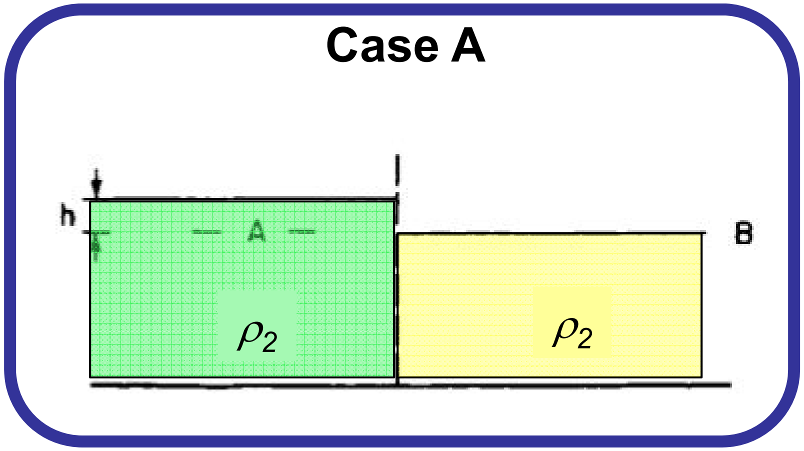
Waves in the Atmosphere and Oceans

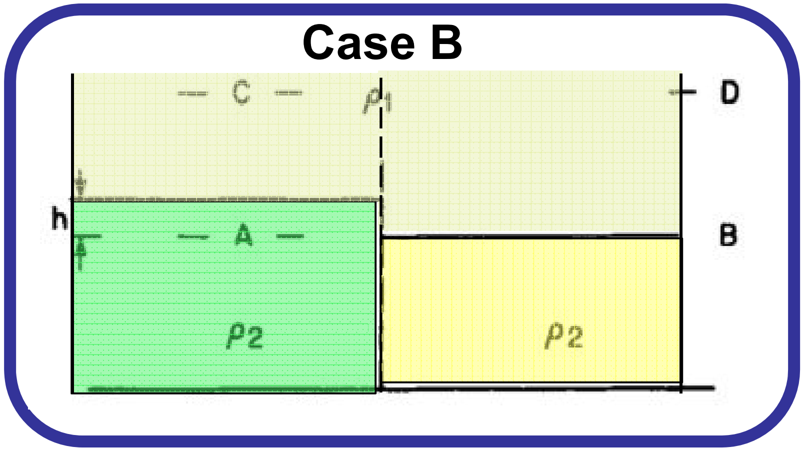
<u>Restoring Force</u>

- ❑Conservation of potential temperature in the presence of positive static stability
  ➔ internal gravity waves
- ❑Conservation of potential vorticity in the presence of a mean gradient of potential vorticity ➔ Rossby waves

- **External gravity wave** (Shallow-water gravity wave)
- **Internal gravity (buoyancy) wave**
- **Inertial-gravity wave**: Gravity waves that have a large enough wavelength to be affected by the earth's rotation.
- **Rossby Wave**: Wavy motions results from the conservation of potential vorticity.
- **Kelvin wave**: It is a wave in the ocean or atmosphere that balances the Coriolis force against a topographic boundary such as a coastline, or a waveguide such as the equator. Kelvin wave is non-dispersive.

Reduced Gravity





Pressure difference between A and B:

$$\Delta P = \rho_2 * g * h$$

Pressure difference between A and B:

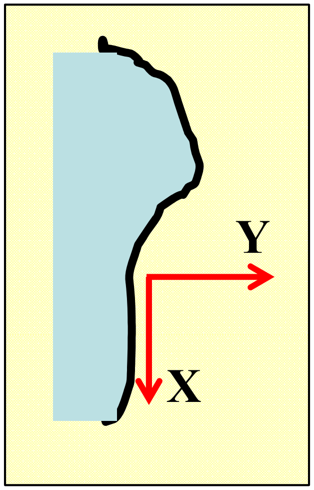
$$\Delta P = (\rho_2 - \rho_1) * g * h$$

buoyancy force = density difference * g

The adjustment process in Case B is exactly the same as in the Case A, except the gravitational acceleration is reduced to a value *g'*, where

$$g' = g(\rho_2 - \rho_1)/\rho_2.$$

Kelvin Waves



Governing Equations

$$\begin{aligned} \frac{du}{dt} - fv &= -g\frac{\partial h}{\partial x} \; ; \\ \frac{dv}{dt} + fu &= -g\frac{dh}{dy} \; ; \\ \frac{dh}{dt} + D\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) &= 0 \; . \end{aligned}$$

A unique boundary condition

$y = 0$ is $v = 0$

$$\begin{pmatrix} u' \\ h' \end{pmatrix} = \mathrm{Re}\left\{ \begin{pmatrix} U(y) \\ H(y) \end{pmatrix} \exp\left[ik\left(x - ct\right)\right]\right\}$$

$$H = const \times \exp\left(-\frac{f}{c}y\right)$$

$$-\frac{f}{g}U = -\frac{f}{c}H$$

$$c = \sqrt{gD}$$

- A Kelvin wave is a type of low-frequency gravity wave in the ocean or atmosphere that balances the Earth's Coriolis force against a topographic boundary such as a coastline, or a waveguide such as the equator.
- Therefore, there are two types of Kelvin waves: coastal and equatorial.
- A feature of a Kelvin wave is that it is non-dispersive, i.e., the phase speed of the wave crests is equal to the group speed of the wave energy for all frequencies.

