APPENDIX

The determination of the interest rate when people hold both currency and deposit accounts

In Section 4.3, we made the simplifying assumption that people only held deposit accounts and did not hold any currency. We now relax this assumption and derive the equilibrium interest rate under the assumption that people hold both deposit accounts and currency.

The easiest way to think about how the interest rate in this economy is determined is still by thinking in terms of the supply and the demand for *central bank money*:

- The demand for central bank money is equal to the demand for currency by people plus the demand for reserves by banks.
- The supply of central bank money is under the direct control of the central bank.
- The equilibrium interest rate is such that the demand and the supply for central bank money are equal.

Figure 4.10 shows the structure of the demand and the supply of central bank money in more detail. (Ignore the equations for the time being. Just look at the boxes.) Start on the left side. The demand for money by people is for both deposit accounts and currency. Because banks have to hold reserves against deposit accounts, the demand for deposit accounts leads to a demand for reserves by banks. Consequently, the demand for central bank money is equal to the demand for reserves by banks plus the demand for currency. Go to the right side. The supply of central bank money is determined by the central bank. Look at the equals sign: the interest rate must be such that the demand and supply of central bank money are equal.

We now go through each of the boxes in Figure 4.7 and ask:

- What determines the demand for deposit accounts and the demand for currency?
- What determines the demand for reserves by banks?
- What determines the demand for central bank money?
- How does the condition that the demand for and the supply of central bank money be equal determine the interest rate?

The demand for money

When people can hold both currency and deposit accounts, the demand for money involves *two* decisions. First, people must decide how much money to hold. Second, they must decide how much of this money to hold in currency and how much to hold in deposit accounts.

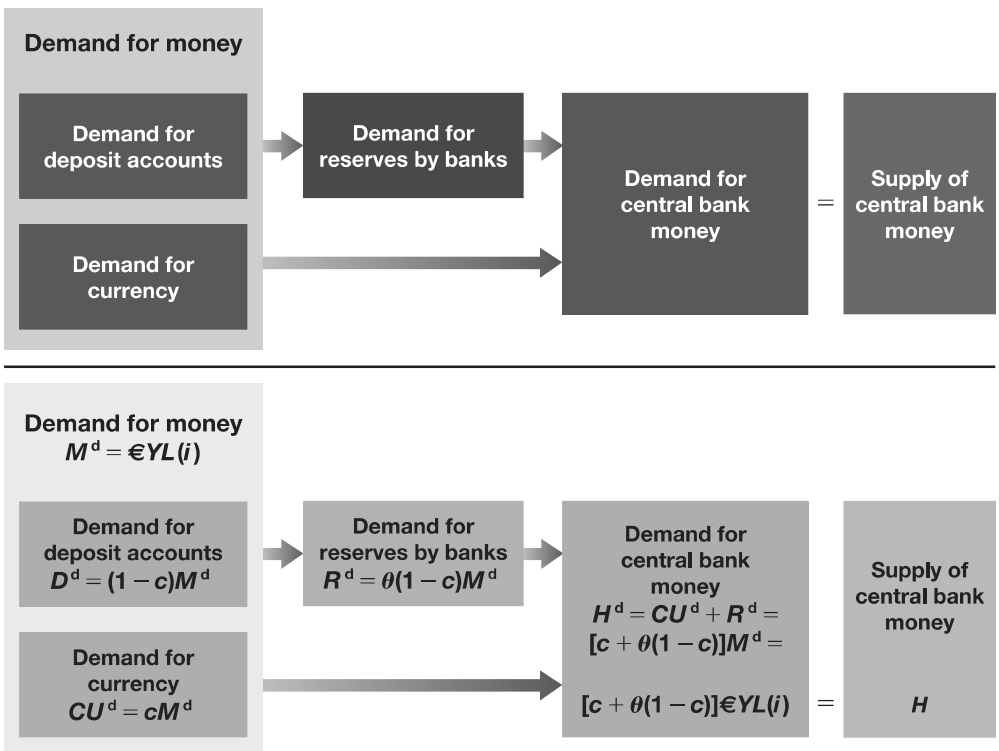

Figure 4.10

Determinants of the demand and the supply of central bank money

It is reasonable to assume that the overall demand for money (currency plus deposit accounts) is given by the same factors as before. People will hold more money the higher the level of transactions and the lower the interest rate on bonds. So we can assume that overall money demand is given by the same equation as before (equation (4.1)):

$$M^d = \in YL(i) \qquad [4A.1]$$

That brings us to the second decision. How do people decide how much to hold in currency and how much in deposit accounts? Currency is more convenient for small transactions (it is also more convenient for illegal transactions). Cheques are more convenient for large transactions. Holding money in your current account is safer than holding cash.

