Digital quantum simulation and Time Evolution

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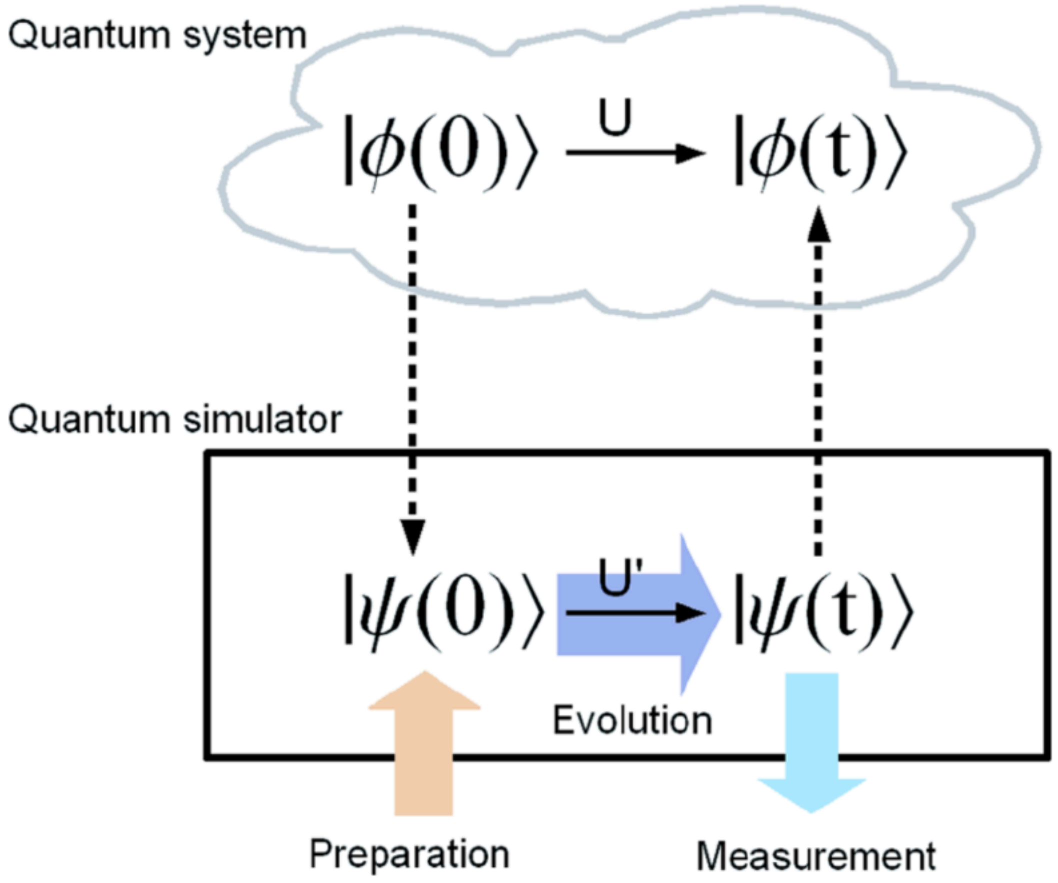
Digital quantum simulation

Time Evolution



- **Analog quantum simulation**: quantum system that emulates a target system
- **Digital quantum simulation**: quantum computation of Time Evolution through a circuit model

Digital quantum simulators

After encoding $|\psi(0)\rangle$ on the computational basis, $|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$ is directly computed through a sequence of single and two qubits gates

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Digital quantum simulation

Suzuki-Trotter decomposition

- Most Hamiltonians of physical interest can be mapped into a sum of L local terms: $\hat{\mathcal{H}} = \sum_k^L \hat{\mathcal{H}}_k$

- Then, map the original Hamiltonian into the QC algebra (i.e. SU2):

