Rotating coordinates

- The Earth is rotating. We measure things relative to this "rotating reference frame".

- Quantity that tells how fast something is rotating:

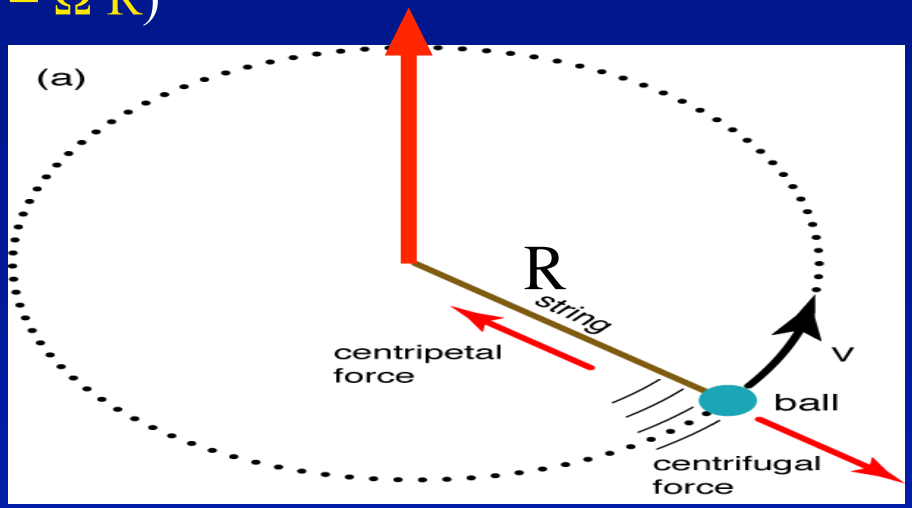
  Angular speed or angular velocity Ω = angle/second

  360° is the whole circle, but express angle in radians (2π radians = 360°)

  For Earth: 2π / 1 day = 2π / 86,400 sec = 0.707×10^{-4} /sec

  Also can show $\Omega = v/R$ where v is the measured velocity and R is the radius to the axis of rotation (therefore $v = \Omega R$)

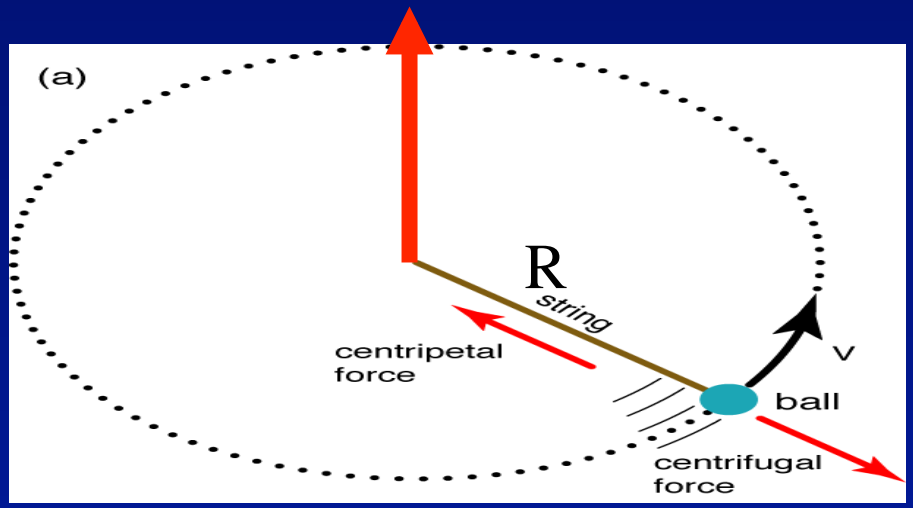
At home: calculate speed you are traveling through space if you are at the equator (radius of earth is about 6371 km), then calculate it at 30N as well. (Do some geometry to figure out the distance from the axis at 30N.)