Let's assume people hold a fixed proportion of their money in currency – call this proportion c – and, by implication, hold a fixed proportion $(1 - c)$ in 'chequeable' deposits. Call the demand for currency CU^d (CU for currency and 'd' for demand). Call the demand for deposit accounts D^d (D for deposits and 'd' for demand). The two demands are given by:

$$CU^d = cM^d \qquad [4A.2]$$

$$D^d = (1 - c)M^d \qquad [4A.3]$$

Equation (4A.2) shows the first component of the demand for central bank money – the demand for currency by the public. Equation (4A.3) shows the demand for deposit accounts.

We now have a description of the first box on the left side of Figure 4.10: Equation (4A.1) shows the overall demand for money. Equations (4A.2) and (4A.3) show the demand for deposit accounts and the demand for currency, respectively.

The demand for deposit accounts leads to a demand by banks for reserves, the second component of the demand for central bank money. Let θ be the reserve ratio, the amount of reserves banks hold per euro of deposit accounts. Let R denote the reserves of banks. Let D denote the euro amount of deposit accounts. Then, by the definition of θ, the following relation holds between R and D:

$$R = \theta D \qquad [4A.4]$$

We saw previously that, in the United States today, the reserve ratio is roughly equal to 10%. Thus, θ is roughly equal to 0.1.

If people want to hold D^d in deposits, then, from equation (4A.4), banks must hold θD^d in reserves. Combining equations (4A.2) and (4A.4), the second component of the demand for central bank money – the demand for reserves by banks – is given by:

$$R^d = \theta(1 - c)M^d \qquad [4A.5]$$

We now have the equation corresponding to the second box on the left side of Figure 4.10.

The demand for central bank money

Call H^d the demand for central bank money. This demand is equal to the sum of the demand for currency and the demand for reserves:

$$H^d = CU^d + R^d \qquad [4A.6]$$

Replace CU^d and R^d by their expressions from equations (4A.2) and (4A.5) to get:

$$H^d = cM^d + \theta(1 - c)M^d = [c + \theta(1 - c)]M^d$$

Finally, replace the overall demand for money, M^d, by its expression from equation (4A.1) to get:

$$H^d = c + \theta(1 - c) = \in YL(i) \qquad [4A.7]$$

This gives us the equation corresponding to the third box on the left side of Figure 4.10.

The determination of the interest rate

We are now ready to characterise the equilibrium. Let H be the supply of central bank money, where H is directly controlled by the central bank; as in the previous section, the central bank can change the amount of H through open market operations. The equilibrium condition is that the supply of central bank money be equal to the demand for central bank money:

$$H = H^d \qquad [4A.8]$$

Or, using equation (4A.7):

$$H^d = c + \theta(1 - c) = \in YL(i) \qquad [4A.9]$$

The supply of central bank money (the left side of equation (4A.9)) is equal to the demand for central bank money (the right side of equation (4A.9)), which is equal to the term in brackets times the overall demand for money.

Look at the term in brackets more closely. Suppose that people held only currency, so $c = 1$. Then, the term in brackets would be equal to one and the equation would be exactly the same as equation (4.2) in Section 4.2 (with the letter H replacing the letter M on the left side, but H and M both standing for the supply of central bank money). In this case, people would hold only currency, and banks would play no role in the supply of money. We would be back to the case we looked at in Section 4.2.

Assume instead that people did not hold currency at all, but held only deposit accounts, so $c = 0$. Then, the term in brackets would be equal to θ and the equation would be exactly the same as equation (4.6) in Section 4.3.

Leaving aside these two extreme cases, note that the demand for central bank money is, as it was in Section 4.2, proportional to the overall demand for money, with the factor of proportionality being $[c + \theta(1 - c)]$ rather than just θ. Thus the implications are very much the same as before. A decrease in central bank money leads to an increase in the interest rate; an increase in central bank money leads to a decrease in the interest rate.