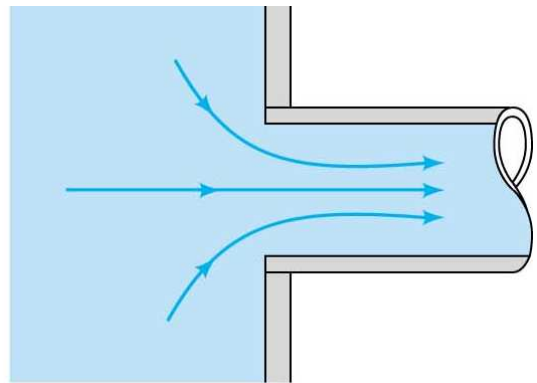
Set D and f to be those for pipe containing component. Usage of this alternative to the loss coefficient is uncommon. No usage in this unit.

The nature of Head losses

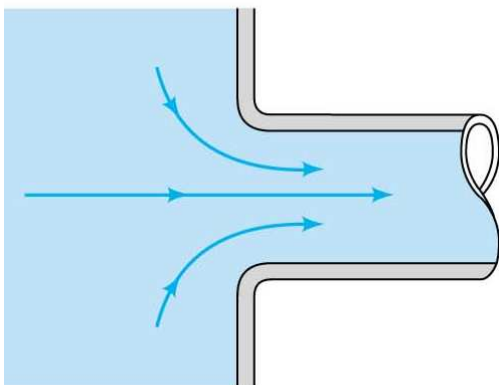
One source of head-loss occurs when the pipe diameters change. These changes can be abrupt or smooth. One of the reasons for head loss is that it is not possible to slow down a fluid easily.



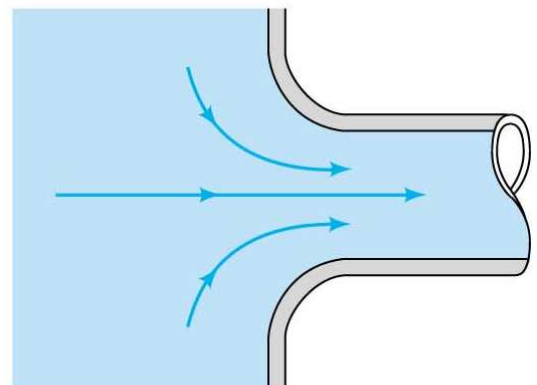
(a)



(b)



(c)



(d)

(a)

(b)

(c)

(d)

$$K_L = 0.8$$

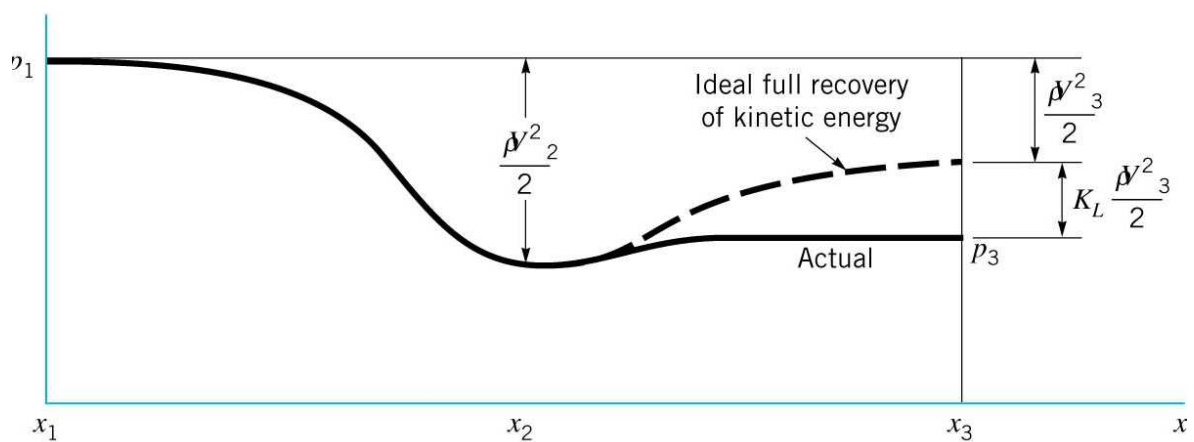
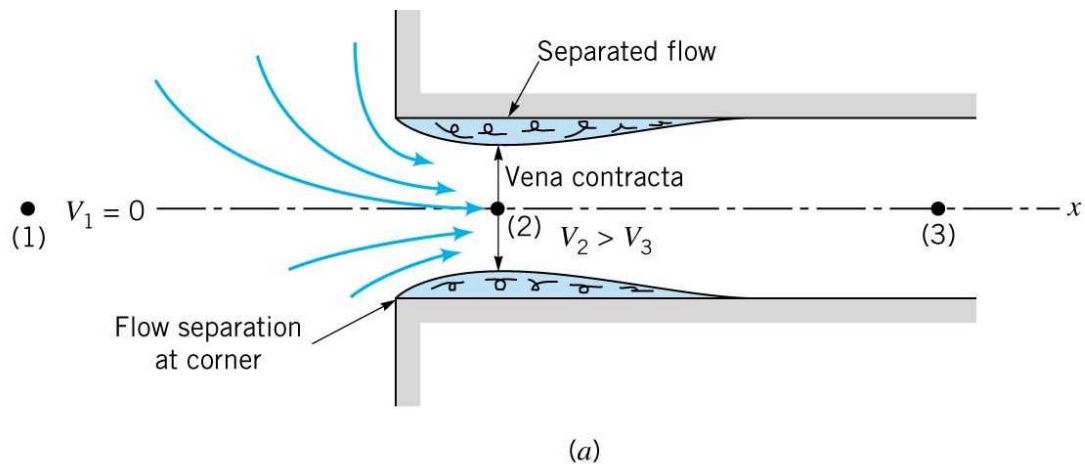
$$K_L = 0.5$$

$$K_L = 0.4$$

$$K_L = 0.2$$

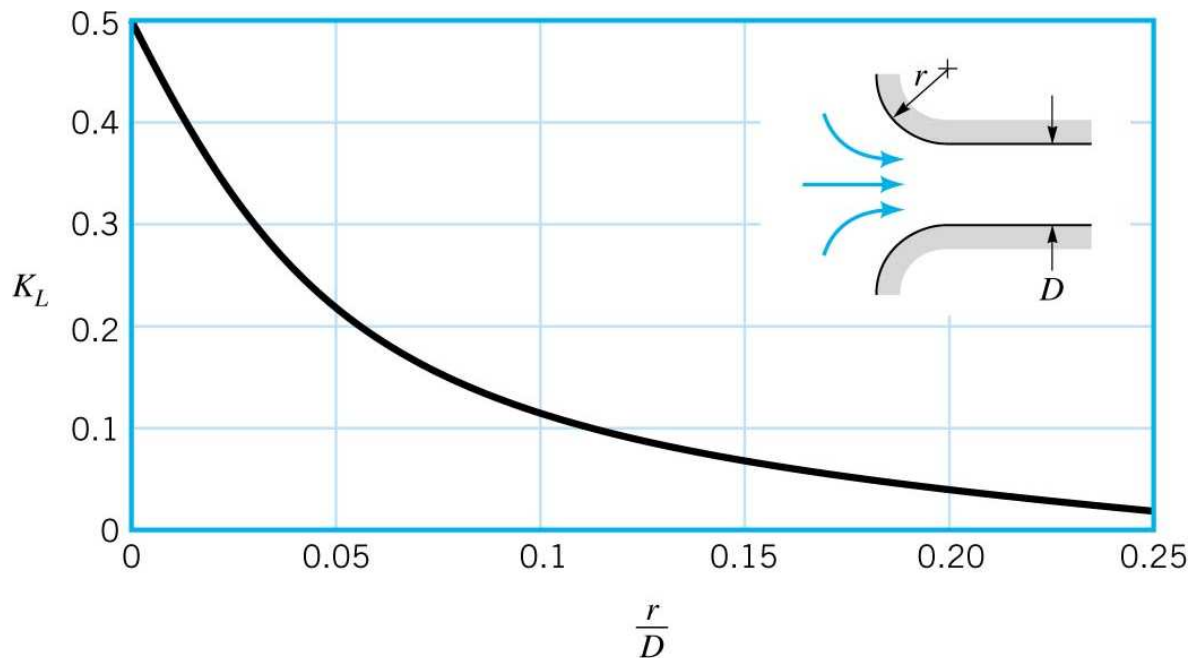
The nature of head losses

One source of head-loss occurs when the pipe diameters change.



The pressure drop associated with increase in fluid speed not entirely recovered by kinetic energy. Most of energy loss is due to shear stress within fluid.

Tank → Pipe Head losses

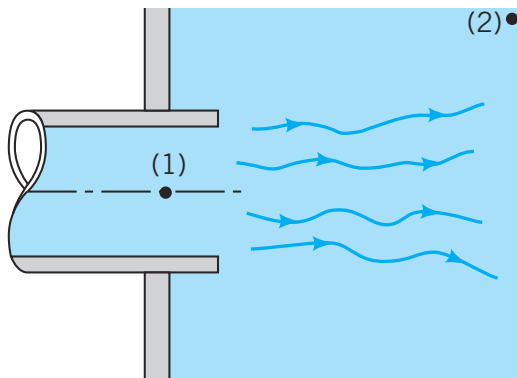


About 50% of the energy is lost when the fluid enters a pipe with a square edged entrance.

