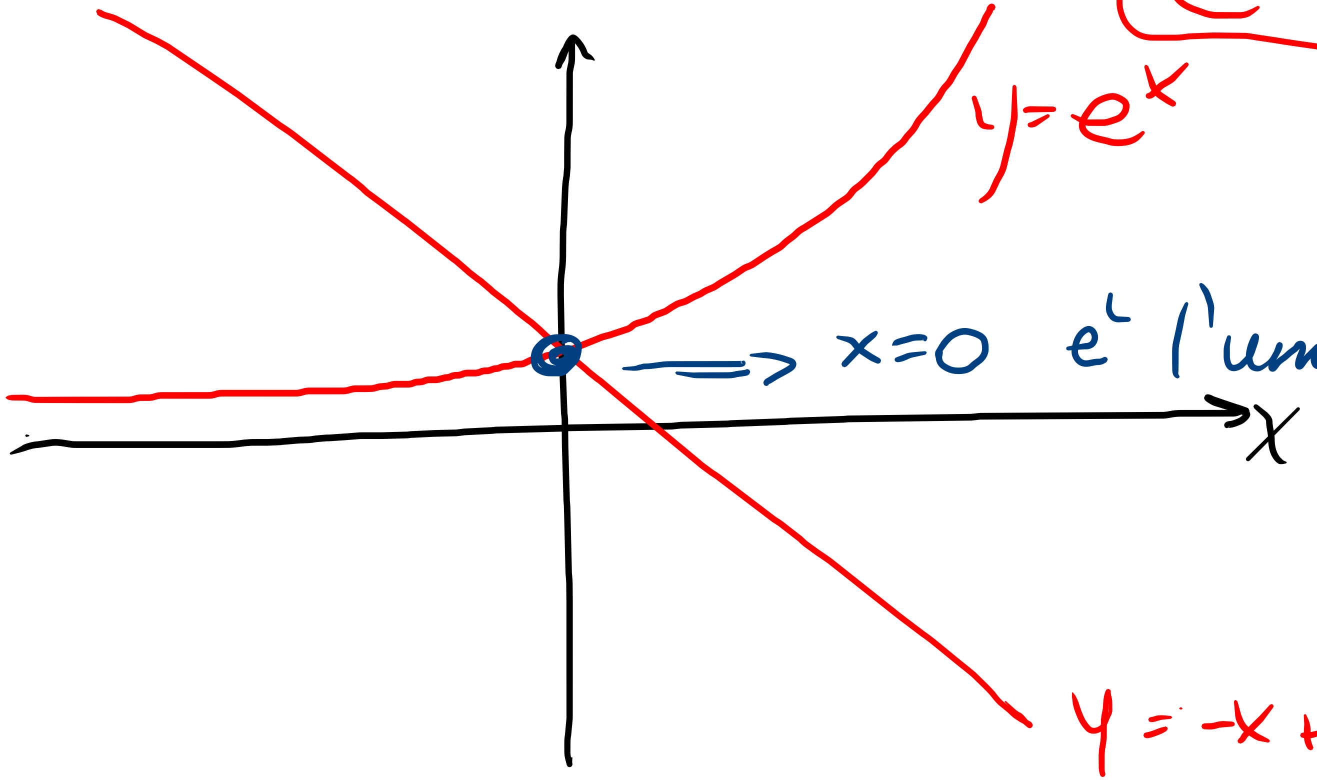
$$f(x) = x(1-e^{-x})$$

$$f'(x) = 1 - e^{-x} + xe^{-x}$$

$$\underbrace{\cancel{e^{-x}}}_{\neq 0}\underbrace{(x-1+e^{x})}=0 \longrightarrow \boxed{e^{x} = -x+1}$$