  - if $[H_l, H_{l'}] = 0$ then $U = \prod_l \exp\{-i \ H_l t\}$ is exact

  - otherwise, it is needed to apply the Suzuki-Trotter decomposition

$$\hat{U}(t) = e^{-i\hat{\mathcal{H}}t} \approx (e^{-i\hat{\mathcal{H}}_1\tau} \cdots e^{-i\hat{\mathcal{H}}_L\tau})^n \qquad \tau = t/n$$

- Program the Time Evolution through simple and two-qubit gates on the real hardware

Steps to Time Evolve your models

Time Evolution through Suzuki-Trotter decomposition

Original Hubbard model fermionic Hamiltonian

$$\mathcal{H} = -t \sum_{<i,j>}^{N} (\hat{b}_i^{\dagger}\hat{b}_j + \hat{b}_j^{\dagger}\hat{b}_i) + V \sum_{i}^{N} (\hat{n}_{i\uparrow}\hat{n}_{i\downarrow})$$

Jordan-Wigner Transform OR Built-in Aqua functions

Spin Hamiltonian

$$\mathcal{H}_{spin}$$

Unitary evolution with Suzuki-Trotter decomposition

Original Ising transverse field model spin Hamiltonian

$$\mathcal{H}_t = b({S_1}^x + {S_2}^x) + j({S_1}^z {S_z}^z)$$

Unitary evolution with Suzuki-Trotter decomposition

Choose your own problem to Time Evolve!

- 2 to n sites Hubbard model
- 2 to n sites Ising model (transverse magnetic field)
- 2 to n sites Heisenberg model
- ...

Heisenberg model, n=2

Time Evolution through Suzuki-Trotter decomposition

Original Heisenberg model Hamiltonian

$$\mathcal{H} = J'(s_{x1}s_{x2} + s_{y1}\, s_{y2} + s_{z1}s_{z2}) + Bg(s_{z1} + s_{z2})$$

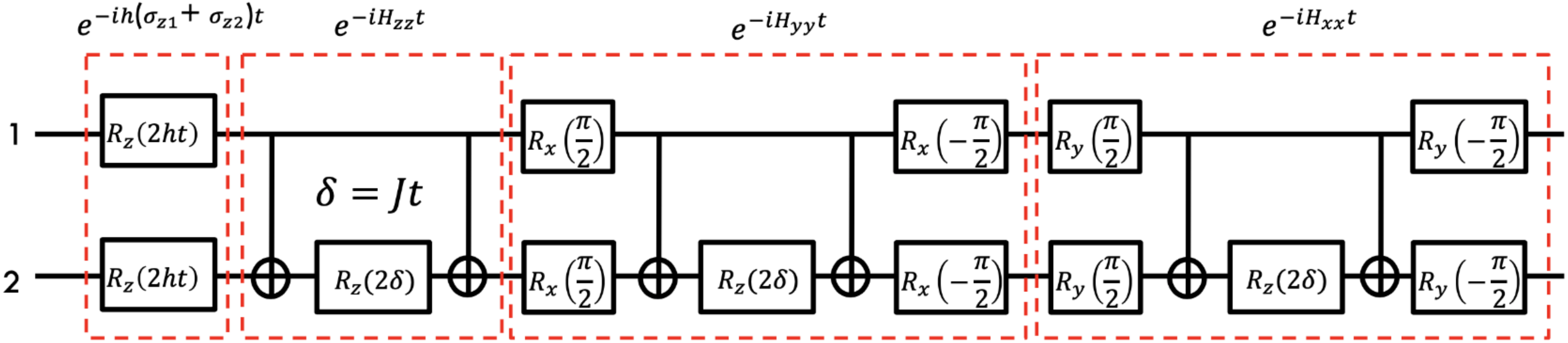
Mapped Hamiltonian

$$H_2 = J(\sigma_{x1}\sigma_{x2} + \sigma_{y1}\, \sigma_{y2} + \sigma_{z1}\sigma_{z2}) + h(\sigma_{z1} + \sigma_{z2})$$

Unitary evolution

$$e^{-iH_2 t} = e^{-iH_{xx}t} e^{-iH_{yy}t}\, e^{-iH_{zz}t} e^{-ih(\sigma_{z1} + \sigma_{z2})t}$$