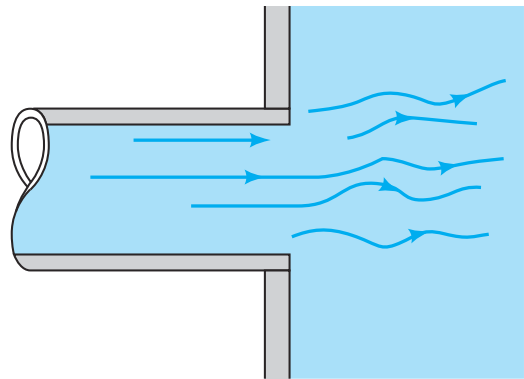
Rounding the entrance corner will reduce the loss coefficient.

If the pipe protrudes into the tank the loss coefficient will be even larger.

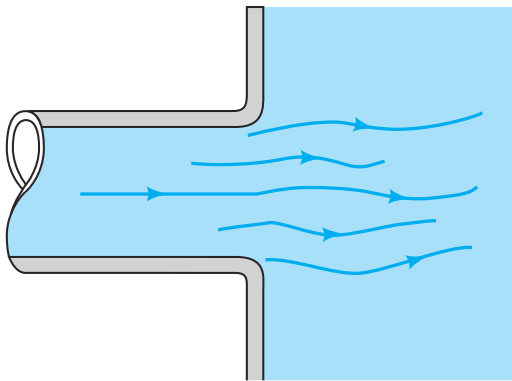
Pipe \rightarrow Tank Head losses



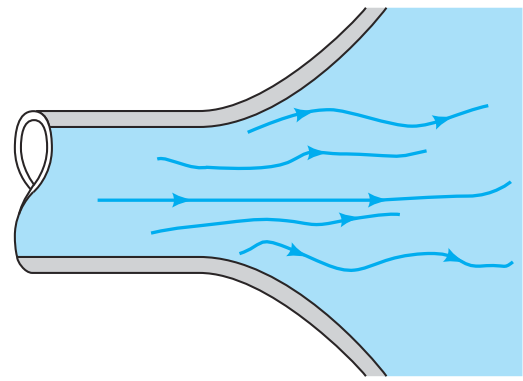
(a)



(b)



(c)

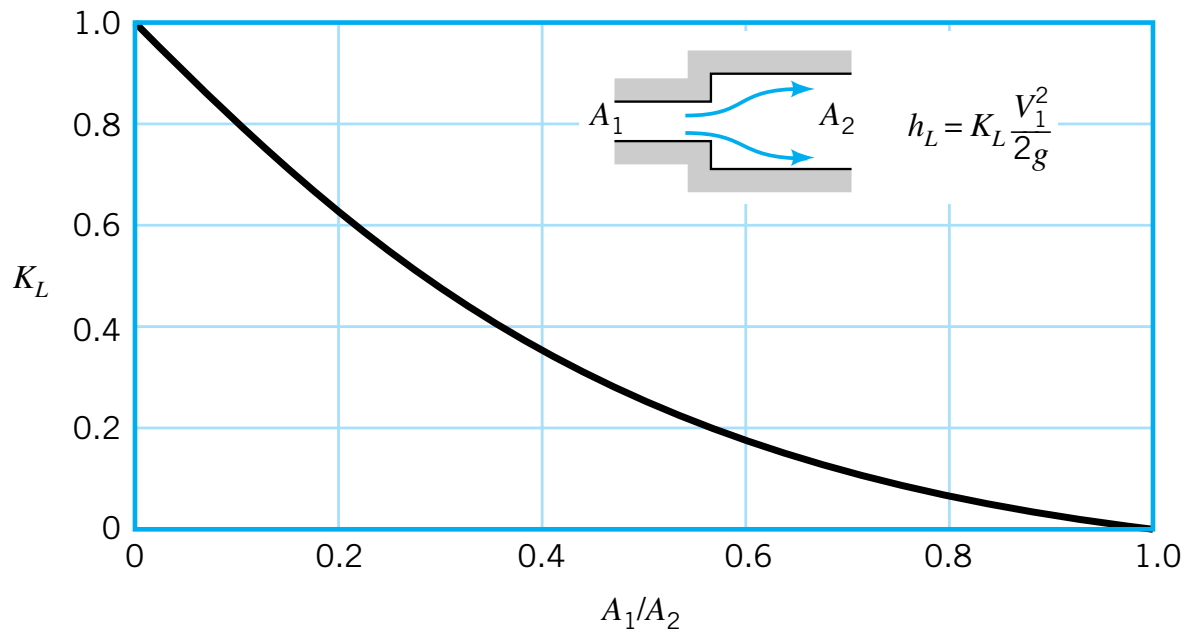


(d)

The head loss when water from a large pipe enters a tank is $K_L = 1$ irrespective of the geometry.

The fluid from the pipe mixes with the fluid and its kinetic energy is dissipated through viscous effects as the fluid eventually comes to rest.

Pipe Expansion



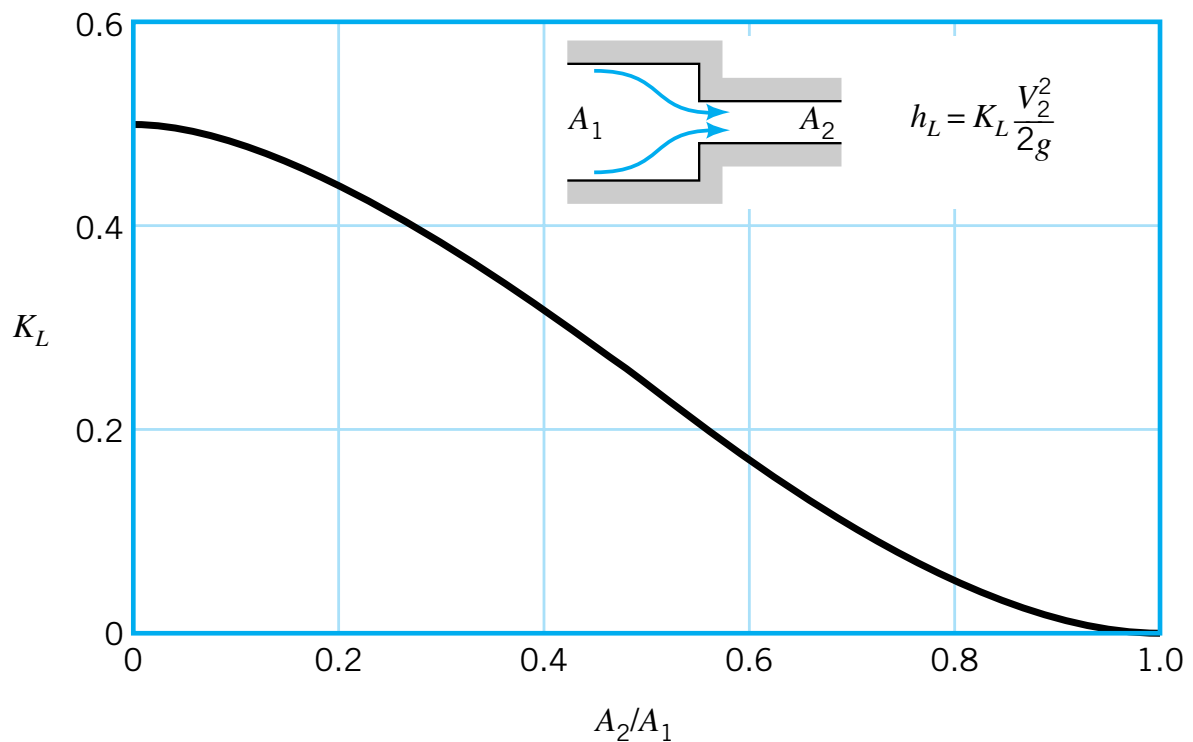
When a pipe undergoes a sudden expansion, we can use mass, momentum and energy equations to derive

$$K_L = \left(1 - \frac{A_1}{A_2}\right)^2$$

The loss coefficient goes to zero as $A_2 \rightarrow A_1$.

The loss coefficient goes to one as $A_2 \rightarrow \infty$.