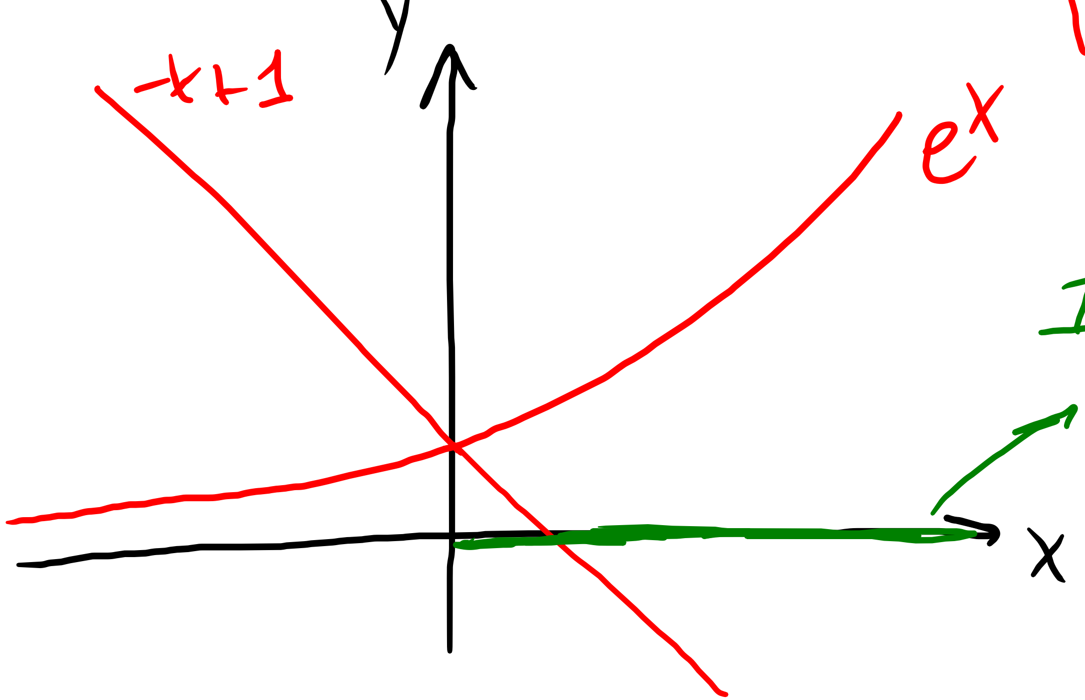
$\Longrightarrow x=0$ è l'unica soluzione!