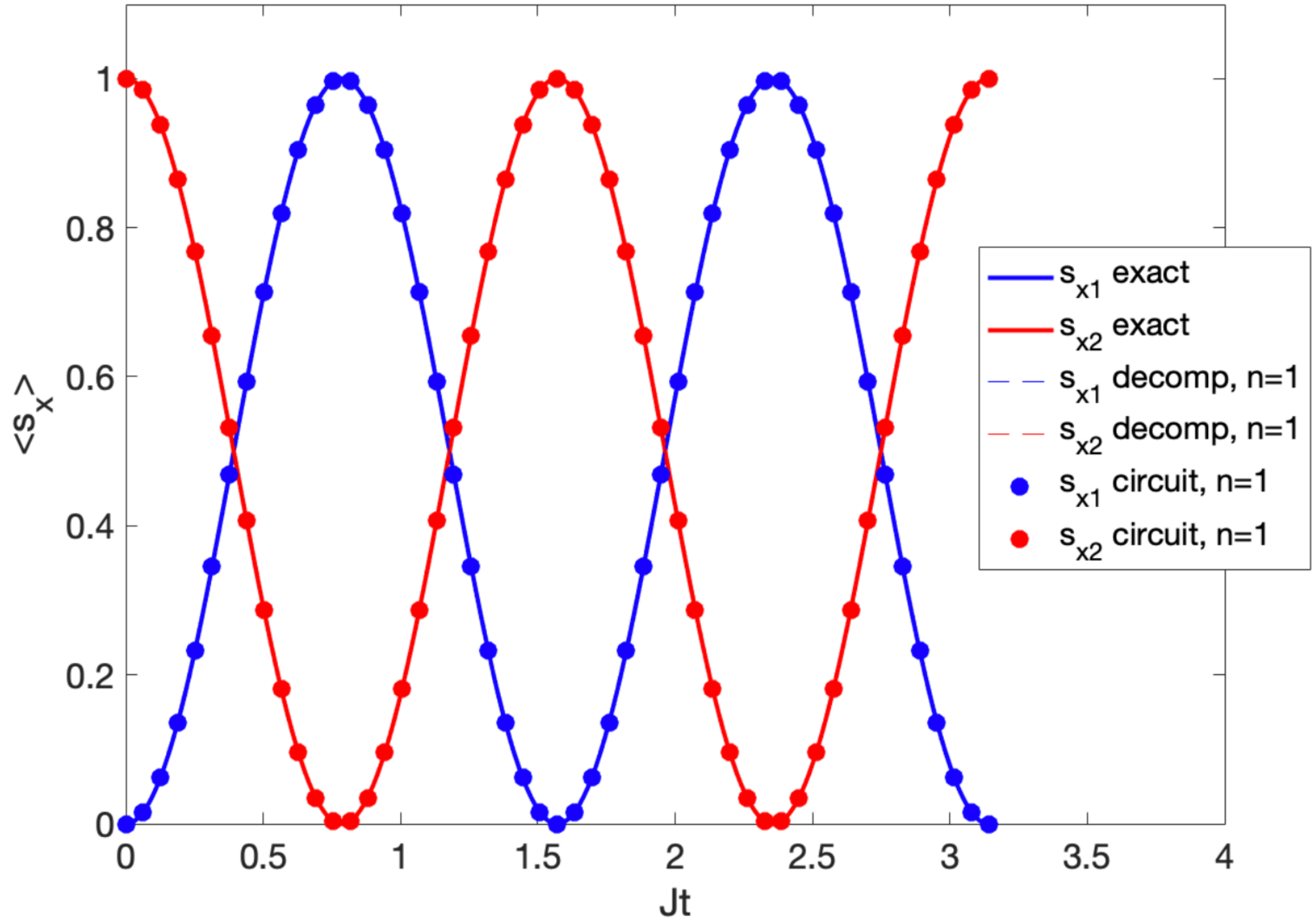
Circuit model for the unitary evolution



Heisenberg model, n=2

Time Evolution through Suzuki-Trotter decomposition

h=0



Mapped Hamiltonian

$$H_2 = J(\sigma_{x1}\sigma_{x2} + \sigma_{y1}\,\sigma_{y2} + \sigma_{z1}\sigma_{z2}) + h(\sigma_{z1} + \sigma_{z2})$$