Costal Kelvin Waves

$$H = const \times \exp\left(-\frac{f}{c}y\right)$$

At the coast $y = 0$ is $v = 0$:

$$c = \sqrt{gD}$$

depth of the fluid

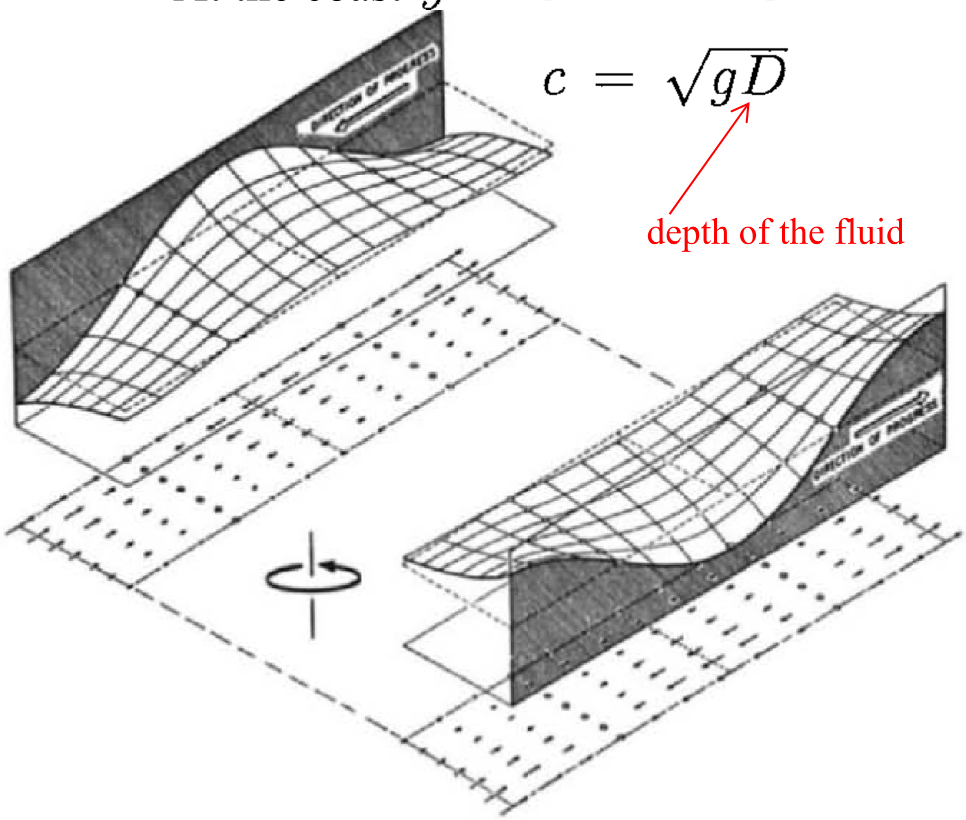


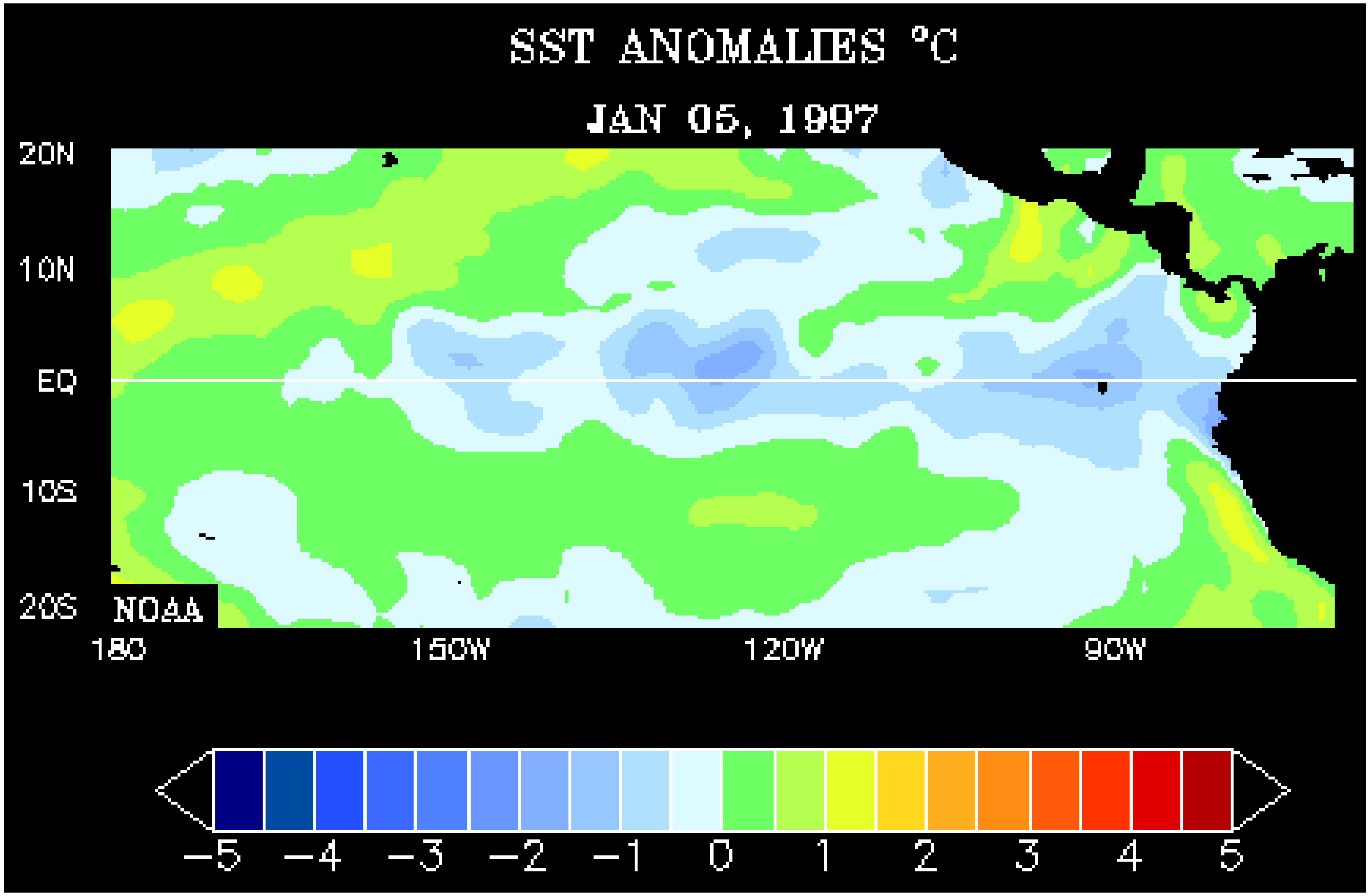
Fig. 10.3. Northern hemisphere Kelvin waves on opposite sides of a channel that is wide compared with the Rossby radius. In each vertical plane parallel to the coast, the currents (shown by arrows) are entirely within the plane and are exactly the same as those for a long gravity wave in a nonrotating channel. However, the surface elevation varies exponentially with distance from the coast in order to give a geostrophic balance. This means Kelvin waves move with the coast on their right in the northern hemisphere and on their left in the southern hemisphere. [From Mortimer (1977).]