$f'(x) \geq 0$

$$\underbrace{e^{-x}}_{\text{sempre positivo}}(x-1+e^{x}) \geq 0$$

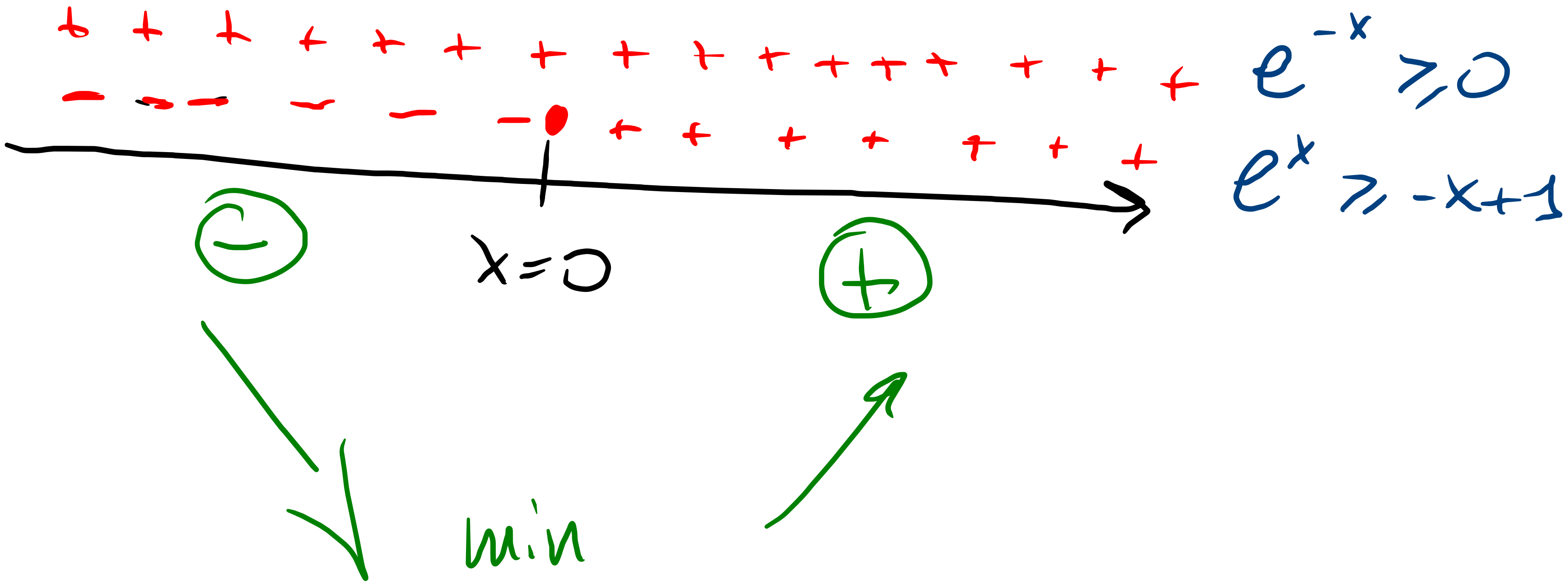
$$(x-1+e^{x}) \geq 0$$

$$\boxed{e^{x} \geq -x+1}$$



In questa parte $e^{x} \geq -x+1$

Quindi soluzione è $x \geq 0$



$e^{-x} \geq 0$
$e^{x} \geq -x+1$
$x=0$
−
+
min

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e \qquad \lim_{x\to 0}\frac{\sin x}{x} = 1$$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2} \qquad \lim_{x\to 0}\frac{e^x-1}{x} = 1$$

$$\lim_{x\to 0}\frac{\ln(1+x)}{x} = 1 \qquad \lim_{x\to 0}\frac{\tan x}{x} = 1$$

$$\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x \qquad x\in\mathbb{R}$$

sost.

$$\underbrace{\frac{x}{n} = \frac{1}{t}}_{\frac{n}{x} = t \Rightarrow n = tx}$$

$$= \lim_{t\to\infty}\left(1+\frac{1}{t}\right)^{tx} = \lim_{t\to\infty}\left[\underbrace{\left(1+\frac{1}{t}\right)^{t}}_{\to e}\right]^x = e^x$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x^2}\right)^x = \nearrow$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x^2}\right)^x = \lim_{x\to\infty}\left\{\left[\left(1+\frac{1}{x^2}\right)^x\right]^x\right\}^{\frac{1}{x}} =$$

$$= \lim_{x\to\infty}\left\{\underbrace{\left(1+\frac{1}{x^2}\right)^{x^2}}_{\to e}\right\}^{\frac{1}{x}} = \lim_{x\to\infty} e^{\underbrace{\frac{1}{x}}_{\to 0}} = 1$$

$$\sqrt[2]{a^2} = a$$

$$\lim_{x\to 0}\left(1+\frac{x}{2}\right)^{\frac{1}{x}} =$$

$$\underbrace{\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} = e}$$

FORMA 1^{∞} $\Rightarrow$ INDETERMINATA

Sost. $\frac{x}{2} = \frac{1}{t}$ $\Longrightarrow$ $\frac{1}{x} = \frac{t}{2}$

$x\to 0$ $\quad$ $t\to\infty$

$$\lim_{t\to\infty}\left(1+\frac{1}{t}\right)^{\frac{t}{2}} = \lim_{t\to\infty}\left[\underbrace{\left(1+\frac{1}{t}\right)^{t}}_{\to e}\right]^{\frac{1}{2}} = \sqrt{e}$$

$$\lim_{x\to 0} \frac{\sin(3x)-\sin(2x)}{\log(x+1)} =$$

limiti notevoli che utilizzerò:

$$= \lim_{x\to 0} \frac{\sin(3x)-\sin(2x)}{\ln(x+1)} \cdot \frac{x}{x} =$$

$$1 \to \lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$$

$$= \lim_{x\to 0} \frac{\sin(3x)}{x} - \frac{\sin(2x)}{x} =$$

$$= \lim_{x\to 0} \frac{\sin(3x)}{x}\frac{3}{3} - \frac{\sin(2x)}{x}\frac{2}{2} = \quad 3-2=1$$

