$$\int_0^1 ch(t^2)e^{t^2}dt$$ Appossimare con numero razionale e con errore $< \frac{1}{100}$

$$ch(t^2)e^{t^2} = \frac{e^{t^2}+e^{-t^2}}{2}e^{t^2} = \frac{1}{2}e^{2t^2} + \frac{1}{2}$$

$$\int_0^1 ch(t^2)e^{t^2} = \frac{1}{2}\int_0^1 e^{2t^2}dt + \frac{1}{2}$$

$\int_0^1 e^{2t^2}dt$ ed approssimiamolo con un numero razionale, con errore $< \frac{1}{50}$

$$\int_0^1 e^{2t^2}\,dt = \sum_{j=0}^{n} \frac{2^j}{j!} \int_0^1 t^{2j}\,dt + \int_0^1 E_n(2t^2)\,dt$$

$$\frac{1}{2j+1}$$

$$e^y = \sum_{j=0}^{n} \frac{y^j}{j!} + E_n(y)$$

Per $y > 0$ $\qquad E_n(y) = \frac{e^{c_{ny}}}{(n+1)!}\, y^{n+1} \qquad 0 < c_{ny} < y$

$$e^{2t^2} = \sum_{j=0}^{n} \frac{2^j t^{2j}}{j!} + E_n(2t^2)$$

$$= \sum_{j=0}^{n} \frac{2^j}{j!\,(2j+1)} + \int_0^1 E(2t^2)\,dt$$

$$\int_0^1 E_n(2t^2)\,dt \qquad E_n(y) = \frac{e^{c_n y}}{(n+1)!}\, y^{n+1}$$

$$= \frac{\int_0^1 e^{c_{n(2t^2)}}\, 2^{n+1}\, t^{2n+2}\, dt}{(n+1)!} \le \frac{2^{n+1}}{(n+1)!}\, e^2 \int_0^1 t^{2n+2}\, dt$$

$$< \boxed{\frac{9\; 2^{n+1}}{(n+1)!\;(2n+3)}} < \frac{1}{50}$$

$$0 < c_{n(2t^2)} < 2t^2 \le 2$$

$$0 \le t \le 1$$

$$o(x^n) + o(x^{n+1}) = o(x^n)$$ nello 0.

Se io devo dimostrare che $f(x) = o(x^n)$, significa che devo dimostrare $\lim_{x \to 0} \frac{f(x)}{x^n} = 0$

Nella fattispecie, io avrò $o(x^n) + o(x^{n+1}) = o(x^n)$ se riesco a dimostrare che $\lim_{x \to 0} \frac{o(x^n) + o(x^{n+1})}{x^n} = 0$

In questo caso

$$\boxed{\lim_{x\to 0} \frac{o(x^n) + o(x^{n+1})}{x^n}} = \lim_{x\to 0} \left[\frac{o(x^n)}{x^n} + \frac{o(x^{n+1})}{x^n} \right]$$

$$= \lim_{x\to 0} \frac{o(x^n)}{x^n} + \lim_{x\to 0} \frac{o(x^{n+1})}{x^n}$$

$$= 0 + \lim_{x\to 0} \frac{o(x^{n+1})}{x^{n+1}} x = \lim_{x\to 0} \frac{o(x^{n+1})}{x^{n+1}} \lim_{x\to 0} x$$

$$= 0 \cdot 0 = 0$$

$$\Longrightarrow o(x^n) + o(x^{n+1}) = o(x^n)$$

$$f(x) = \begin{cases} \displaystyle\int_x^{2x} \frac{\lg(1+t)}{t^2}\,dt & \text{per } x > 0 \\ \displaystyle\int_0^{x} e^{[2t]}\,dt & \text{per } x \le 0 \end{cases}$$

1) $$f(0) = \int_0^0 e^{[2t]}\,dt = 0 = \lim_{x\to 0^-} \int_0^x e^{[2t]}\,dt = \lim_{x\to 0^-} f(x)$$

$$f(0^+) = \lim_{x\to 0^+} f(x) = \lg 2$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \int_x^{2x} \frac{\lg(1+t)}{t^2}\,dt =$$

$$= \lim_{x\to 0^+} \int_x^{2x} \frac{t - \frac{t^2}{2} + o(t^2)}{t^2}\,dt = \lim_{x\to 0^+} \int_x^{2x} \left[\frac{1}{t} - \frac{1}{2} + o(1)\right] dt$$

$$= \lg 2 + \lim_{x\to 0^+} \int_x^{2x} \left(-\frac{1}{2} + o(1)\right) dt$$

$$= \lg(2)$$

2) $\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \int_x^{2x} \frac{\lg(1+t)}{t^2}\, dt$