- *Coastal Kelvin waves always propagate with the shoreline* on the right in the northern hemisphere and on the left in the southern hemisphere.
- In each vertical plane to the coast, the currents (shown by arrows) *are entirely within the plane* and are exactly the same as those for a long gravity wave in a non-rotating channel.
- However, the surface elevation varies exponentially with distance from the coast in order to give a geostrophic balance.

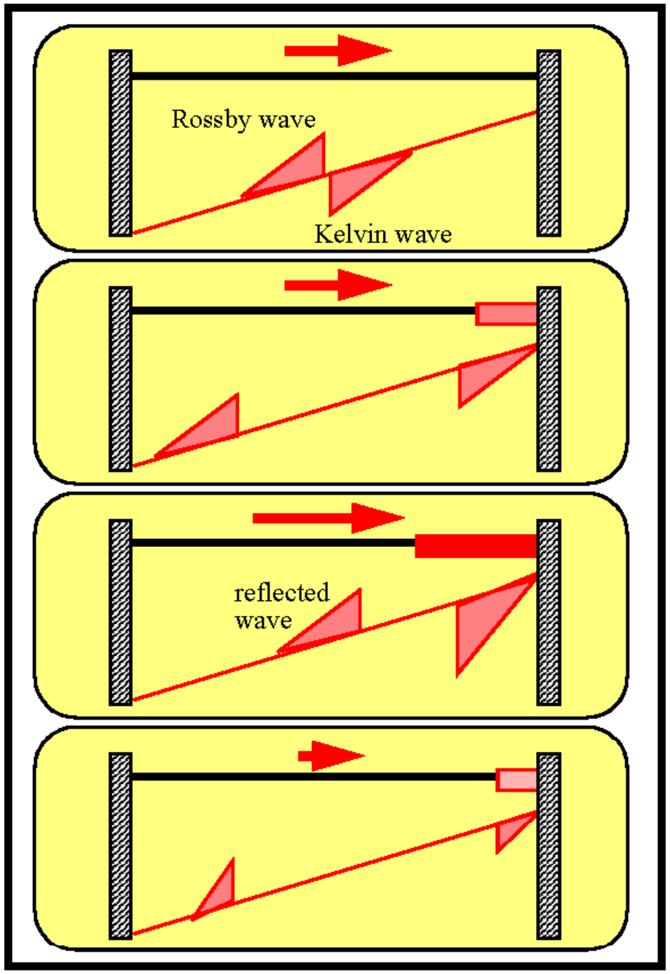
Equatorial Kelvin Waves

- The equator acts analogously to a topographic boundary for both the Northern and Southern Hemispheres, which make the equatorial Kelvin wave to behaves very similar to the coastally-trapped Kelvin wave.
- Surface equatorial Kelvin waves travel very fast, at about 200 m per second. Kelvin waves in the thermocline are however much slower, typically between 0.5 and 3.0 m per second.
- They may be detectable at the surface, as sea-level is slightly raised above regions where the thermocline is depressed and slightly depressed above regions where the thermocline is raised.
- The amplitude of the Kelvin wave is several tens of meters along the thermocline, and the length of the wave is thousands of kilometres.
- Equatorial Kelvin waves can only travel eastwards.