$1 \qquad 1$

$$\lim_{\boxed{x\to-\infty}}\left(x+\sqrt{x^2+x}\right) =$$

$\sqrt[2]{x^2} = |x|$

Quindi se $x<0$ e $\sqrt[2]{x^2} = -x$

FORMA $-\infty+\infty$ INDETERMINATA

Attenzione al segno prima della radice

$$= \lim_{x\to-\infty}\left(x+\sqrt{x^2+x}\right)\cdot\frac{x-\sqrt{x^2+x}}{x-\sqrt{x^2+x}} =$$

$$= \lim_{x\to-\infty}\frac{\cancel{x^2}-(\cancel{x^2}+x)}{x-\sqrt{x^2+x}} = \lim_{x\to-\infty}\frac{-x}{x-\sqrt{x^2\left(1+\frac{1}{x}\right)}}$$

$$= \lim_{x\to-\infty}\frac{-x}{x-(-x)\sqrt{1+\frac{1}{x}}} = \lim_{x\to-\infty}\frac{-\cancel{x}}{\cancel{x}\left(1+\sqrt{1+\frac{1}{x}}\right)} = -\frac{1}{2}$$

$\frac{1}{x}\to 0$

STUDIO FUNZIONE

$$f(x) = x^2(\log|x| - 1)$$

Dominio:

$$|x| > 0 \qquad D: \mathbb{R} - \{0\}$$

Segno:

$$x^2(\log|x| - 1) \geq 0 \qquad x^2 \geq 0$$

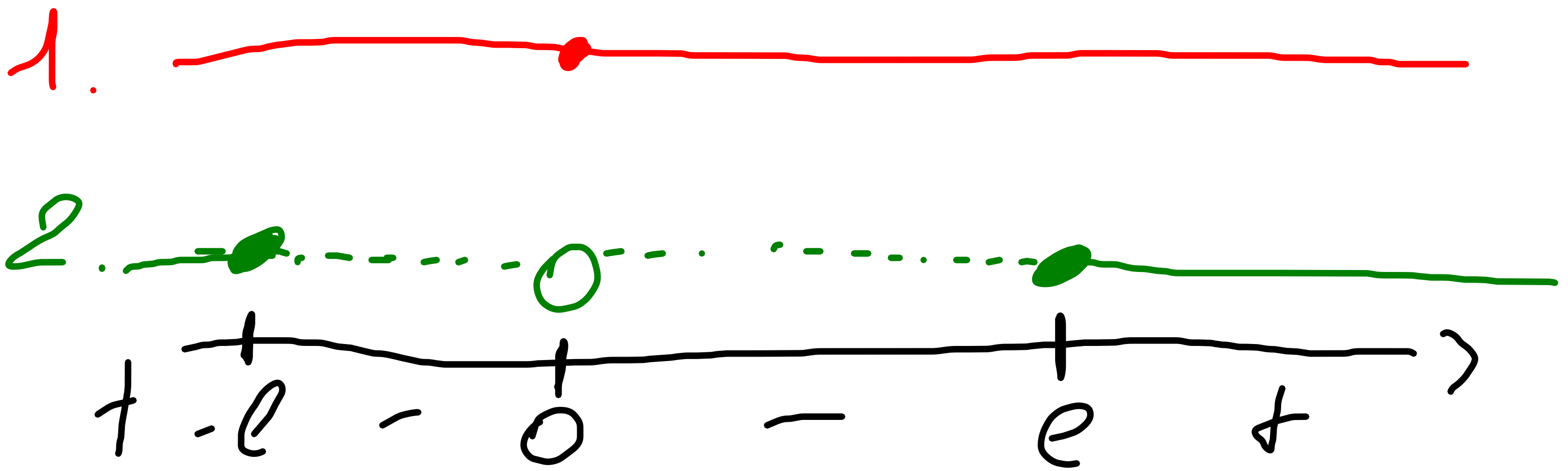
$$\log|x| - 1 \geq 0$$

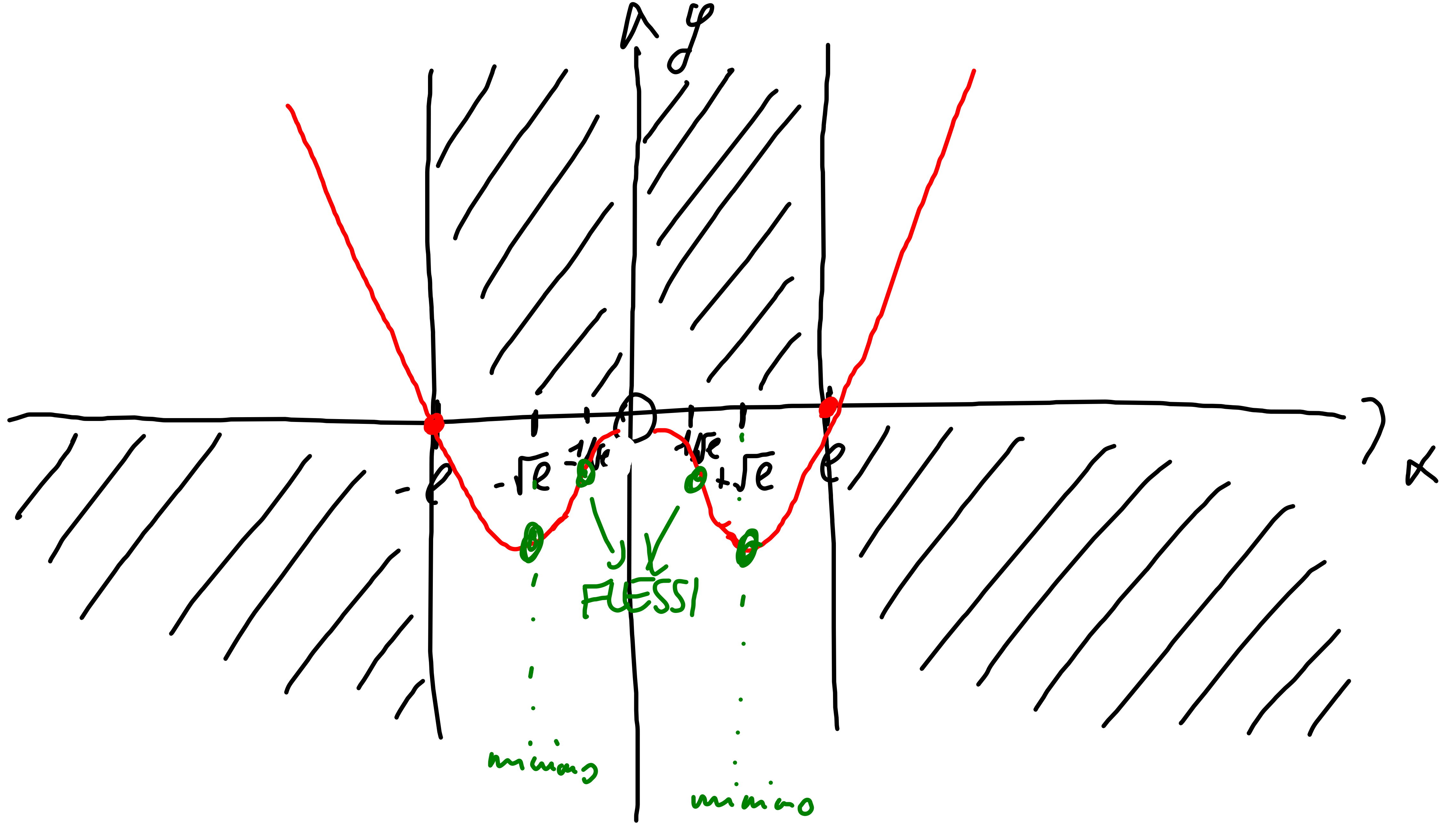
$$\log|x| \geq 1 = \log e$$

$$e^{(\log|x|)} \geq e^{(1)}$$

$$\Downarrow$$

$$|x| \geq e \Rightarrow \begin{array}{l} x \geq e \\ -x \geq e \rightarrow x \leq -e \end{array}$$





y
x
-e
$-\sqrt{e}$
$-1/\sqrt{e}$
0
$1/\sqrt{e}$
$+\sqrt{e}$
e
FLESSI
minimo
minimo

