

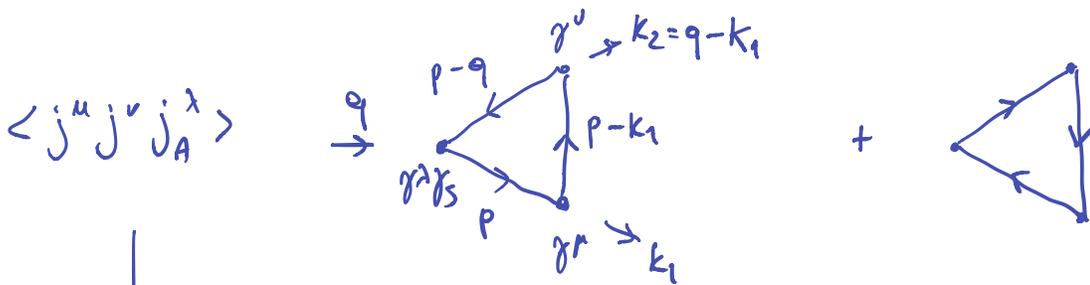
CHIRAL ANOMALY IN 4d

$$p = \bar{\psi} \gamma_5 \psi$$

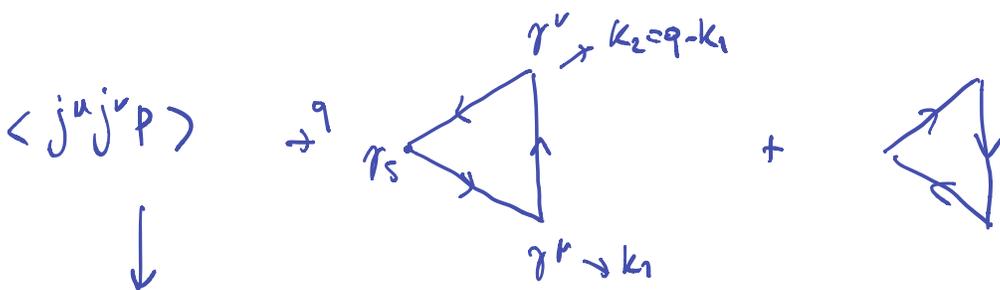
Ci calcoliamo i correlatori $\langle j^\mu j^\nu j_A^\lambda \rangle$ e $\langle P j^\mu j^\nu \rangle$

AWI : $q_\lambda T^{\mu\nu\lambda} = 2m T^{\mu\nu}$

VWI : $k_{1\mu} T^{\mu\nu\lambda} = 0 = k_{2\nu} T^{\mu\nu\lambda}$



$$T^{\mu\nu\lambda} = i \int \frac{d^4 p}{(2\pi)^4} (-) \text{tr} \left[\frac{i}{\not{p} - m} \gamma^\lambda \gamma_5 \frac{i}{\not{p} - \not{q} - m} \gamma^\nu \frac{i}{\not{p} - \not{k}_1 - m} \gamma^\mu \right] + \left(\begin{matrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{matrix} \right)$$



$$T^{\mu\nu} = i \int \frac{d^4 p}{(2\pi)^4} (-) \text{tr} \left[\frac{i}{\not{p} - m} \gamma_5 \frac{i}{\not{p} - \not{q} - m} \gamma^\nu \frac{i}{\not{p} - \not{k}_1 - m} \gamma^\mu \right] + \left(\begin{matrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{matrix} \right)$$

Avremo bisogno della seguente identità:

$$\not{q} \gamma_5 = \gamma_5 (\not{p} - \not{q} - m) + (\not{p} - m) \gamma_5 + 2m \gamma_5$$

$$g_{\lambda} T^{\mu\nu\lambda} = 2m T^{\mu\nu} + \underbrace{R_1^{\mu\nu} + R_2^{\mu\nu}}_{1^{\circ}} \quad \underbrace{\quad}_{2^{\circ}}$$