Pipe Contraction

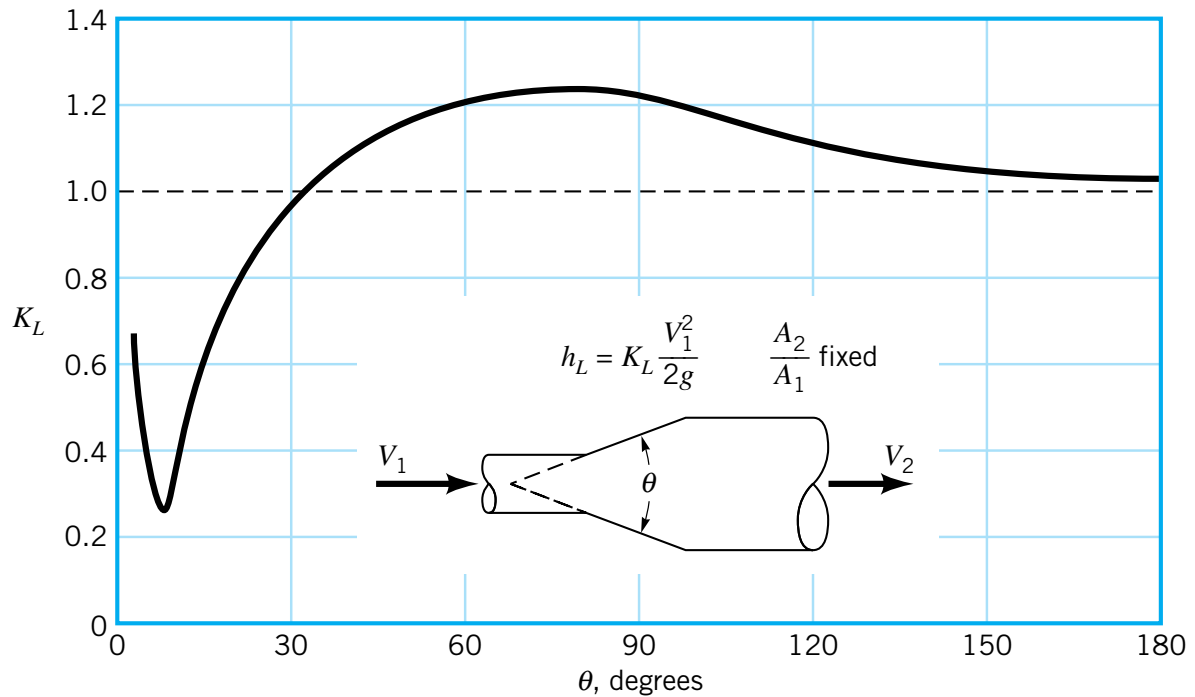


The loss coefficient for a contraction with a sharp-edge entrance and exist is similar to a tank when $A_2/A_1 \rightarrow 0$.

The loss coefficient decreases when the sizes of the two pipes are close together. Obviously,

$$\lim_{A_2/A_1 \rightarrow 1} K_L = 0$$

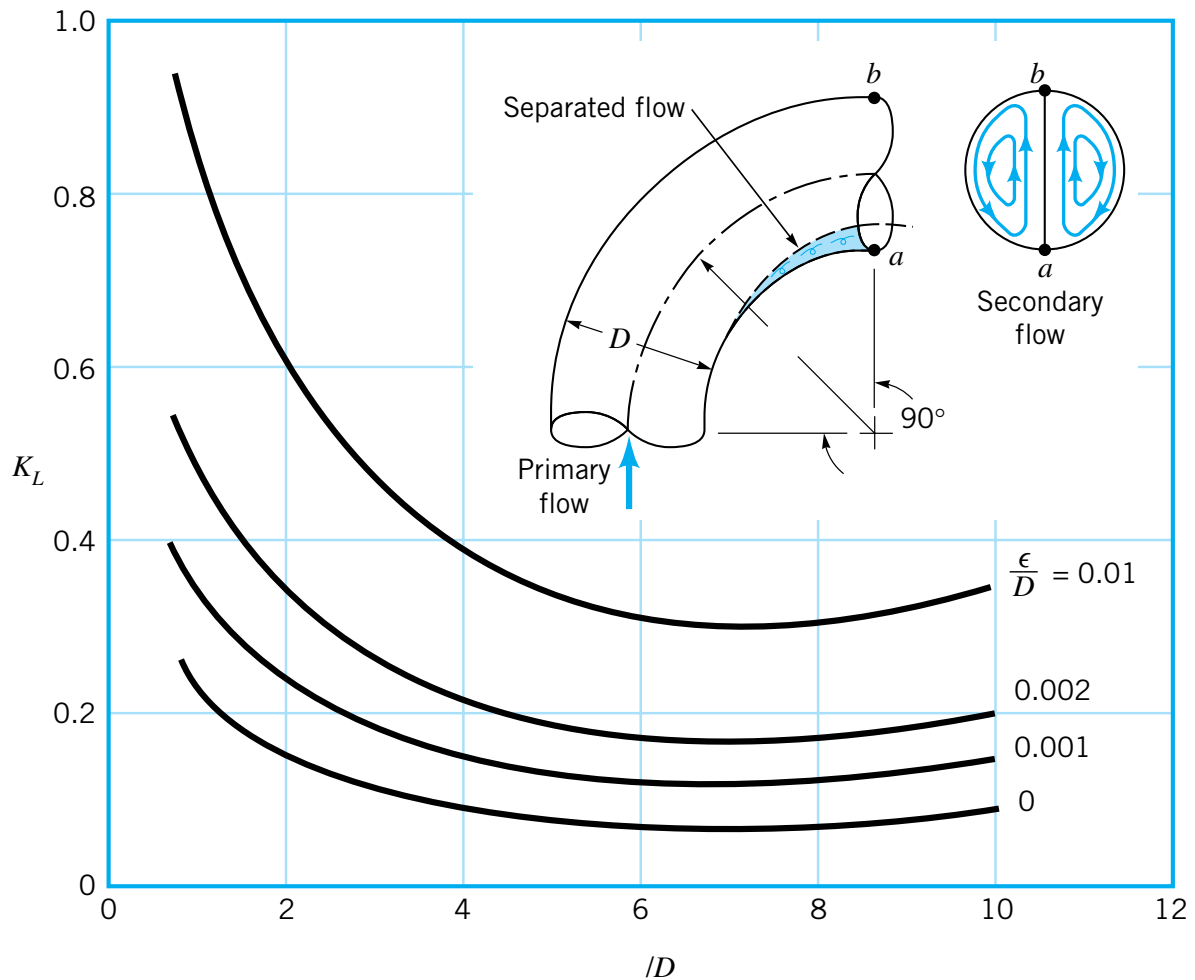
Gradual pipe diameter changes



Losses can be decreased by having a *gradual* contraction or expansion. The angle θ cannot be too small (e.g. $\theta = 1^\circ$ means the diffuser will be excessively long) or too large.

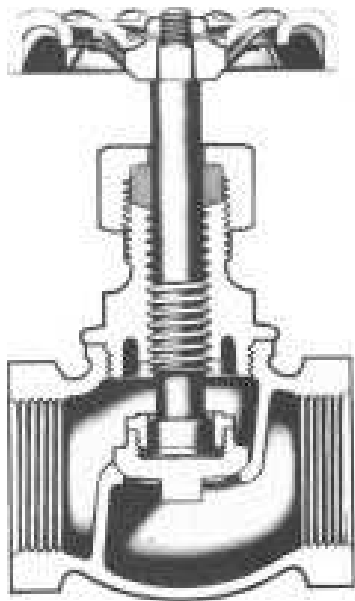
The loss coefficient for conical contractors can be quite small $K_L = 0.02$ for $\theta = 30^\circ$.

Pipe Bends

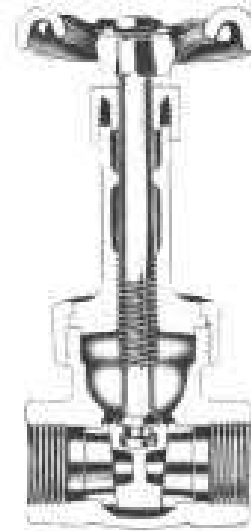


The loss factor is generally larger for curved pipes than for straight pipes. The diagram shows loss factor for large Re . This loss factor is additional to that of the equivalent straight line length of the pipe.