Intersezioni:

asse x : $(e;0)$

$(-e;0)$

Non ci sono int. con asse y

$$f(x) = x^2(\log|x| - 1)$$

$$\lim_{x\to+\infty} x^2(\log|x|-1) = \lim_{x\to+\infty} x^2(\log x - 1) = +\infty$$

$$\lim_{x\to-\infty} x^2(\log|x|-1) = \lim_{x\to-\infty} x^2(\log(-x)-1) = +\infty$$

$$\lim_{x\to 0} x^2(\log|x|-1) =$$ Forma Indeterminata $0\cdot\infty \Longrightarrow 0$

con De L'Hop

$$\lim_{x\to 0} \frac{\log|x|-1}{\frac{1}{x^2}} \Longrightarrow \lim_{x\to 0} \frac{\frac{1}{\cancel{x}}}{-\frac{2}{x^{\cancel{3}2}}} =$$

FORMA $\frac{\infty}{\infty}$

$$= \lim_{x\to 0} -\frac{1}{2}x^2 = 0$$

$$f(x) = x^2(\log|x| - 1)$$

$$f'(x) = 2x(\log|x| - 1) + x^{\cancel{2}}\left(\frac{1}{\cancel{x}}\right) = x(2\log|x| - 1)$$

$$f'(x) = 0 \Rightarrow x^{\cancel{2}}(2\log|x| - 1) = 0$$

$$x^{\cancel{2}} = 0$$

$$2\log|x| - 1 = 0$$

$$2\log|x| = 1 \quad \log|x| = 1/2$$

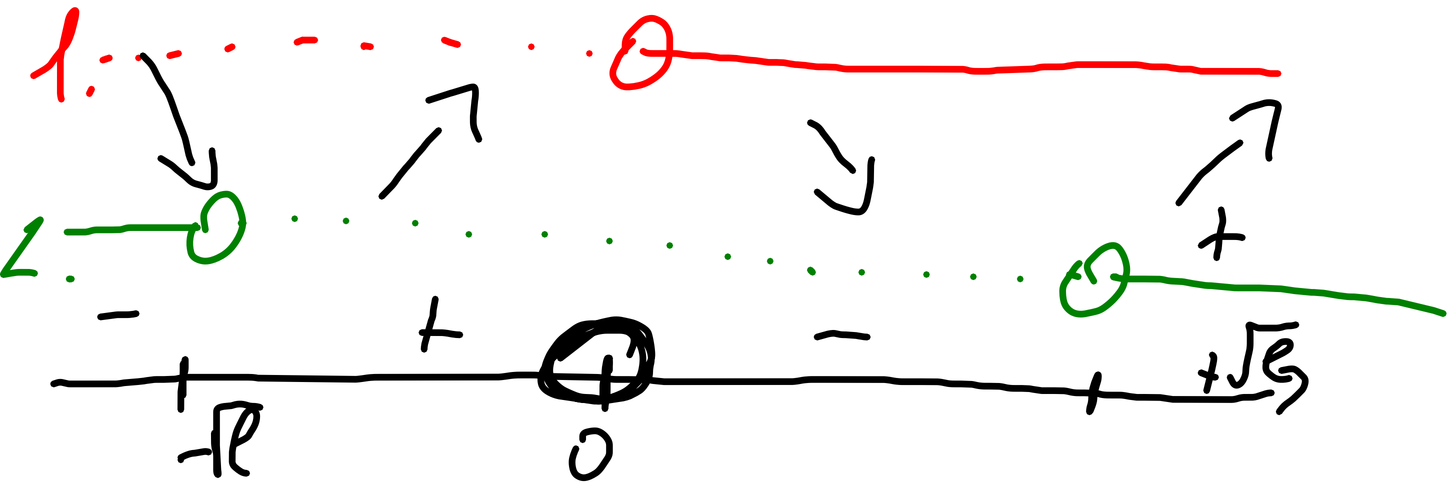
$$e^{\log|x|} = e^{1/2} \qquad |x| = \sqrt{e} \qquad x = \sqrt{e}$$