$$R_1^{\mu\nu} = \int \frac{d^4 p}{(2\pi)^4} \text{tr} \left(\frac{1}{\not{p} - \not{k}_2 - m} \gamma_5 \gamma^\nu \underbrace{\frac{1}{\not{p} - \not{q} - m}}_{= \not{k}_1 + \not{k}_2} \gamma^\mu - \frac{1}{\not{p} - m} \gamma_5 \gamma^\nu \frac{1}{\not{p} - \not{k}_1 - m} \gamma^\mu \right)$$

$$R_2^{\mu\nu} = \int \frac{d^4 p}{(2\pi)^4} \text{tr} \left(\frac{1}{\not{p} - \not{k}_1 - m} \gamma_5 \gamma^\mu \frac{1}{\not{p} - \not{q} - m} \gamma^\nu - \frac{1}{\not{p} - m} \gamma_5 \gamma^\mu \frac{1}{\not{p} - \not{k}_2 - m} \gamma^\nu \right)$$

Se $R_1 = R_2 = 0 \Rightarrow$ AWI è soddisfatta

→ Formalmente $R_1 \rightarrow 0$ se shiftiamo $p \rightarrow p + k_2$ nel 1° integrale e $p \rightarrow p - k_2$ nel 2° integrale

$$R_1^{\mu\nu} = \int \frac{d^4 p}{(2\pi)^4} \text{tr} \left(\frac{1}{\not{p} - m} \gamma_5 \gamma^\nu \frac{1}{\not{p} - \not{k}_1 - m} \gamma^\mu - \frac{1}{\not{p} - \not{k}_2 - m} \gamma_5 \gamma^\nu \frac{1}{\not{p} - \not{q} - m} \gamma^\mu \right)$$

$$= -R_1^{\mu\nu} \Rightarrow R_1^{\mu\nu} = 0$$

Stesso cont. in $R_2^{\mu\nu}$ ($p \rightarrow p \pm k_1$)

div. lineare
 → siccome l'integrale diverge, le completazioni interne all'integrale non sono permesse ($\infty - \infty$)

→ dobbiamo regolarizzare la teoria.

Per pto conto useremo la regolarizzazione di PAULI-VILLARS:

Introduciamo una specie ausiliaria di fermioni Ψ con

STATISTICA OPPOSTA e massa M ; alla fine del conto

bisogna prendere il lim. $M \rightarrow \infty$ (statistica errata)



$$\hookrightarrow \circ T_{reg}^{\mu\nu\lambda} = T^{\mu\nu\lambda}(m) - T^{\mu\nu\lambda}(M)$$

$$T_{phys}^{\mu\nu\lambda} \equiv \lim_{M \rightarrow \infty} T_{reg}^{\mu\nu\lambda}$$

$$\circ T_{phys}^{\mu\nu} = \lim_{M \rightarrow \infty} T_{reg}^{\mu\nu} = \lim_{M \rightarrow \infty} (T^{\mu\nu}(m) - T^{\mu\nu}(M)) = T^{\mu\nu}(m)$$

$T^{\mu\nu}$ è convergente e $T^{\mu\nu}(M) \sim \frac{1}{M} \rightarrow 0$
 $M \rightarrow \infty$

Questa regolarizzazione fa una scelta specifica in
 chi preserva tra AWI e VWI:

$$\begin{aligned} K_{1\mu} T_{phys}^{\mu\nu\lambda} &= 0 \\ K_{2\nu} T_{phys}^{\mu\nu\lambda} &= 0 \end{aligned} \quad \rightarrow \quad \text{VWI è PRESERVATA}$$

(Eunosticam. $L_{\mathcal{G}}$ non rompe $U(1)_V$)

AWI?