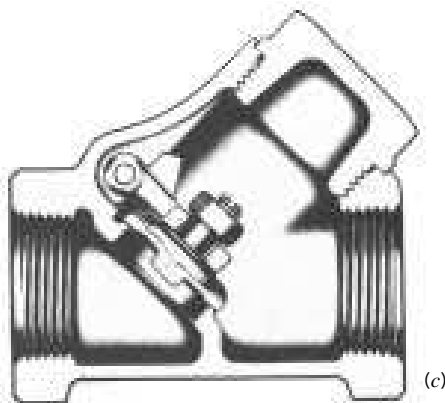
Valve Types



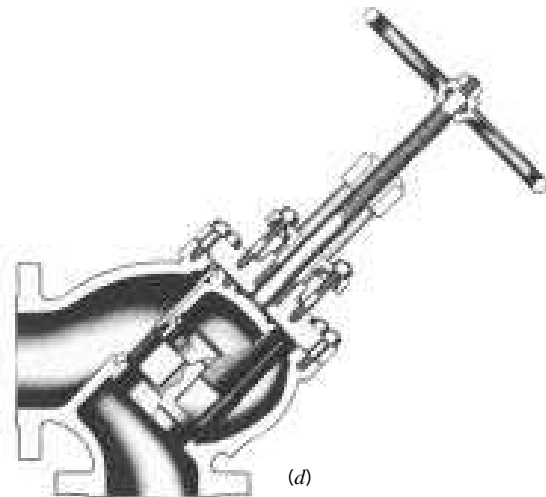
(a)



(b)



(c)



(d)

(a) Globe Valve

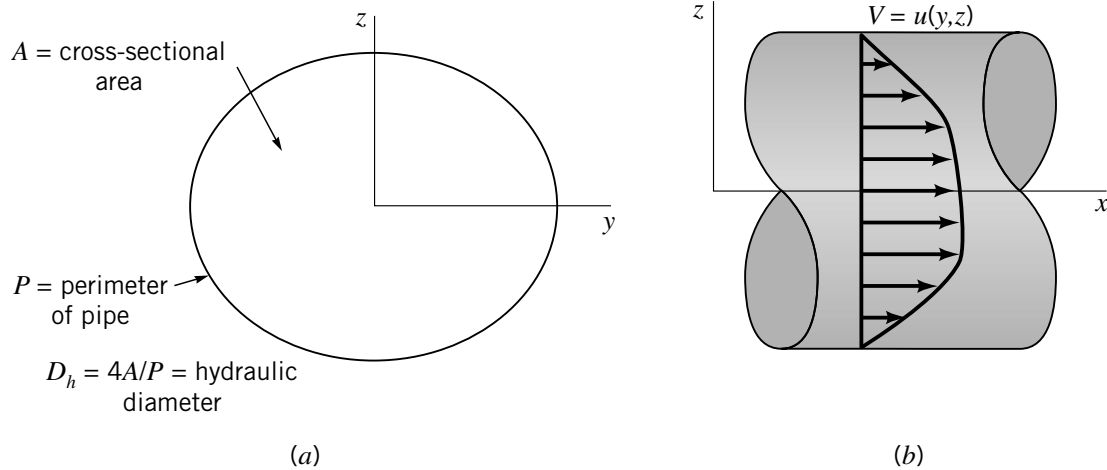
(b) Gate valve

(c) Swing Check Valve

(d) Stop Check Valve

Tables of loss coefficients exist for common pipe fittings.

Non-circular conduits



The laminar friction factor is written $f = \frac{C}{\text{Re}_h}$ where C depends on conduit shape. The definition of Re_h is

$$\text{Re}_h = \frac{\rho D_h v}{\mu}$$

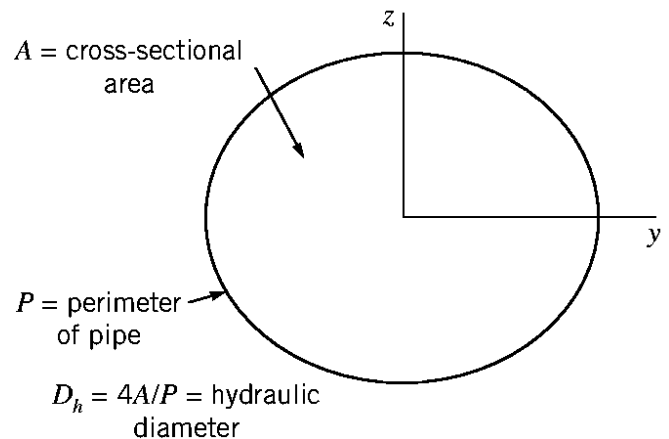
where the hydraulic diameter D_h is

$$D_h = \frac{4A}{P}$$

A is the cross sectional area of pipe P is the length of the wetted perimeter

Non-circular conduits

The Moody chart is used with D replaced by D_h , and Re replaced by Re_h .



Tables of values exist for different shaped conduits.

The hydraulic diameter, D_h is used in the definitions for turbulent flow.

$$h_L = f \frac{1}{2} v^2 \frac{\rho}{\gamma} \frac{l}{D_h} = f \frac{v^2 l}{2gD_h}$$

with roughness $= \varepsilon/D$. This procedure for turbulent flow is accurate to about 15%.

Pipe flow scenarios

The real world pipe flow design scenarios are divided into 3 types, I, II and III.

I We know the fluid, pipe size and desired flow rate.

We need to determine the pressure drop or head loss. In effect we want to know how large a pump needs to be installed.

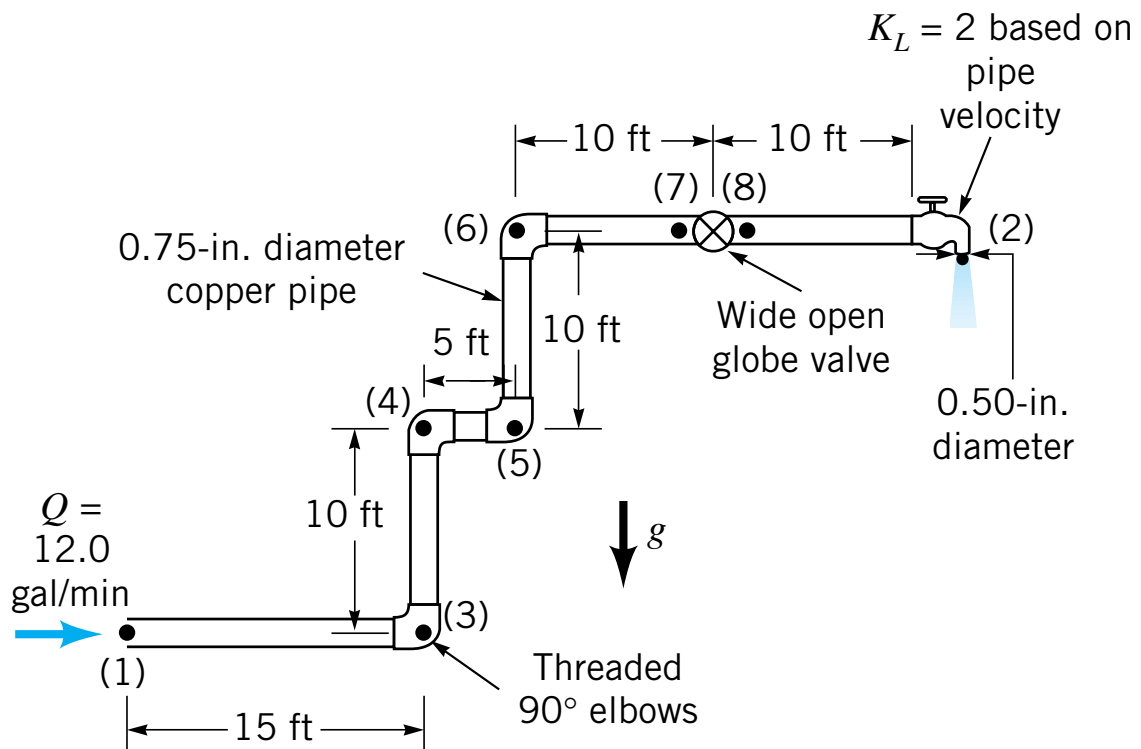
II We know the fluid, pipe size and head loss. We want to determine the flow rate.

III We know the fluid, flow rate and head loss. We want to determine pipe size.

Type II and III scenarios are more complicated to solve since they involve a non-linear equation. Need to adjust v and there is complicated dependence between f and v .

Type I example

Water at $15.6\text{ }^{\circ}\text{C}$ flow from the basement to the second floor through a 19.0 mm -diameter copper pipe at a rate of $Q = 7.57 \times 10^{-4}\text{ m}^3/\text{s}$ and exits through a faucet of 12.7 mm diameter.



Determine the pressure at (1) if (a) All losses are neglected (b) Only major losses are included (c) All losses are included.

Type I example: continued

Estimate fluid velocity to get flow rate.

$$v_1 = \frac{Q}{A_1} = \frac{7.57 \times 10^{-4}}{\pi(9.5 \times 10^{-3})^2} = 2.67 \text{ m/s}$$

The Reynolds number is

$$\text{Re} = \frac{\rho D v}{\mu} = \frac{10^3 19.0 \times 10^{-3} 2.67}{1.12 \times 10^{-3}} = 45300$$

The flow is turbulent flow. The equation to be applied is

$$\frac{p_1}{\gamma} + \alpha_1 \frac{v_1^2}{2g} + z_1 - h_L = \frac{p_2}{\gamma} + \alpha_2 \frac{v_2^2}{2g} + z_2$$

For turbulent flow set $\alpha_1 = \alpha_2 \approx 1$.

The faucet is a free jet, so $p_2 = 0$ (gauge)

Set $z_1 = 0.0 \text{ m}$ and $z_2 = 6.10 \text{ m}$

The exit velocity $v_2 = Q/A_2 = 5.98 \text{ m/s}$

Type I example: No head loss

Set $h_L = 0.0 \text{ m}$. So energy equation gives

$$\begin{aligned} \frac{p_1}{\gamma} &= \frac{p_2}{\gamma} + \frac{v_2^2 - v_1^2}{2g} + (z_2 - z_1) \\ \frac{p_1}{9800} &= 0 + \frac{5.98^2 - 2.67^2}{19.6} + (6.10) \\ \frac{p_1}{9800} &= 7.56 \\ \Rightarrow p_1 &= 74.1 \text{ kPa} \end{aligned}$$

About **80%** of the pressure drop (from $z_1 \rightarrow z_2$) is due to the elevation increase while **20%** is due to the velocity increase.