1997-98 El Nino

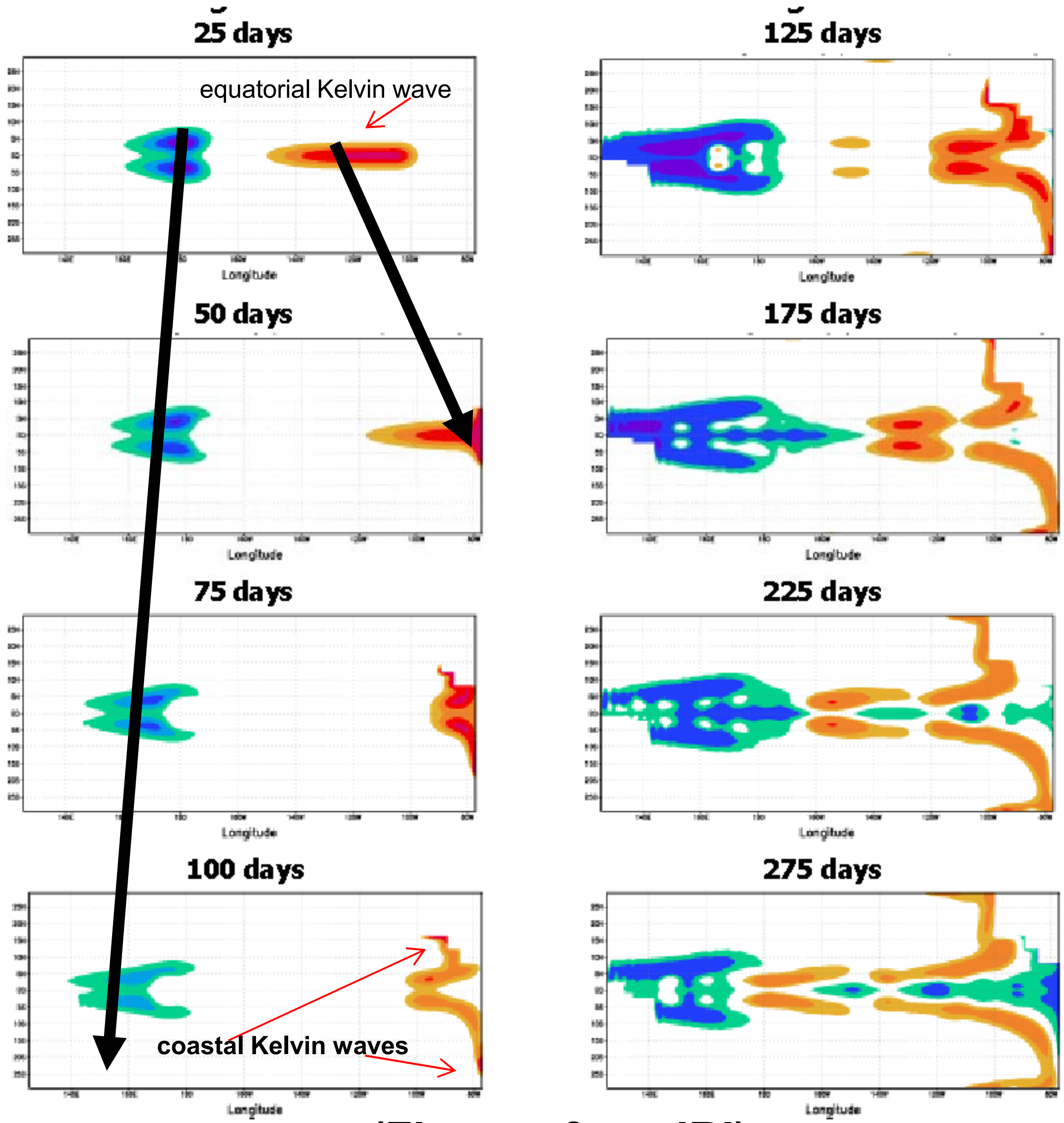


Delayed Oscillator Theory



- Wind forcing at the central Pacific: produces a downwelling Kevin wave propagating eastward and a upwelling Rossby wave propagating westward.
- wave propagation: the fast kelvin wave causes SST warming at the eastern basin, while slow Rossby wave is reflected at the western boundary.
- wave reflection: Rossby wave is reflected as a upwelling Kelvin wave and propagates back to the eastern basin to reverse the phase of the ENSO cycle.
- ENSO period: is determined by the propagation time of the waves.

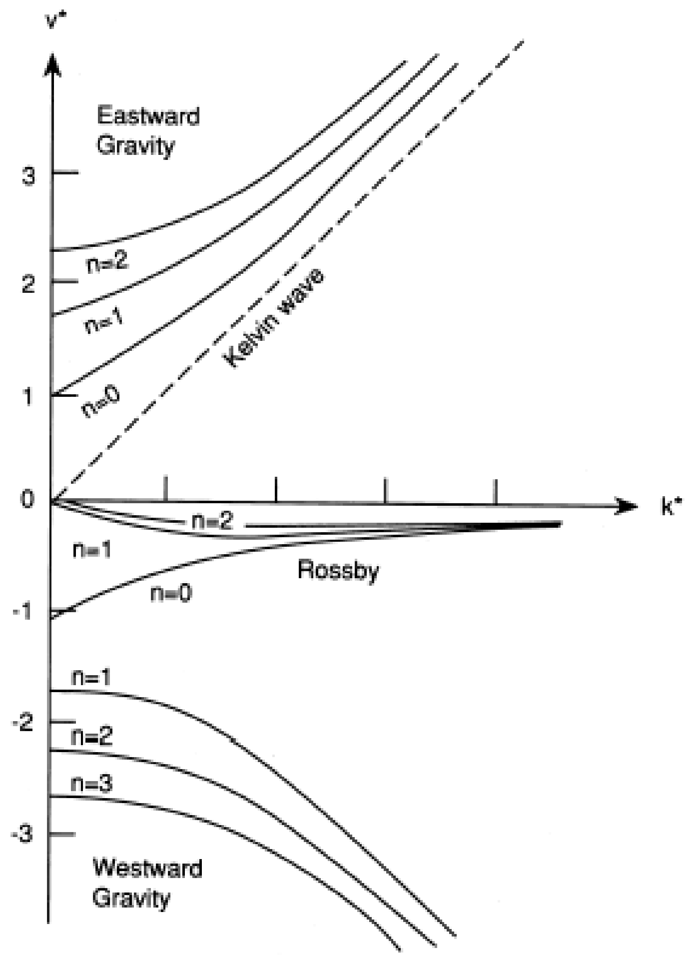
Wave Propagation and Reflection



(Figures from IRI)

- ❑ It takes Kevin wave (phase speed = 2.9 m/s) about 70 days to cross the Pacific basin (17,760km).
- ❑ It takes Rossby wave about 200 days (phase speed = 0.93 m/s) to cross the Pacific basin.