Ora $T_{reg}^{\mu\nu\lambda}$ è finito; in particolare $R_{1\eta\gamma}^{\mu\nu}$ e $R_{2\eta\gamma}^{\mu\nu}$ sono
 finiti \rightarrow quindi ora posto per le mescolamenti
 all'interno dell'integrale $\Rightarrow R_{1\eta\gamma}^{\mu\nu} = 0 = R_{2\eta\gamma}^{\mu\nu}$

$$\Rightarrow q_\lambda T_{reg}^{\mu\nu\lambda} = 2m T^{\mu\nu}(m) - 2M T^{\mu\nu}(M)$$

$$\Rightarrow q_\lambda T_{phys}^{\mu\nu\lambda} = 2m T^{\mu\nu}(m) - \lim_{M \rightarrow \infty} 2M T^{\mu\nu}(M)$$

ora calcoleremo q_λ
 (violazione di AWI
 $\neq 0$)

$$T^{\mu\nu}(M) = \int \frac{d^4 p}{(2\pi)^4} (-) \text{tr} \frac{1}{\not{p}-M} \gamma_5 \frac{1}{\not{p}-\not{q}-M} \gamma^\nu \frac{1}{\not{p}-\not{k}_1-M} \gamma^\mu$$

$$+ \left(\begin{matrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{matrix} \right)$$

$$\frac{1}{a_1 a_2 a_3} = 2 \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{1}{[a_1 x_2 + a_2 (1-x_1-x_2) + a_3 x_1]^3}$$

$$= - \int \frac{d^4 p}{(2\pi)^4} 2 \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{\text{tr} [(\not{p}+M) \gamma_5 (\not{p}-\not{q}+M) \gamma^\nu (\not{p}-\not{k}_1+M) \gamma^\mu]}{[(p^2-M^2)x_2 + (p-q)^2-M^2(1-x_1-x_2) + (p-k_1)^2-M^2 x_1]^3}$$

$$+ \left(\begin{matrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{matrix} \right)$$

$$\text{Tr} \gamma_5 \gamma^{d_1} \dots \gamma^{d_n} = 0 \quad \mu \quad n \neq 4$$

$$= 4i \epsilon^{d_1 \dots d_4} \quad \mu \quad n=4$$

$$\text{tr} (\not{p} \gamma_5 (\not{p}-\not{q}) \gamma^\nu \gamma^\mu) + \text{tr} (\not{p} \gamma_5 \gamma^\nu (\not{p}-\not{k}_1) \gamma^\mu) + \text{tr} (\gamma_5 (\not{p}-\not{q}) \gamma^\nu (\not{p}-\not{k}_1) \gamma^\mu)$$

$$= \text{tr} (\gamma_5 \not{p} \not{q} \gamma^\nu \gamma^\mu) + \text{tr} (\gamma_5 \not{p} \gamma^\nu \not{k}_1 \gamma^\mu) - \text{tr} (\gamma_5 \not{p} \gamma^\nu \not{k}_1 \gamma^\mu)$$

$$- \text{tr} (\gamma_5 \not{q} \gamma^\nu \not{p} \gamma^\mu) + \text{tr} (\gamma_5 \not{q} \gamma^\nu \not{k}_1 \gamma^\mu) =$$

$$= 4i \epsilon^{\beta\alpha\mu\nu} k_{1\alpha} k_{2\beta} \quad \begin{matrix} \text{||} \\ k_1+k_2 \end{matrix} \quad \epsilon^{\alpha\beta\mu\nu} = \epsilon^{\beta\alpha\mu\nu} = \epsilon^{\mu\nu\alpha\beta}$$

$$= 4i \epsilon^{\beta\mu\alpha\nu} k_{2\alpha} k_{1\beta}$$

$$= - \int \frac{d^4 p}{(2\pi)^4} 2 \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{M(-4i \epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta})}{[x_2 p^2 - x_2 M^2 + (1-x_1-x_2)(p^2 - 2pq + q^2 - M^2) + x_1(p^2 - 2pk_1 + k_1^2 - M^2)]^3} + \left(\begin{matrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{matrix} \right)$$

$$p^2 - M^2 - 2p(q(1-x_1-x_2) + k_1 x_1) + q^2(1-x_1-x_2) + k_1^2 x_1$$

$$= p^2 - 2pk - M^2 \quad k \equiv q(1-x_1-x_2) + k_1 x_1 \quad \bar{M} \equiv p^2 - q^2(1-x_1-x_2) - k_1^2 x_1$$

$$= 8iM \epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int \frac{d^4 p}{(2\pi)^4} \frac{1}{(p^2 - 2pk - \bar{\pi}^2)^3} + \left(\begin{matrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{matrix} \right)$$

$$p^0 = i\ell^0 \\ \Downarrow \\ p^2 = -\ell^2$$

$$\rightarrow -i \int \frac{d^4 \ell}{(2\pi)^4} \frac{1}{(\ell^2 + 2p\ell + \bar{\pi}^2)^3} = \frac{-i \Gamma(3-2)}{(\bar{\pi}^2 - k^2)^{3-2} (4\pi)^2 \Gamma(3)}$$