Type I example: Major head loss

The total length of the copper pipe is 18.3 m (60 ft).

Copper pipes are drawn tubing so $\varepsilon = 0.0015 \text{ mm}$.

Therefore $\varepsilon/D = 7.9 \times 10^{-5}$. Friction factor from Moody is 0.0215 (It is practically smooth).

So head loss is

$$\begin{aligned} h_{Lmajor} &= \frac{flv^2}{2Dg} \\ &= \frac{0.0215 \times 18.3 \times 2.67^2}{2 \times 0.019 \times 9.80} \\ &= 7.53 \text{ m} \end{aligned}$$

Simply add this head loss to

$$\begin{aligned} \frac{p_1}{\gamma} &= 7.56 + h_{Lmajor} = 7.56 + 7.53 \\ \Rightarrow p_1 &= 15.09 \times 9.8 = 148 \text{ kPa} \end{aligned}$$

Type I example: Minor head loss

Now include impact of bends and valves. There are 4 90° elbows, each with $K_L = 1.5$.

The open globe valve has $K_L = 10$.

The loss coefficient of the faucet is $K_L = 2$.

To get the minor head losses, one simply adds up the individual losses from each component

$$h_{Lminor} = \sum_i h_{Lminor_i}$$

$$h_{Lminor} = \sum_i K_{L_i} \frac{v^2}{2g}$$

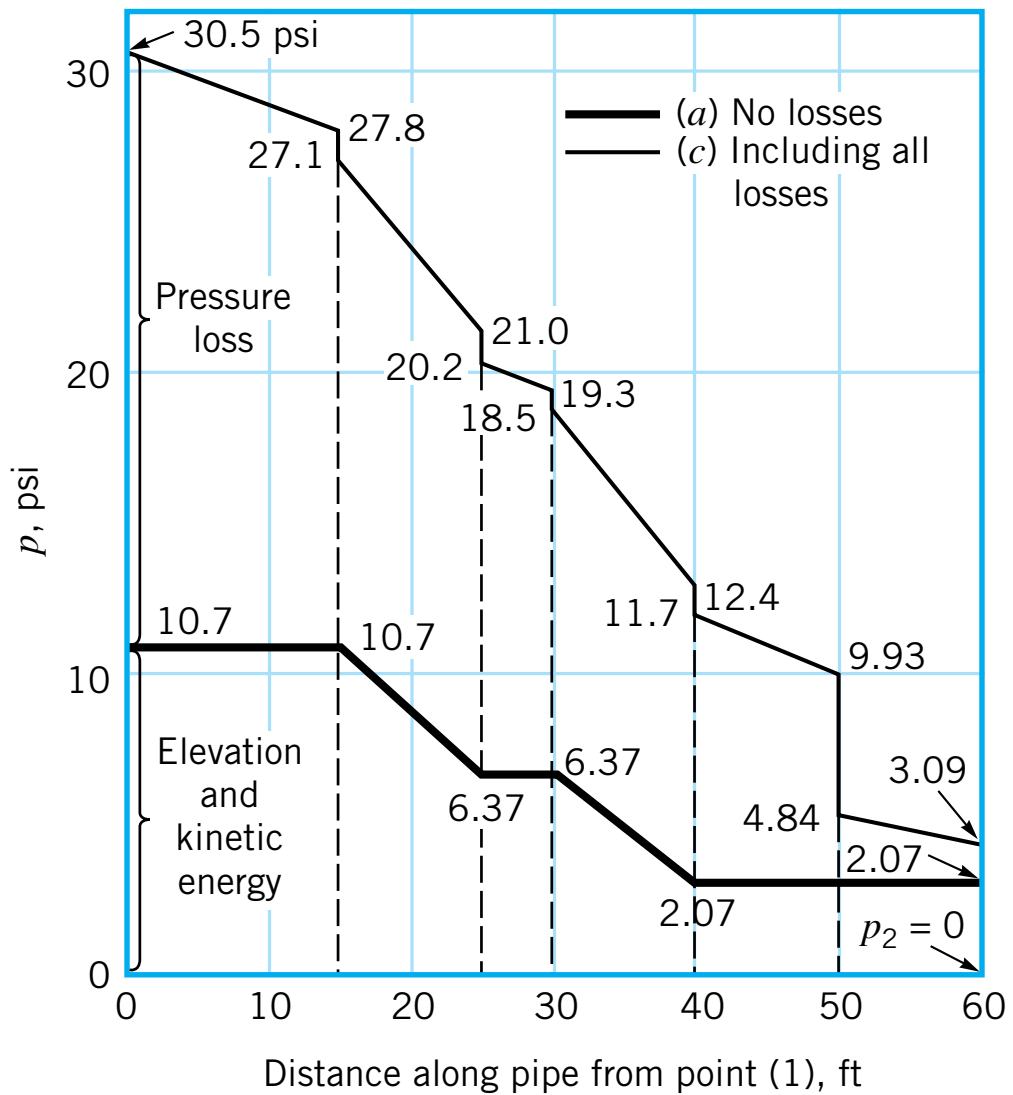
$$h_{Lminor} = (10 + 4 \times 1.5 + 2) \frac{v^2}{2g} = 18.0 \frac{v^2}{2g}$$

$$h_{Lminor} = 18.0 \frac{2.67^2}{19.6} = 6.55 \text{ m}$$

So the total head loss is $7.56 + 7.53 + 6.55 = 21.64 \text{ m}$

. The corresponds to a pressure difference of $21.64 \times 9.8 = 212.1 \text{ kPa}$. The gauge pressure $p_1 = 212.1 \text{ kPa}$.

Type I example: Graphical Representation

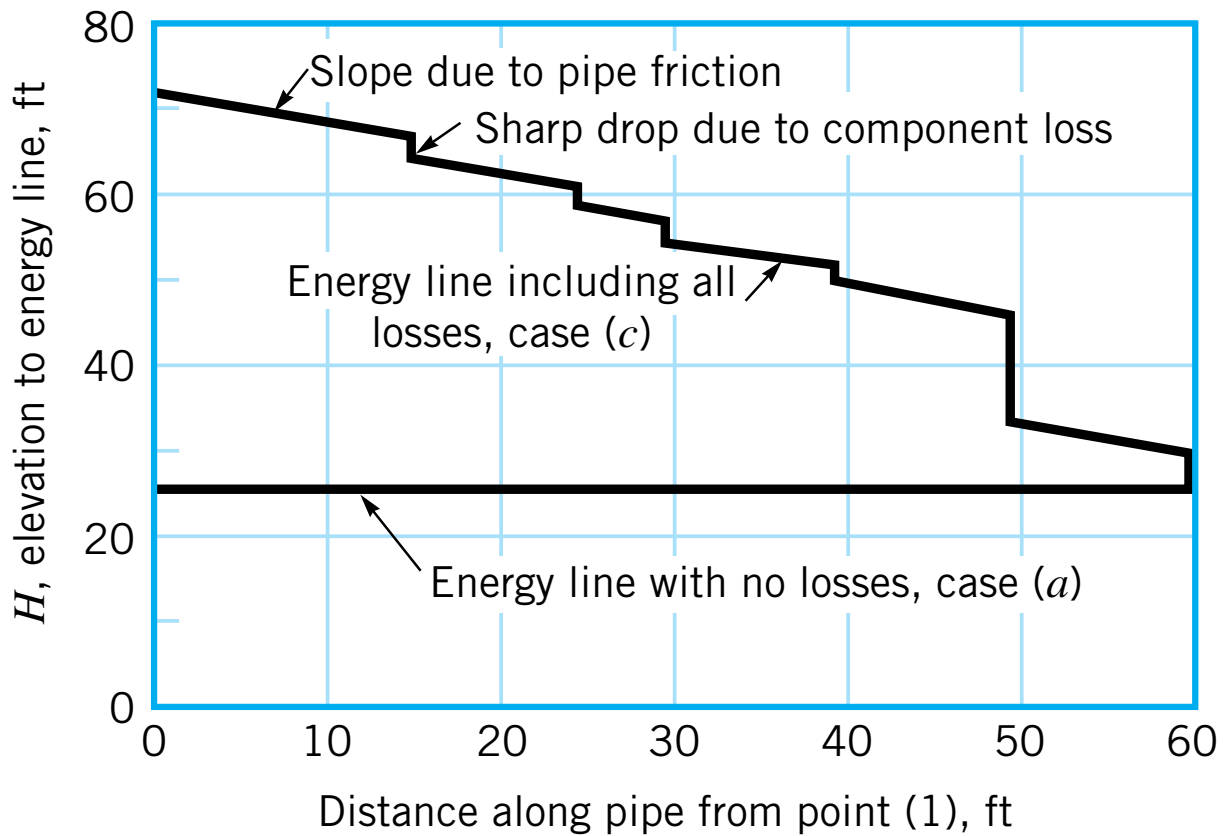


Location: (1) (3) (4) (5) (6) (7) (8) (2)

The pressure rise from (2) consists of

- Linear increases in pressure over the length of the drawn copper tubing.
- Sudden jumps in pressure through the components.