Equatorial Waves



- Equatorial waves are an important class of eastward and westward propagating disturbances in the atmosphere and in the ocean that are trapped about the equator (i.e., they decay away from the equatorial region).
- Diabatic heating by organized tropical convection can excite atmospheric equatorial waves, whereas wind stresses can excite oceanic equatorial waves.
- Atmospheric equatorial wave propagation can cause the effects of convective storms to be communicated over large longitudinal distances, thus producing remote responses to localized heat sources.

Equatorial β-Plane Approximation

- *f-plane approximation*: On a rotating sphere such as the earth, f varies with the sine of latitude; in the so-called f-plane approximation, this variation is ignored, and a value of *f* appropriate for a particular latitude is used throughout the domain.
- *β-plane approximation*: *f* is set to vary linearly in space.
- The advantage of the beta plane approximation over more accurate formulations is that it does not contribute nonlinear terms to the dynamical equations; such terms make the equations harder to solve.
- *Equatorial β-plane approximation*:

$$\begin{cases} \cos\varphi \approx 1, \\ \sin\varphi \approx y/a. \\ f \approx \beta y \end{cases}$$

and $\beta = 2\Omega/r = 2.3 \times 10^{-11} \ \mathrm{m^{-1}\,s^{-1}}$

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Shallow-Water Equation on an Equatorial β-Plane

Linearized shallow-water equations

$$\partial u'/\partial t - \beta y v' = -\partial \Phi' \partial x$$

$$\partial v'/\partial t + \beta y u' = -\partial \Phi' \partial y$$

$$\partial \Phi'/\partial t + g h_e \left(\partial u'/\partial x + \partial v'/\partial y\right) = 0$$

Assume wave-form solutions

$$\begin{pmatrix} u' \\ v' \\ \Phi' \end{pmatrix} = \begin{bmatrix} \hat{u}(y) \\ \hat{v}(y) \\ \hat{\Phi}(y) \end{bmatrix} \exp[i(kx - \nu t)]$$

$$-i\nu\hat{u} - \beta y \hat{v} = -ik\hat{\Phi}$$

$$-i\nu\hat{v} + \beta y \hat{u} = -\partial\hat{\Phi}/\partial y$$

$$-i\nu\hat{\Phi} + g h_e\left(ik\hat{u} + \partial\hat{v}/\partial y\right) = 0$$

$$\frac{\partial^2\hat{v}}{\partial y^2} + \left[\left(\frac{\nu^2}{g h_e} - k^2 - \frac{k}{\nu}\beta\right) - \frac{\beta^2 y^2}{g h_e}\right]\hat{v} = 0$$

Only if this constant equal to an odd integer that the boundary condition (v=o at y= 0) can be satisfied.

Dispersion relationship for equatorial waves

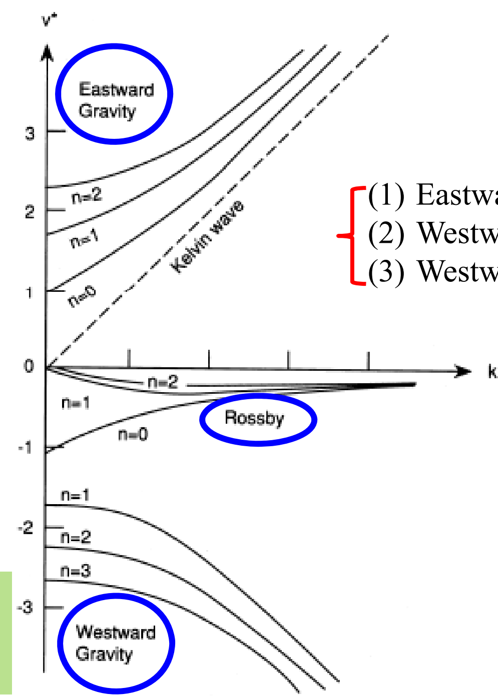
$$\frac{\sqrt{g h_e}}{\beta}\left(-\frac{k}{\nu}\beta - k^2 + \frac{\nu^2}{g h_e}\right) = 2n + 1; \quad n = 0, 1, 2, \ldots$$

The index n corresponds to the number of nodes in the meridional velocity profile in the domain |y| < ∞.

Only these waves that satisfy the condition that the wave amplitudes decay far from the equator (where the beta-plane approximation becomes invalid.)

$$\frac{\sqrt{g h_e}}{\beta}\left(-\frac{k}{\nu}\beta - k^2 + \frac{\nu^2}{g h_e}\right) = 2n + 1; \quad n = 0, 1, 2, \ldots$$

This cubic dispersion equation permit three groups of equatorially trapped waves:

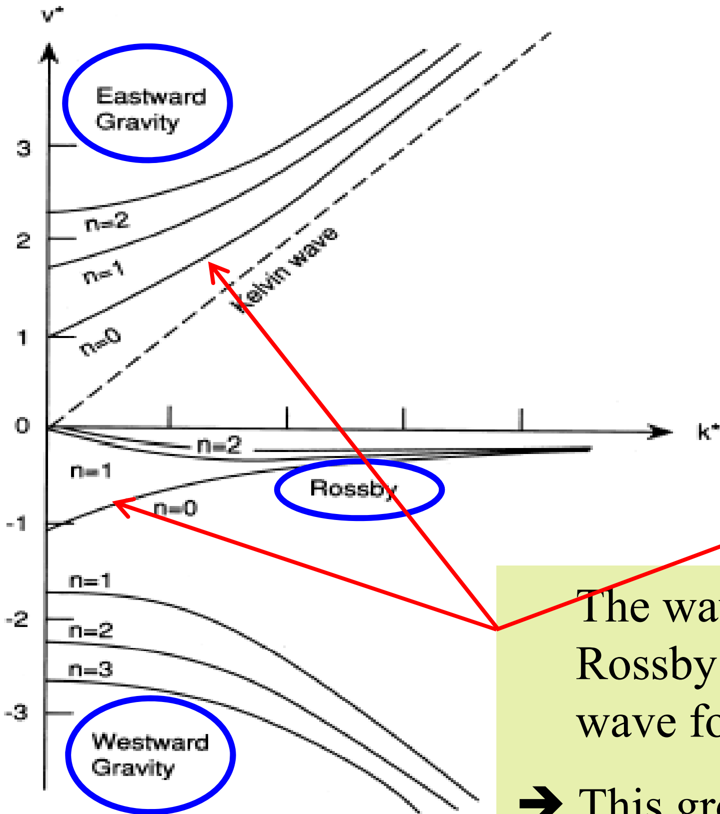


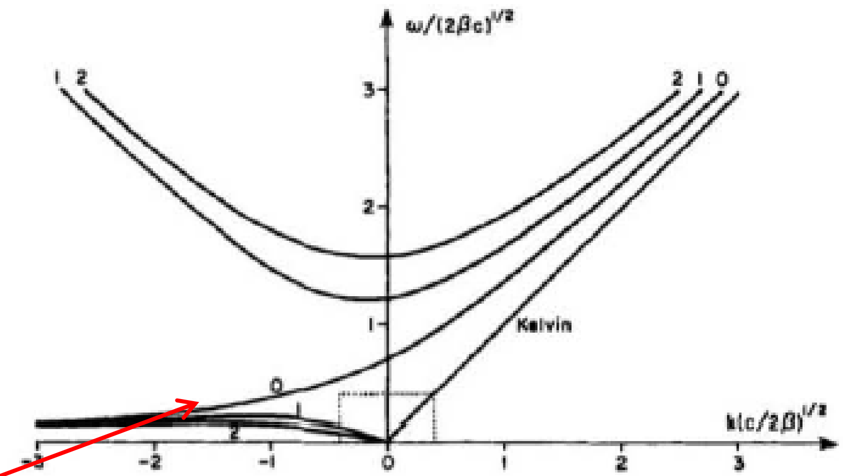
(1) Eastward-moving gravity waves
(2) Westward-moving gravity waves
(3) Westward-moving Rossby waves

ESS228
Prof. Jin-Yi Yu

Equatorial Waves with n=0

(Mixed Rossby-Gravity Waves)



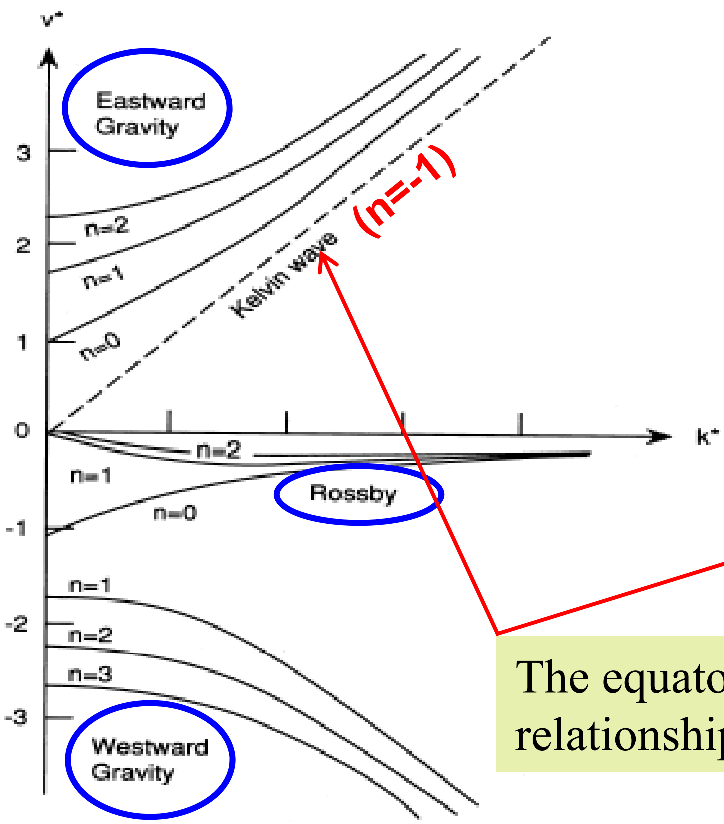


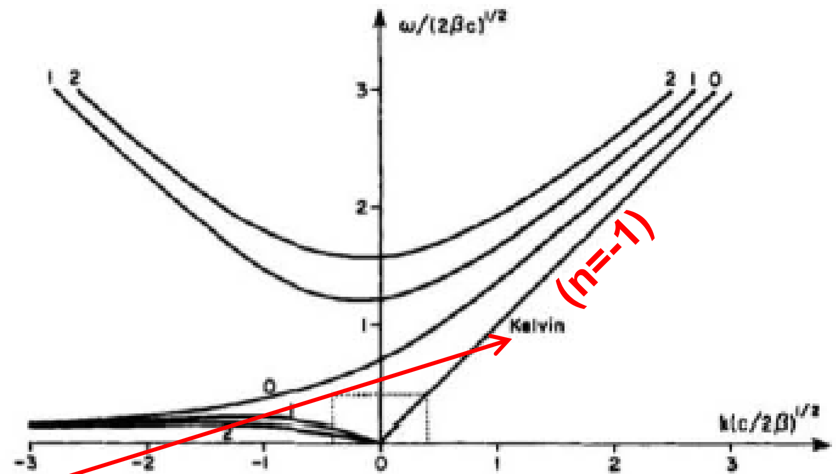
The wave solution with n=0 is special, which behaves like a Rossby wave for large negative k but behaves like a gravity wave for large positive k.

