$$\begin{cases} x_1 = R\cos\varphi & \dot{x}_1 = -R\dot{\varphi}\,\mathrm{sen}\,\varphi \\ y_1 = R\,\mathrm{sen}\,\varphi & \dot{y}_1 = R\dot{\varphi}\cos\varphi \\ z_1 = 0 \end{cases} \qquad \begin{cases} x_2 = R\cos\psi & \dot{x}_2 = -R\dot{\psi}\,\mathrm{sen}\,\psi \\ y_2 = R\,\mathrm{sen}\,\psi & \dot{y}_2 = R\dot{\psi}\cos\psi \\ z_2 = \gamma R\,\mathrm{sen}\,\psi & \dot{z}_2 = \quad R\dot{\psi}\cos\psi \end{cases}$$

1) $$T = \frac{m}{2}R^2\dot{\varphi}^2 + \frac{m}{2}R^2\dot{\psi}^2\left(1 + \quad \cos^2\psi\right) \qquad a = mR^2\begin{pmatrix} 1 & \\ & 1+\quad\cos^2\psi \end{pmatrix}$$

$$d^2 = R^2\left\{(\cos\varphi - \cos\psi)^2 + (\mathrm{sen}\,\varphi - \mathrm{sen}\,\psi)^2 + \quad \mathrm{sen}^2\psi\right\}$$

$$= R^2\left\{1 + 1 - 2\cos\varphi\cos\psi - 2\,\mathrm{sen}\,\varphi\,\mathrm{sen}\,\psi + \quad \mathrm{sen}^2\psi\right\} =$$

$$= R^2\left\{2 - 2\cos(\varphi-\psi) + \quad \mathrm{sen}^2\psi\right\}$$

$$V = mg\,R\,\mathrm{sen}\,\psi + KR^2\left\{1 - \cos(\varphi-\psi) + \frac{1}{2}\,\mathrm{sen}^2\psi\right\}$$

2) $$L = \frac{m}{2}R^2\dot{\varphi}^2 + \frac{m}{2}R^2\dot{\psi}^2(1 + \quad \cos^2\psi) - mg\,R\,\mathrm{sen}\,\psi - KR^2\left\{1 - \cos(\varphi-\psi) + \frac{1}{2}\,\mathrm{sen}^2\psi\right\}$$

$$\frac{d}{dt}\frac{\partial L}{\partial\varphi} = mR^2\ddot{\varphi} \qquad \ddot{\varphi} = -\frac{K}{m}\,\mathrm{sen}(\varphi-\psi)$$

$$\frac{\partial L}{\partial\varphi} = -KR^2\,\mathrm{sen}(\varphi-\psi)$$

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\psi}} = \frac{d}{dt}\left[mR^2\dot{\psi}(1+\gamma^2\cos^2\psi)\right] = mR^2\ddot{\psi}(1 + \quad \cos^2\psi) - 2mR^2\quad\dot{\psi}^2\,\mathrm{sen}\,\psi\cos\psi$$

$$\frac{\partial L}{\partial\psi} = -mR^2\dot{\psi}^2\quad\cos\psi\,\mathrm{sen}\,\psi - mg\,R\cos\psi - KR^2\,\mathrm{sen}(\psi-\varphi) - KR^2\quad\mathrm{sen}\,\psi\cos\psi$$

$$\ddot{\psi}(1 + \quad \cos^2\psi) = \dot{\psi}^2\,\mathrm{sen}\,\psi\cos\psi - \frac{g}{R}\cos\psi$$

$$-\frac{k}{m}\,\mathrm{sen}(\psi-\varphi) - \frac{k}{m}\quad\mathrm{sen}\,\psi\cos\psi$$

3) $$V = mg\, R \operatorname{sen}\psi + KR^2 \left\{ 1 - \cos(\varphi-\psi) + \frac{1}{2}\operatorname{sen}^2\psi \right\}$$

$$\frac{\partial V}{\partial \varphi} = KR^2 \operatorname{sen}(\varphi-\psi) \qquad \varphi - \psi = K\pi$$

$$\frac{\partial V}{\partial \psi} = mg\, R\cos\psi - KR^2 \operatorname{sen}(\varphi-\psi) + KR^2 \operatorname{sen}\psi\cos\psi$$

$$KR^2 \cos\psi \left(\operatorname{sen}\psi + \frac{mg}{KR} \right) = 0$$

$$\psi = \frac{\pi}{2}, -\frac{\pi}{2} \qquad \operatorname{sen}\psi = -\frac{mg}{KR} \swarrow \exists \text{ se } mg \leq KR$$

$$\partial^2 V = \begin{pmatrix} KR^2\cos(\varphi-\psi) & -KR^2\cos(\varphi-\psi) \\ -KR^2\cos(\varphi-\psi) & -mg\, R\operatorname{sen}\psi + KR^2\cos(\varphi-\psi) + KR^2(1-2\operatorname{sen}^2\psi) \end{pmatrix}$$