Type I example: Energy grade line

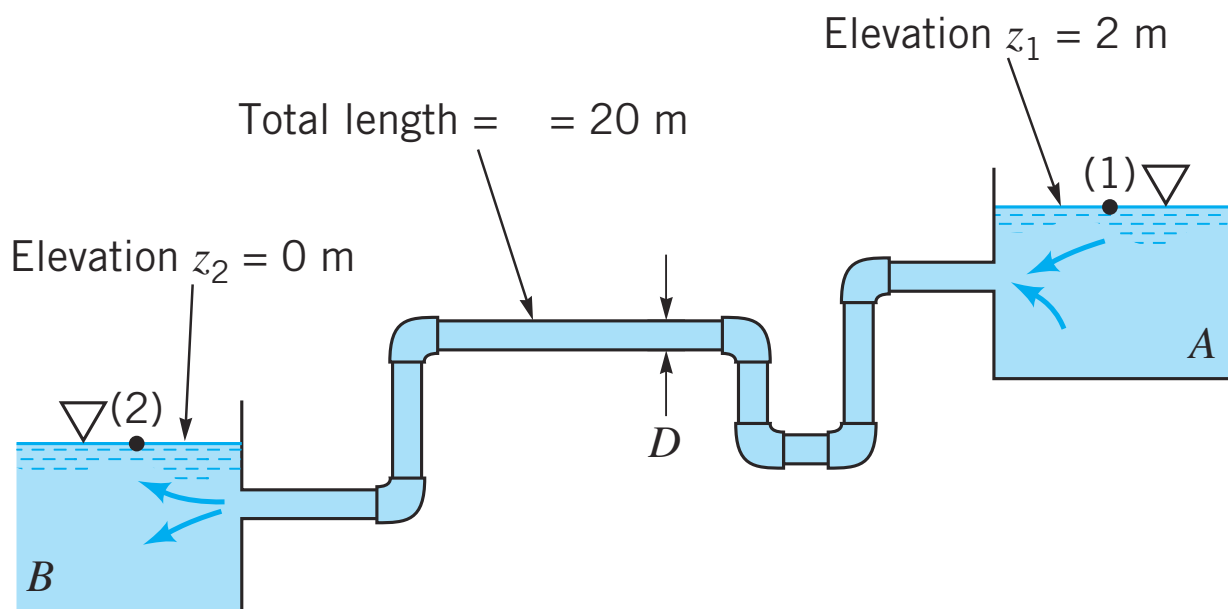


The energy line with no head losses would be horizontal.

The energy line consists of steady increases, indicating a constant energy dissipation along the pipe, and sudden jumps indicating losses through valves and bends.

Type III with minor losses. Example

Water at $10\text{ }^{\circ}\text{C}$ with kinematic viscosity $1.31 \times 10^{-6}\text{ m}^2/\text{s}$ is to flow from A to B through a cast-iron pipe $\varepsilon = 0.26\text{ mm}$ at a rate of $0.0020\text{ m}^3/\text{s}$. The system contains a sharp-edged entrance and six threaded 90° elbows. Determine the pipe diameter that is needed.

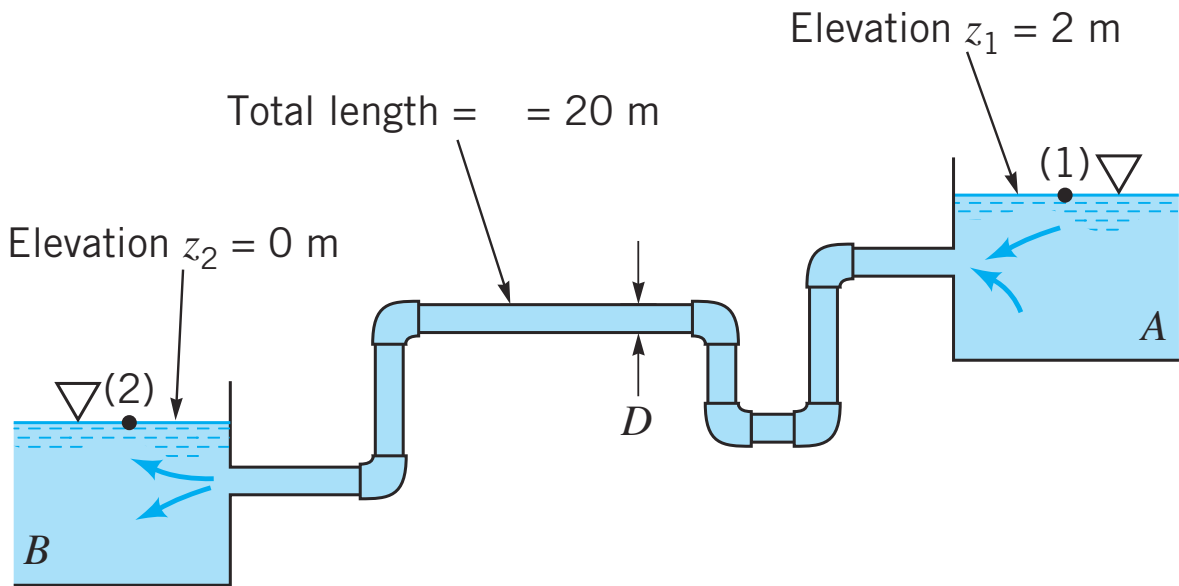


Will use the energy equation

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_L$$

With reference points at (1) and (2) so that $p_1 = p_2 = 0$, $z_2 = 0$ and $v_1 = v_2 = 0$.

Type III with minor losses. Example



The energy equation simplifies

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_L$$

$$z_1 = h_L$$

$$z_1 = \frac{v^2}{2g} \left(\frac{f l}{D} + \sum_i K_{L_i} \right)$$

where v is water velocity in pipe.

$$v = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{2.55 \times 10^{-3}}{D^2}$$

Type III with minor losses. Example

Head-loss terms

- Six 90° elbows. $K_L = 6 \times 1.5 = 9.0$
- Tank \rightarrow Pipe. $K_L = 0.5$
- Pipe \rightarrow Tank. $K_L = 1.0$
- Total: $\sum K_L = 10.5$

The energy equation becomes

$$z_1 = \frac{v^2}{2g} \left(\frac{f\ell}{D} + \sum_i K_{L_i} \right)$$

$$2.0 = \frac{6.50 \times 10^{-6}}{2 \times 9.80 \times D^2} \left(\frac{f \times 20}{D} + 10.5 \right)$$

The problem with this equation is that the friction factor depends on D in a complicated manner. The friction factor depends on the roughness ε/D and Reynolds number $\rho v D / \mu$.

What we want to do is choose D so that the head loss is 2.0 m . This is a non-linear equation.

Solving the pipe sizing problem

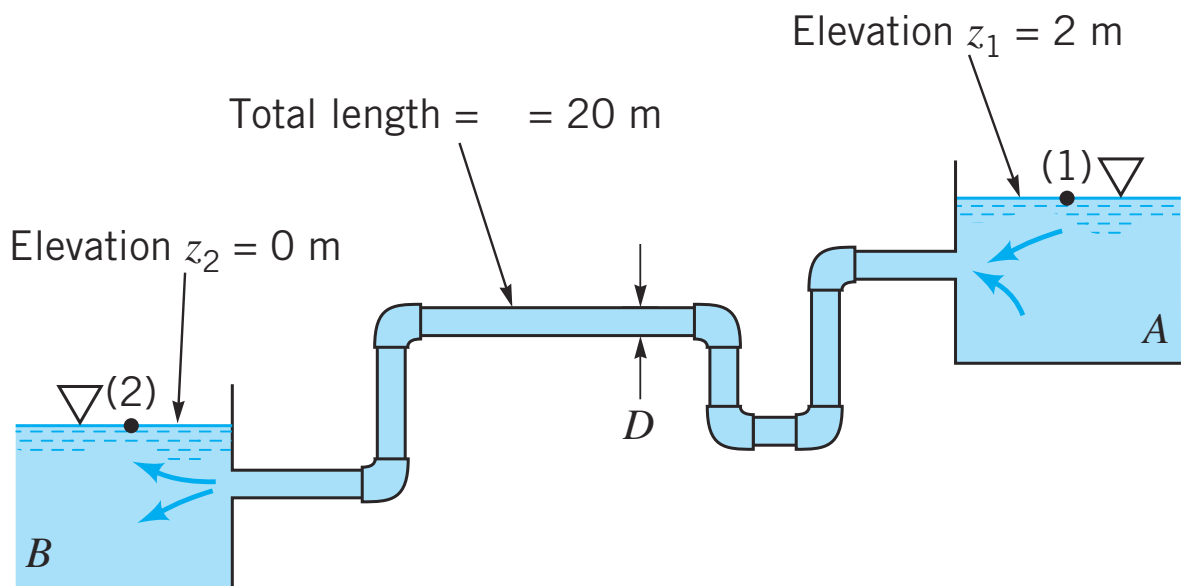
Want to solve

$$2.0 = \frac{6.50 \times 10^{-6}}{2 \times 9.80 \times D^2} \left(\frac{f \times 20}{D} + 10.5 \right)$$

The head-loss (right hand side) will get smaller as D gets larger. Procedure

- Need to bootstrap the problem (supply initial guess). Set $f = 0.0200$ (a reasonable value for many pipe problems) get an initial value of D , namely D_0 .
- Plug initial estimate of D_0 into *RHS*. Use $D_0 \rightarrow \varepsilon/D \rightarrow f \rightarrow h_L$. Use Moody diagram or approximate formula for specific flow regime.
- If $h_L < 2.0$, then D needs to be decreased. If $h_L > 2.0$, then D needs to be increased. Use $h_L \propto 1/D^2$ scaling to get next estimate
- Keep record of h_L vs D . When have enough points, plot h_L vs D and determine where $h_L = 2.0$ m is true. Note, no point in getting D to better than 1% accuracy.