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 + 2kp + b^2)^A} = \frac{\Gamma(A-d/2)}{(b^2 - p^2)^{A-d/2} (4\pi)^{d/2} \Gamma(A)}$$

$$= \frac{-i}{(4\pi)^2} \frac{1}{2(\bar{\pi}^2 - k^2)}$$

$$= \frac{1}{4\pi^2} \epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{M}{M^2 + \underbrace{f(x_1, x_2)}_{\text{non dip da } M}} + \left(\begin{matrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{matrix} \right)$$

↑
stessa espressione
del primo termine
ma con $f(x_1, x_2)$
sostituita da
una diversa

$$\lim_{\pi \rightarrow \infty} 2M T^{\mu\nu}(\pi) = \frac{1}{4\pi^2} \epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \lim_{\pi \rightarrow \infty} \left(\frac{2M^2}{M^2 + f(x_1, x_2)} + \frac{2M^2}{\pi^2 + g(x_1, x_2)} \right)$$

$$= \frac{1}{\pi^2} \epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 = 4$$

$$= \int_0^1 dx_1 (1-x_1) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$g_\lambda T_{\text{phys}}^{\mu\nu\lambda} = 2m T^{\mu\nu}(m) - \frac{1}{2\pi^2} \epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta}$$

ANOMALIA

$$\begin{aligned}
\partial_\lambda^z \langle j^\mu(x) j^\nu(y) j_A^\lambda(z) \rangle &= -i \int \frac{d^4q}{(2\pi)^4} \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} e^{-ik_1x - ik_2y + iqz} \\
&\quad \cdot iq_\lambda T^{\mu\nu\lambda}(q, k_1, k_2) \\
&= \int \frac{d^4q}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} e^{-ik_2(y-x) + iq(z-x)} \left(-\frac{1}{2\pi^2} \epsilon^{\mu\nu\alpha\beta} iq_\alpha (-i)k_{2\beta} \right) \\
&= -\frac{1}{2\pi^2} \epsilon^{\mu\nu\alpha\beta} \partial_\alpha^z \delta(z-x) \partial_\beta^y \delta(y-x)
\end{aligned}$$

$$\begin{aligned}
\partial_\lambda^z \langle j_A^\lambda(z) \rangle &= \partial_\lambda^z \langle j_A^\lambda(z) \rangle_{\text{free}} + \dots - \frac{1}{2} \int dx_1 dx_2 A_{S_1}(x_1) A_{S_2}(x_2) \cdot \\
&\quad \cdot \partial_\lambda \langle j^{S_1}(x_1) j^{S_2}(x_2) j_A^\lambda(z) \rangle + \dots
\end{aligned}$$

\uparrow
 campo di gauge
 esterni accoppiato alla
 corrente vettoriale j^μ

$$\begin{aligned}
&= - \int dx_1 dx_2 A_{S_1}(x_1) A_{S_2}(x_2) \left(-\frac{1}{4\pi^2} \right) \epsilon^{S_1 S_2 \alpha\beta} \partial_\alpha^z \delta(z-x_1) \partial_\beta^{x_2} \delta(x_2-x_1) \\
&= \frac{1}{16\pi^2} \epsilon^{S_1 S_2 \alpha\beta} F_{\alpha S_1} F_{\beta S_2} \\
&= -\frac{1}{16\pi^2} \epsilon^{\alpha S_1 \beta S_2} F_{\alpha S_1} F_{\beta S_2} \\
&= -\frac{1}{16\pi^2} \epsilon^{\mu_1 \nu_1 \mu_2 \nu_2} F_{\mu_1 \nu_1} F_{\mu_2 \nu_2}
\end{aligned}$$