$\varphi - \psi = 0$
$$\partial^2 V = \begin{pmatrix} KR^2 & -KR^2 \\ -KR^2 & 2KR^2 - R\operatorname{sen}\psi\,(mg + 2KR\operatorname{sen}\psi) \end{pmatrix}$$

$$\psi = \frac{\pi}{2} \qquad b\left(\frac{\pi}{2},\frac{\pi}{2}\right) = \begin{pmatrix} KR^2 & -KR^2 \\ -KR^2 & 2KR^2 - R(mg + 2KR) \end{pmatrix}$$

$$\det = \cancel{KR^2} - \cancel{2}K R^2 - Rmg < 0 \qquad \text{instabile}$$

$$\psi = -\frac{\pi}{2} \qquad b\left(-\frac{\pi}{2},-\frac{\pi}{2}\right) = \begin{pmatrix} KR^2 & -KR^2 \\ -KR^2 & \cancel{2KR^2} + R(mg - \cancel{2KR}) \end{pmatrix}$$

$$\det = KR^3(mg - KR) \qquad \text{stab. se } mg > KR$$

$$\text{instab. se } mg < KR$$

$\varphi - \psi = \pi$

$$\partial^2 V = \begin{pmatrix} -KR^2 & KR^2 \\ KR^2 & -R\sin\psi\,(mg + 2KR\sin\psi) \end{pmatrix}$$

$\psi = -\frac{\pi}{2}$ $\quad b\left(+\frac{\pi}{2}, -\frac{\pi}{2}\right) = \begin{pmatrix} -KR^2 & KR^2 \\ KR^2 & R(mg - 2KR) \end{pmatrix}$ instab.

$\det = KR^3 \, (KR - mg)$ instab. ~~$mg > KR$~~

~~(stab. $mg < KR$)~~

$\psi = +\frac{\pi}{2}$ $\quad b\left(-\frac{\pi}{2}, +\frac{\pi}{2}\right) = \begin{pmatrix} -KR^2 & KR^2 \\ KR^2 & -R(mg + 2KR) \end{pmatrix}$

$\det = KR^2\,(mg + KR) > 0$ instab

4) $(\varphi, \psi) = \left(-\frac{\pi}{2}, -\frac{\pi}{2}\right)$

$$A = mR^2 \begin{pmatrix} 1 & \\ & 1 + \cos^2\psi \end{pmatrix}_{\psi = \pi/2} = mR^2 \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} KR^2 & -KR^2 \\ -KR^2 & mgR \end{pmatrix}$$

$$B - \lambda A = \begin{pmatrix} KR^2 - \lambda mR^2 & -KR^2 \\ -KR^2 & mgR - \lambda mR^2 \end{pmatrix}$$

$$\det(B - \lambda A) = mgKR^3 - \lambda mKR^4 - \lambda m^2 g R^3 + \lambda^2 m^2 R^4 - K^2R^4 = 0$$

$$\lambda^2 - \lambda\left(\frac{K}{m} + \frac{g}{R}\right) + \frac{K}{m}\frac{g}{R} - \left(\frac{K}{m}\right)^2 = 0$$

$$\lambda_{1,2} = \frac{1}{2}\left(\frac{k}{m} + \frac{g}{R}\right) \pm \frac{1}{2}\sqrt{\underbrace{\left(\frac{k}{m}+\frac{g}{R}\right)^2 - 4\frac{k}{m}\frac{g}{R} + \frac{4k^2}{m^2}}_{\left(\frac{k}{m}-\frac{g}{R}\right)^2 + 4\left(\frac{k}{m}\right)^2}}$$

$$\frac{g}{R} = 3\frac{k}{m}$$

$$\hookrightarrow \lambda_{1,2} = 2\frac{k}{m} \pm \sqrt{2}\frac{k}{m} = \frac{k}{m}\left(2 \pm \sqrt{2}\right)$$

5) ...

6)

$$L = \frac{m}{2}R^2\dot{\varphi}^2 + \frac{m}{2}R^2\dot{\psi}^2 - kR^2\left\{1 - \cos(\varphi - \psi)\right\}$$

1. Energia $\quad \frac{m}{2}R^2\dot{\varphi}^2 + \frac{m}{2}R^2\dot{\psi}^2 + kR^2\left(1 - \cos(\varphi - \psi)\right)$

2. Simmetria $\quad \varphi \to \varphi + \alpha \qquad \frac{\partial L}{\partial \dot{\varphi}} = mR^2\dot{\varphi}$

$\psi \to \psi + \alpha \qquad \frac{\partial L}{\partial \dot{\psi}} = mR^2\dot{\psi}$

$$P = \sum_h \frac{\partial L}{\partial \dot{q}_h}\frac{\partial q_h^\alpha}{\partial \alpha} = \frac{\partial L}{\partial \dot{\varphi}} + \frac{\partial L}{\partial \dot{\psi}} = mR^2(\dot{\varphi} + \dot{\psi})$$