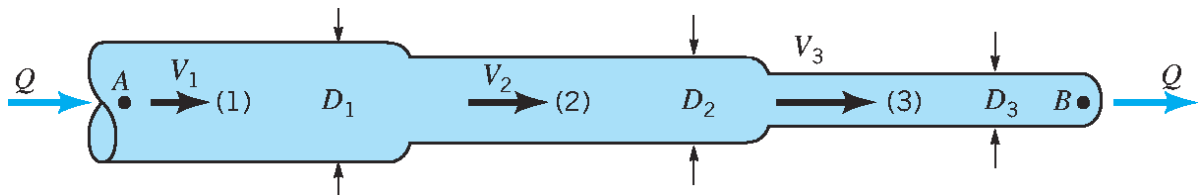
Solving the pipe sizeing problem



The actual solution occurs when self-consistency occurs

Note, pipes come with certain standard diameters. Choose a pipe diameter that is larger than exact diameter extracted from equation.

Pipe systems in Series



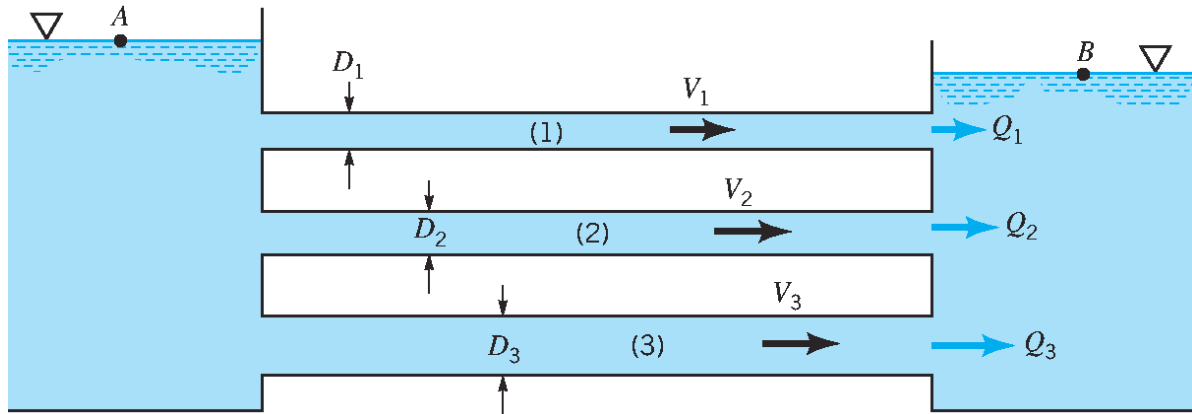
The flow-rate along the pipe is constant for steady-state flow.

$$Q = Q_1 = Q_2$$

The total head loss is obtained by adding up the head loss along the individual segments

$$h_{L1-3} = h_{L1} + h_{L2} + h_{L3}$$

Pipe systems in Parallel



The flow-rate along the pipe is constant for steady-state flow.

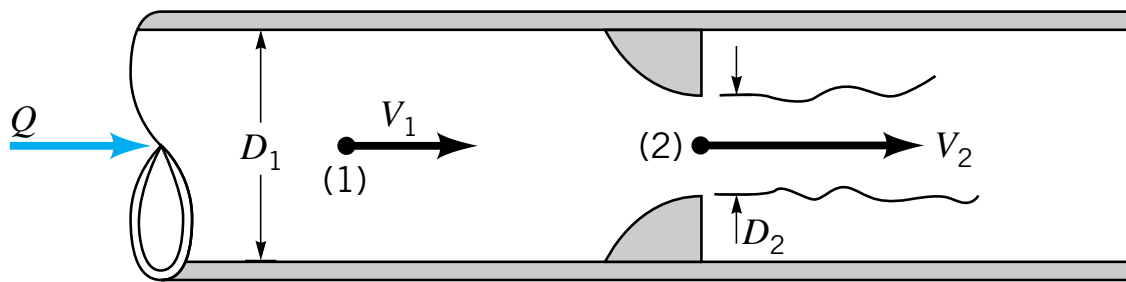
$$Q_T = Q_1 + Q_2 + Q_3$$

Consider the points A and B . The head loss between these points must be the same irrespective of the pipe chosen

$$h_{L_{AB}} = h_{L_1} = h_{L_2} = h_{L_3}$$

More complicated pipe networks will require more sophisticated method of analysis.

Flow measurements



According to Bernoulli, the flow-rate between (1) and (2) can be written

$$Q_{ideal} = A_2 v_2 = A_2 \sqrt{\frac{2(p_1 - p_2)}{1 - \beta^4}}$$

where $\beta = D_2/D_1$ is ratio of pipe diameters

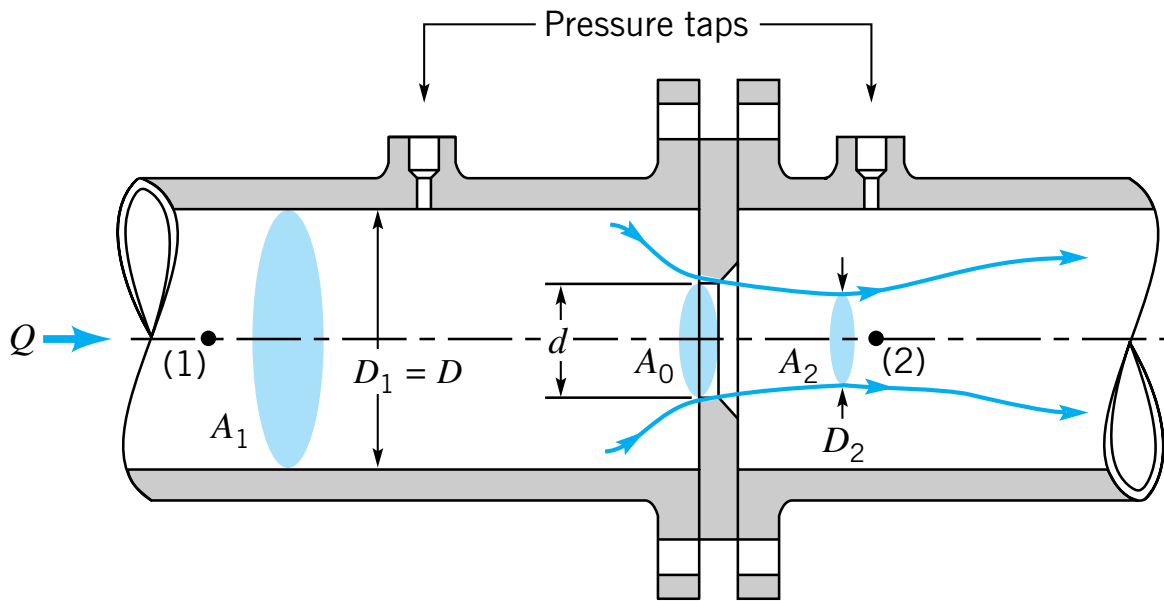
However, the energy equation is

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_L$$

There is no compact expression for h_L . Making a universal identity for the flow rate incorporating the head loss in the constriction is not possible.

Empirical coefficients, valid for specific flow meters are used to correct the ideal expression for the flow rate.