$$x = -\sqrt{e}$$

$$f'(x) > 0 \quad x(2\log|x| - 1) > 0 \qquad x > 0$$

minimo in $\pm\sqrt{e}$

$$2\log|x| > 0 \Rightarrow |x| > \sqrt{e}$$

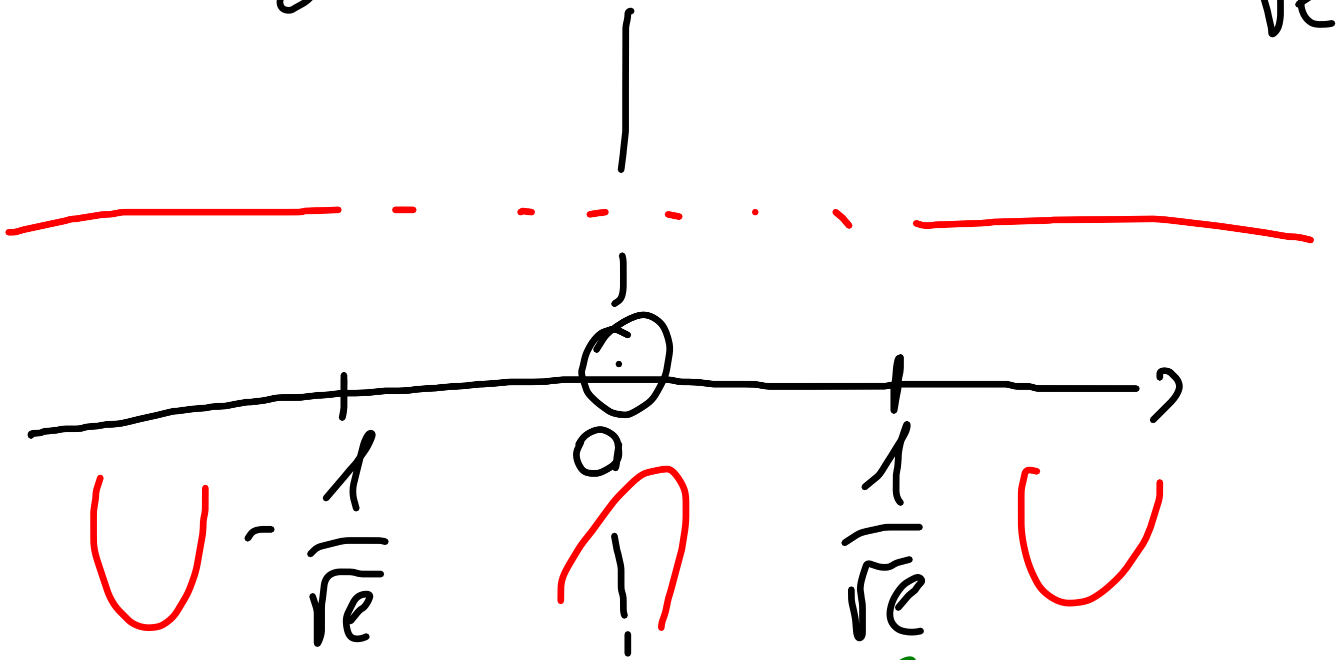


$$f'(x) = x(2\log|x|-1)$$

$$f''(x) = (2\log|x|-1) + \cancel{x}(2/\cancel{x}) = 2\log|x|+1$$

$2\log|x|+1=0 \quad 2\log|x| = -1 \quad \log|x| = -1/2 \quad x = \pm\frac{1}{\sqrt{e}}$

$$2\log|x| + 1 > 0 \qquad |x| > \frac{1}{\sqrt{e}}$$



Flesso discendente

Flesso Ascendente