Initial state

(|00> + |01>) /√2

Results

$< \sigma_{x1} > (t)$; $< \sigma_{x2} > (t)$

TO DO: change h

Native and decomposed gates implemented on IBM Q

Native gates

$$U_1(\lambda) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\lambda} \end{pmatrix}$$

$$U_3(\vartheta, \lambda, \varphi) = \begin{pmatrix} \cos\frac{\vartheta}{2} & -e^{i\lambda}\sin\frac{\vartheta}{2} \\ e^{i\varphi}\sin\frac{\vartheta}{2} & e^{i(\lambda+\varphi)}\cos\frac{\vartheta}{2} \end{pmatrix}$$

$$U_2(\varphi, \lambda) = \begin{pmatrix} \frac{1}{\sqrt{2}} & -e^{i\lambda}\frac{1}{\sqrt{2}} \\ e^{i\varphi}\frac{1}{\sqrt{2}} & e^{i(\lambda+\varphi)}\frac{1}{\sqrt{2}} \end{pmatrix}$$

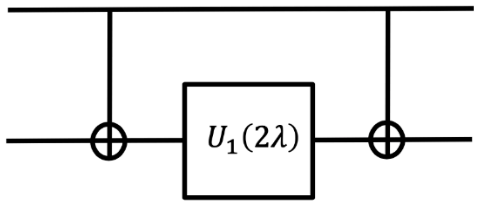
Single qubit rotations

$$CX = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

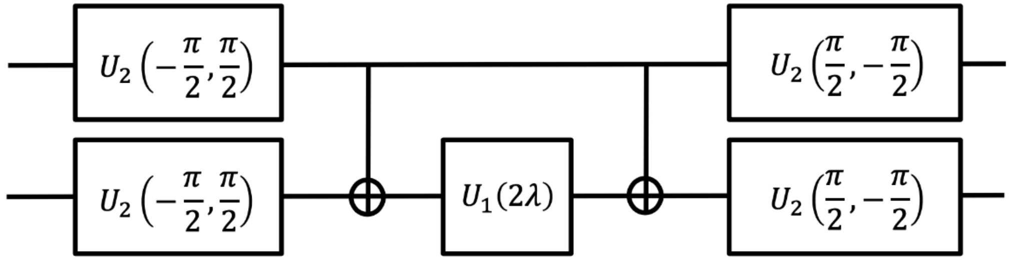
CNOT: two-qubit entangling gate

Decomposed gates

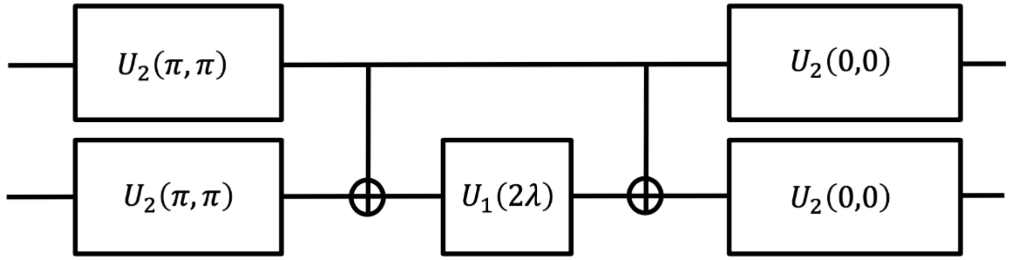
$e^{-i\lambda\sigma_1^z\sigma_2^z} =$



$e^{-i\lambda\sigma_1^y\sigma_2^y} =$



$e^{-i\lambda\sigma_1^x\sigma_2^x} =$



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