Orifice meter



Non-ideal effect occur due to

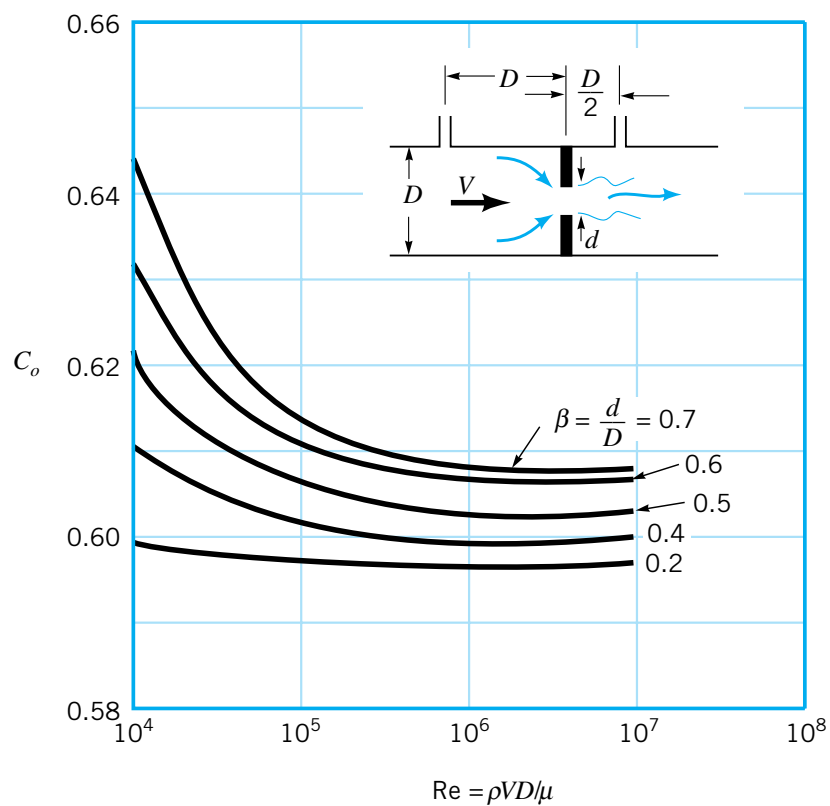
- Vena contracta effect. The diameter for the flow stream is slightly less than the diameter of the orifice.
- Turbulent motion and viscosity effects act to produce a head loss through the orifice.

$$Q = C_o Q_{ideal} = C_o A_o \sqrt{\frac{2(p_1 - p_2)}{1 - \beta^4}}$$

where A_o is the area of the orifice

Orifice meter

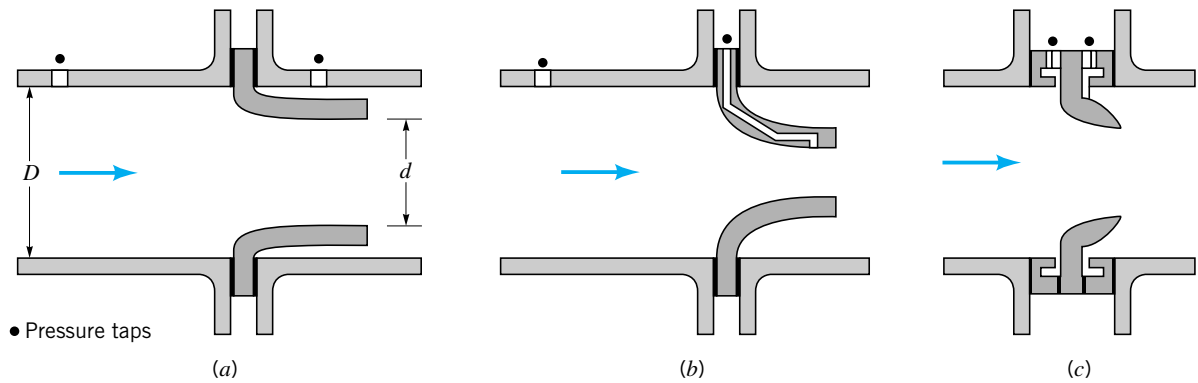
$$Q = C_0 A_o \sqrt{\frac{2(p_1 - p_2)}{1 - \beta^4}}$$



The contraction coefficient is a function of Re and β . Meters constructed under precise conditions and would come with discharge coefficient data.

Nozzle meter

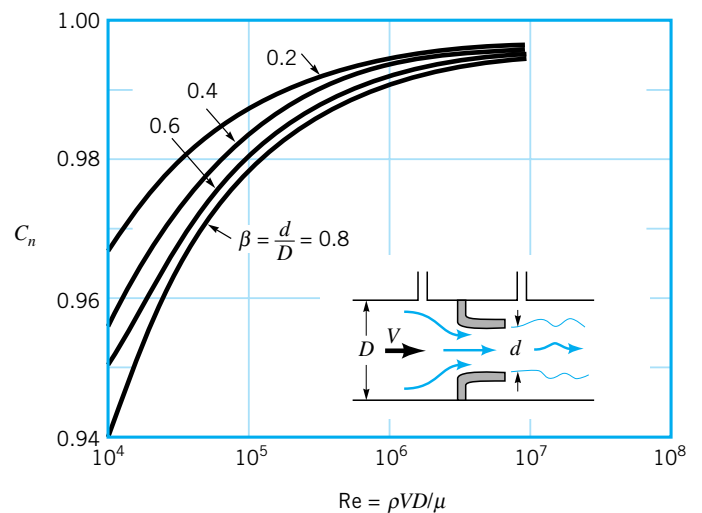
The nozzle meter has a tapered orifice



$$Q = C_n A_n \sqrt{\frac{2(p_1 - p_2)}{1 - \beta^4}}$$

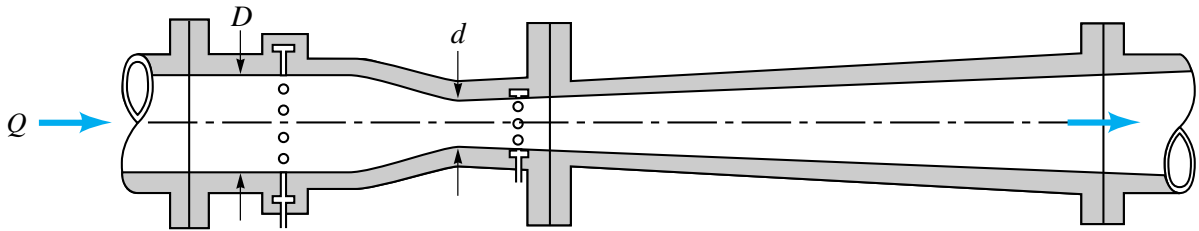
$$C_n > C_o$$

The head loss associated with nozzle is smaller than orifice. Vena Contracta effect is smaller.

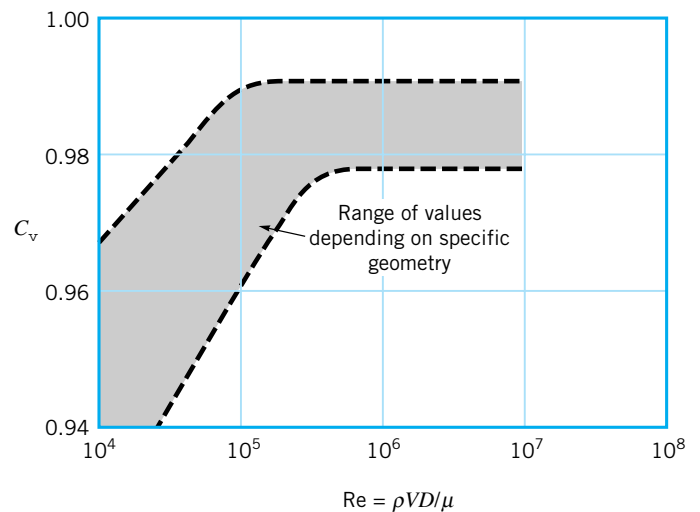


Venturi meter

The Venturi meter measures the pressure in a tapered pipe.



The head loss associated with constriction are small and the Vena Contracta is minimal.



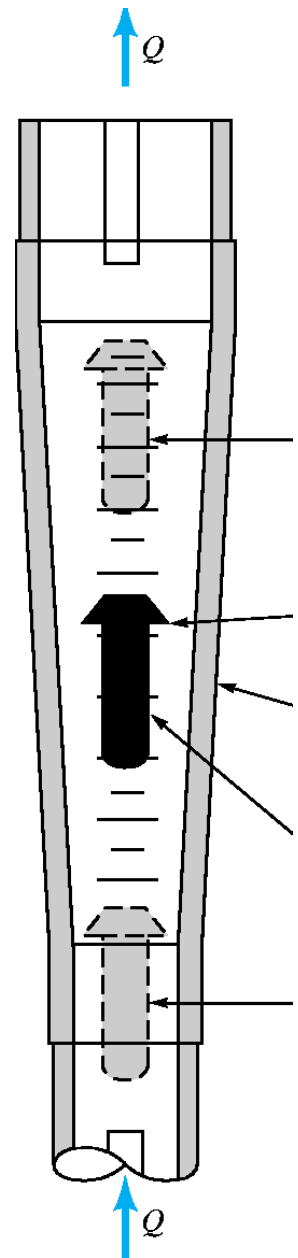
$$Q = C_v A_n \sqrt{\frac{2(p_1 - p_2)}{1 - \beta^4}}$$

The Venturi meters are the most precise and the most expensive.

The Rotameter

The rotameter is common and inexpensive.

- Float contained within a tapered transparent tube.
- As flow rate is increased, the float moves up the tube to find a new equilibrium position, (when buoyancy, float weight and fluid drag are in balance).
- The flow rate is then read from a calibrated scale.



The Volume flow meters

Volume flow meters

Other flow measuring devices

- Turbine flowmeters
- Ultrasonic flow meters
- Doppler-effect Ultrasonic flow meters
- Electromagnetic Flow meters
- Laser Doppler Velocimetry