So che per $t \to +\infty$, $\lg(1+t) = o(t^{\varepsilon}) \quad \forall \varepsilon > 0$

cioè $\lim_{t \to +\infty} \frac{\lg(1+t)}{t^{\varepsilon}} = 0$ $\qquad \varepsilon = \frac{1}{2}$

$$\int_x^{2x} \frac{\lg(1+t)}{t^2}\, dt = \int_x^{2x} \frac{o(t^{\frac{1}{2}})}{t^2}\, dt = \int_x^{2x} \frac{t^{\frac{1}{2}} o(1)}{t^2}\, dt$$

$$= \int_x^{2x} \frac{o(1)}{t^{\frac{3}{2}}}\, dt < \int_x^{2x} \frac{1}{t^{\frac{3}{2}}}\, dt < \int_x^{+\infty} t^{-\frac{3}{2}}\, dt = \frac{x^{-\frac{3}{2}+1}}{\frac{3}{2} - 1} \xrightarrow{x \to +\infty} 0$$

$$\lim_{x \to +\infty} f(x) = 0$$

3) $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \int_0^x e^{[2t]}dt = -\lim\limits_{x\to-\infty} \int_x^0 e^{[2t]}dt$

$= -\int_{-\infty}^0 e^{[2t]}dt$ se l'integrale esiste.

~~Sia~~ $\int_{-\infty}^0 e^{[2t]}dt = \lim\limits_{y\to-\infty} \int_{-\infty}^0 e^{[2t]}dt = \lim\limits_{n\to+\infty} \int_{-n}^0 e^{[2t]}dt$

2n intervalli

$\int_{-n}^0 e^{[2t]}dt$

$= \int_{-n}^{-n+\frac{1}{2}} e^{[2t]}dt + \int_{-n+\frac{1}{2}}^{-n+1} e^{[2t]} + \dots + \int_{-1}^{-\frac{1}{2}} e^{[2t]}dt + \int_{-\frac{1}{2}}^{0} e^{[2t]}dt$

$$\int_{-n}^{0} e^{[2t]}\,dt = \int_{-n}^{-n+\frac{1}{2}} e^{[2t]} + \int_{-n+\frac{1}{2}}^{-n+1} e^{[2t]} + \dots + \int_{-1}^{-\frac{1}{2}} e^{[2t]} + \int_{-\frac{1}{2}}^{0} e^{-2t} =$$

$$= \sum_{j=1}^{2n} \int_{-n+\frac{j-1}{2}}^{-n+\frac{j}{2}} e^{[2t]}\,dt = \sum_{j=1}^{2n} e^{-2n+j-1} \cdot \frac{1}{2} =$$

Per $-n+\frac{j-1}{2} < t < -n+\frac{j}{2}$

$$-2n+j-1 < 2t < -2n+j \Rightarrow [2t] = -2n+j-1$$

$$= \frac{e^{-2n-1}}{2} \sum_{j=1}^{2n} e^{j} = \frac{e^{-2n-1}}{2} \sum_{j=0}^{2n} e^{j} - \frac{e^{-2n-1}}{2} =$$

$$= \frac{e^{-2n-1}}{2} \frac{1-e^{2n-1}}{1-e} - \frac{e^{-2n-1}}{2}$$

$$\int_{-n}^{0} e^{[2t]}\,dt = \frac{e^{-2n-1}}{2}\,\frac{\cancel{1} - e^{2n-1}}{\cancel{1} - e} + o(1) =$$

$$= \frac{e^{-2n-1} - e^{-2}}{2(1-e)} + o(1) = \frac{e^{-2}}{2(e-1)} + o(1)$$

$$\lim_{x \to -\infty} f(x) = -\lim_{n \to +\infty} \int_{-n}^{0} e^{[2t]}\,dt = -\frac{e^{-2}}{2(e-1)}$$

$$a > 0$$

$$\lim_{x \to 0^+} \int_x^{2x} \lg^2\left(1 + \frac{1}{t^a}(1 + \sin t)\right) dt = 0$$

$$\sin t = o(1)$$

$$\lg\left(1 + t^{-a} + t^{-a} \sin(t)\right) =$$

$$= \lg\left(1 + t^{-a} + t^{-a} o(1)\right) = \lg\left(t^{-a}\left(t^a + 1 + o(1)\right)\right)$$