➔ This group of waves is called "mixed Rossby-Gravity Waves".

Equatorial Waves with "n=-1"

(Equatorial Kelvin Waves)





The equatorially trapped Kelvin wave satisfies the dispersion relationship of the equatorial beta-plane with n=-1.

Equatorial Kelvin Waves

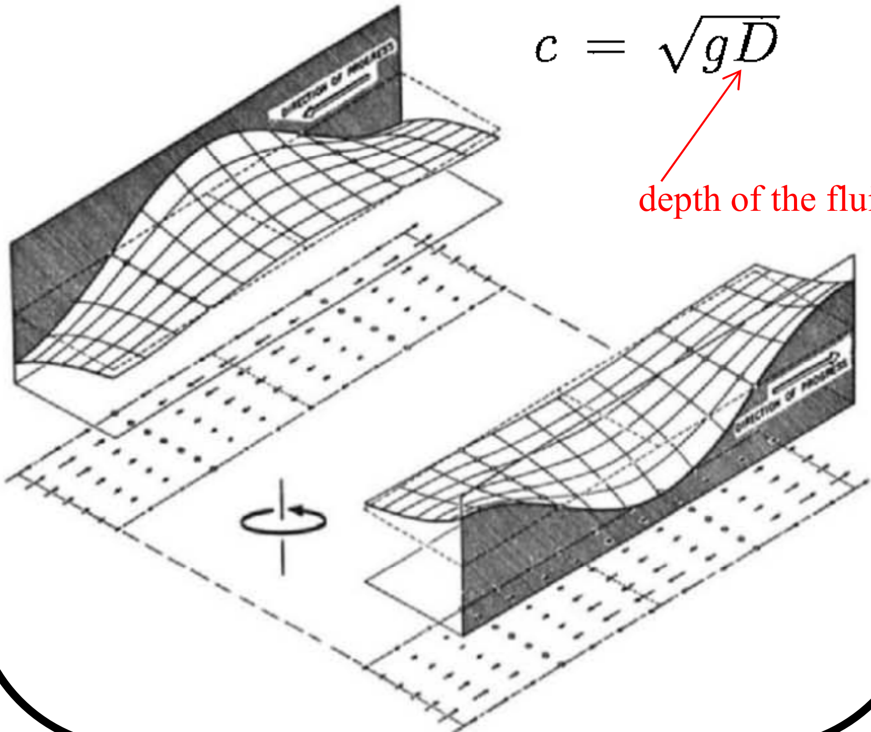
Coastal Kelvin Wave

$$H = const \times \exp\left(-\frac{f}{c}y\right)$$

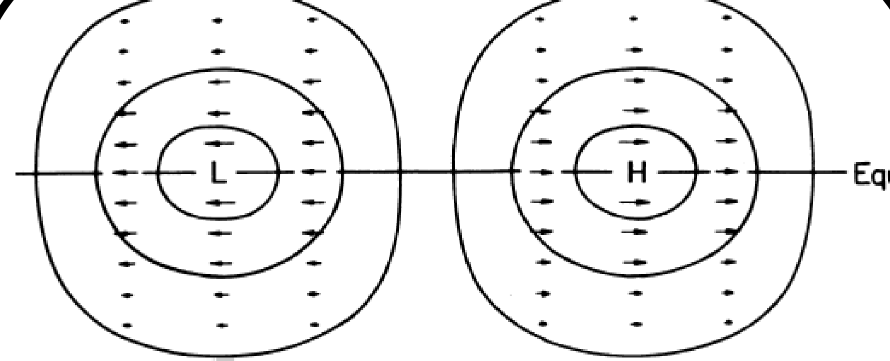
At the coast $y = 0$ is $v = 0$:

$$c = \sqrt{gD}$$

depth of the fluid



Equatorial Kelvin Wave

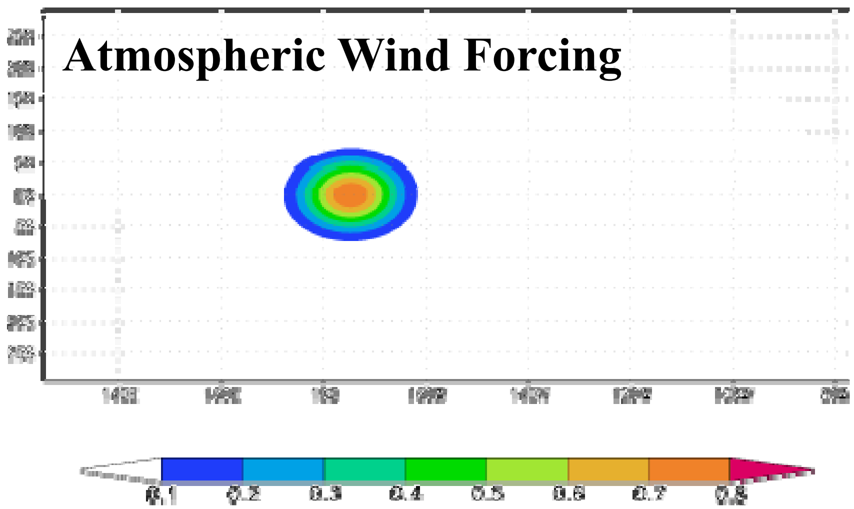


$$\hat{u} = u_o \exp\left(-\beta y^2/2c\right)$$

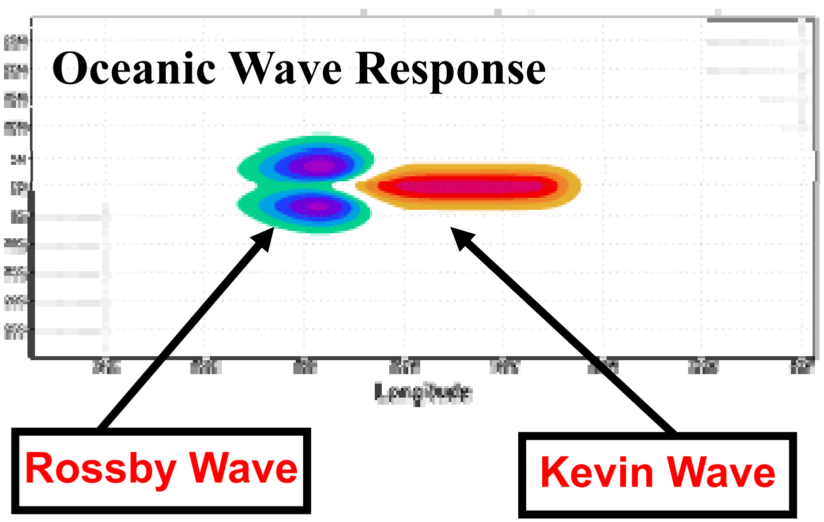
$$Y_K = |2c/\beta|^{1/2}$$ → e-decaying width

for a phase speed c = 30ms−1 gives $Y_K \approx$ 1600 km.

Delayed Oscillator: Wind Forcing

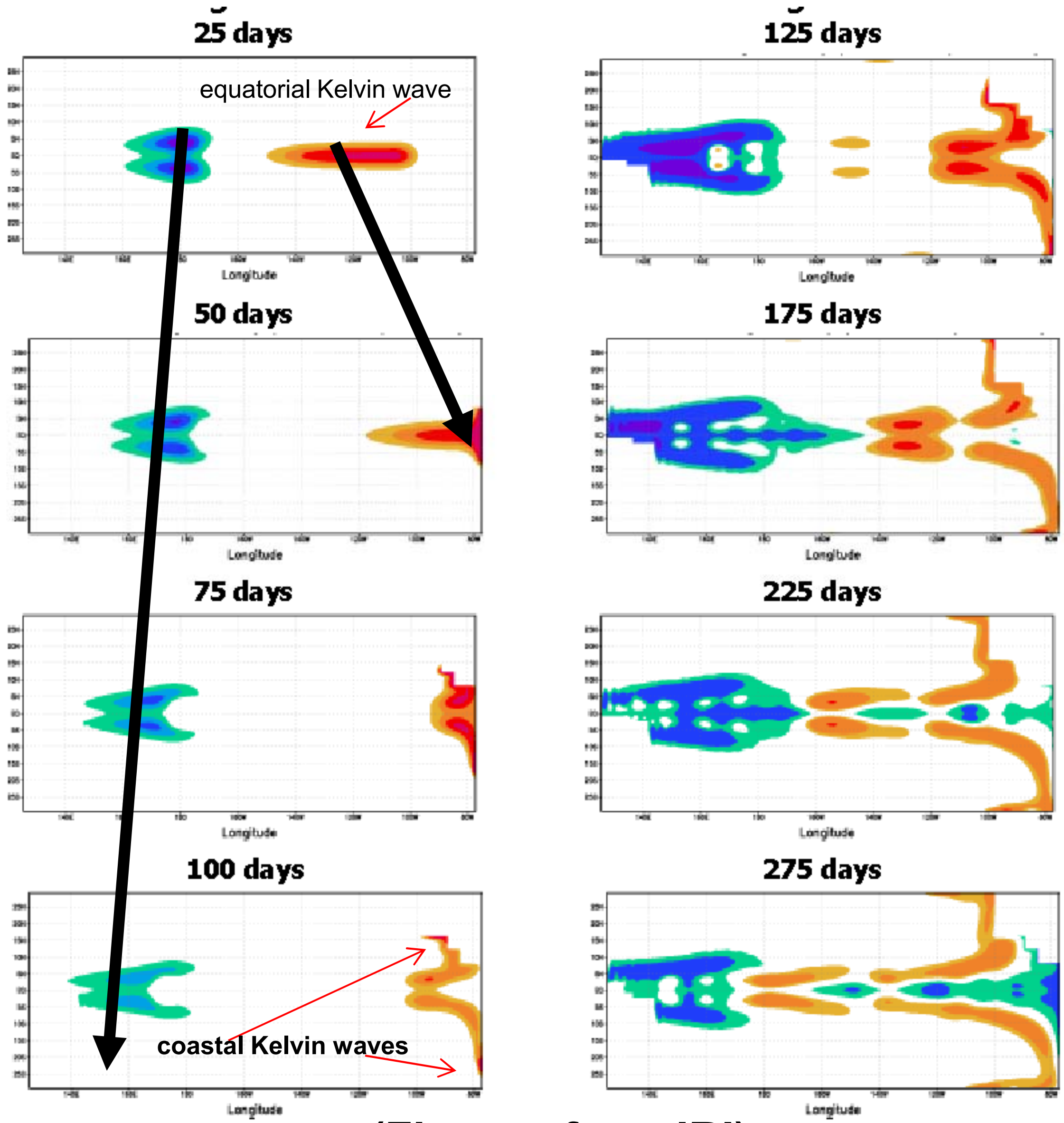


(Figures from IRI)



- The delayed oscillator suggested that oceanic Rossby and Kevin waves forced by atmospheric wind stress in the central Pacific provide the phase-transition mechanism (I.e. memory) for the ENSO cycle.
- The propagation and reflection of waves, together with local air-sea coupling, determine the period of the cycle.

Wave Propagation and Reflection



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