$$= \lg\left(t^{-a}(1 + o(1))\right) = \lg\left(t^{-a}\right) + \lg\left(1 + o(1)\right) =$$

$$= -a \lg t + o(1)$$

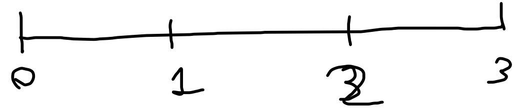
$$\int_x^{2x} \left(-a \lg t + o(1)\right)^2 dt = \int_x^{2x} \left(a^2 \lg^2 t - 2a \lg t \; o(1) + o(1)\right) dt$$

$$= \underbrace{a^2 \int_x^{2x} \lg^2 t \, dt}_{\substack{\downarrow \; x\to 0 \\ 0}} + \underbrace{\int_x^{2x} \lg t \; o(1) \; dt}_{\substack{\downarrow \; x\to 0 \\ 0}} + \underbrace{\int_x^{2x} o(1) \; dt}_{o(1)}$$

$$\lim_{t\to 0^+} \frac{\lg^2(t)}{t^{-\frac{1}{4}}} = 0 \qquad \Rightarrow 0 < \int_x^{2x} \lg^2 t \, dt < \int_x^{2x} o(1) \; t^{-\frac{1}{4}} < \int_x^{2x} t^{-\frac{1}{4}} dt$$

$$\lg^2(t) = o\left(t^{-\frac{1}{4}}\right) = t^{-\frac{1}{4}} o(1) \qquad = \frac{4 t^{\frac{3}{4}}}{3} \Bigg]_x^{2x} \xrightarrow{x \to 0} 0$$

$$\int_0^3 x^2 [x] \sin x \, dx =$$



$$= \underbrace{\int_0^1 x^2 \underset{\underset{0}{\parallel}}{[x]} \sin x \, dx}_{?} + \int_1^2 x^2 \sin x \, dx + 2\int_2^3 x^2 \sin x \, dx$$

$$\int x^2 \sin x \, dx$$

$$\int_0^{+\infty} \frac{\sin x}{x^{\frac{3}{2}}} \lg\left(1+x+e^{\sqrt[4]{x}}\right) dx = \int_0^1 \frac{\sin x}{x^{3/2}} \lg\left(1+x+e^{\sqrt[4]{x}}\right) dx$$

$$+ \int_1^{+\infty} \frac{\sin x}{x^{\frac{3}{2}}} \lg\left(1+x+e^{\sqrt[4]{x}}\right) dx$$

Due problemi:

1) singolarità nello 0

2) cosa succede a $+\infty$

1) $\frac{\sin x}{x^{\frac{3}{2}}} \lg\left(1+x+e^{\sqrt[4]{x}}\right) = \frac{1}{x^{\frac{1}{2}}} \underbrace{\frac{\sin x}{x}}_{=1+o(1)} \lg(2+o(1)) \in L[0,1]$

$$\int_1^{+\infty} \frac{\sin x}{x^{3/2}} \lg\left(1+x+e^{\sqrt[4]{x}}\right) dx$$

ass convergente.

$$\lg\left(1+x+e^{\sqrt[4]{x}}\right) = \lg\left(e^{\sqrt[4]{x}}\left(1+e^{-\sqrt[4]{x}}+e^{-\sqrt[4]{x}}\,x\right)\right) =$$

$$= \sqrt[4]{x} + \lg\left(1+o(1)\right) = \sqrt[4]{x} + o(1)$$

$$= \int_1^{+\infty} \frac{\sin x}{x^{3/2}} \left(\sqrt[4]{x} + o(1)\right) dx = \int_1^{+\infty} \frac{\sin x}{x^{5/4}}\, dx +$$

$$+ \int_1^{\infty} \frac{\sin x}{x^{3/2}}\, o(1)\, dx$$

$$\lg(1+t) = t - \frac{t^2}{2} + o(t^2)$$

$$\lg(1+t) = \int_0^t \frac{1}{1+s}\, ds = \int_0^{t} \left[\sum_{j=0}^{n} (-1)^j s^j + o(s^n) \right] ds$$

$$= \sum_{j=0}^{n} (-1)^j \int_0^t s^j\, ds + \int_0^t o(s^n)\, ds$$

$$= \sum_{j=0}^{n} (-1)^j \frac{t^{j+1}}{j+1} + o(t^{n+1})$$