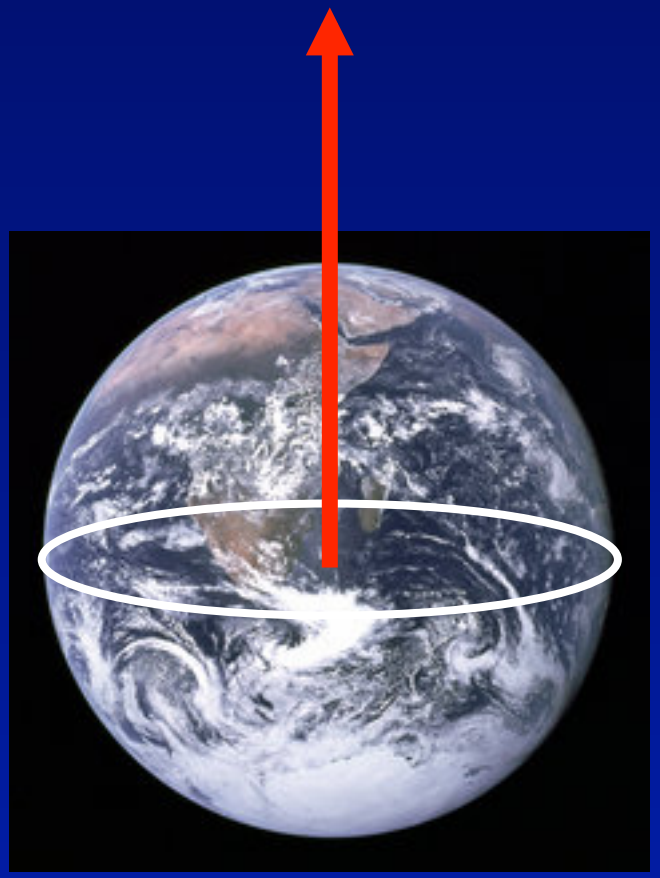


Rotating coordinates

- Vector that expresses direction of rotation and how fast it is rotating:

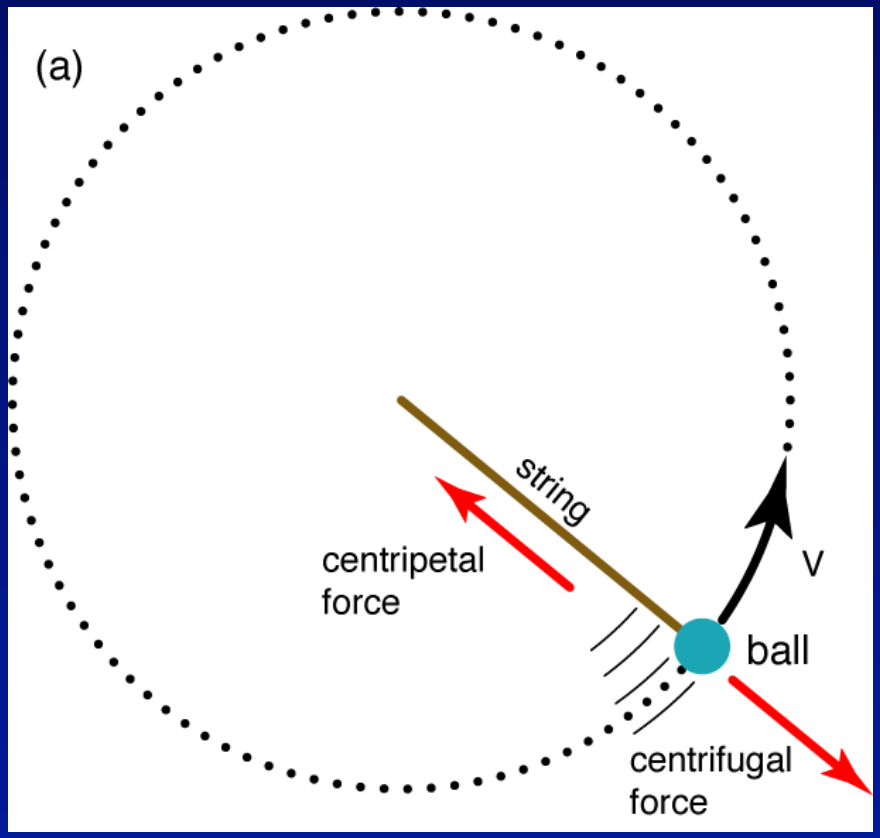
  vector pointing in direction of thumb using right-hand rule, curling fingers in direction of rotation





Centripetal and Centrifugal forces

(now looking straight down on the rotating plane)



Centripetal force is the actual force that keeps the ball "tethered" (here it is the string, but it can be gravitational force)

Centrifugal force is the pseudo-force (apparent force) that one feels due to lack of awareness that the coordinate system is rotating or curving

centrifugal acceleration = $\Omega^2 R$ (outward) (Units of m/sec^2)

Effect of centrifugal force on ocean and earth

Centrifugal force acts on the ocean and earth. It is pointed outward away from the rotation axis.

Therefore it is maximum at the equator (maximum radius from axis) and minimum at the poles (0 radius).

$\Omega = 0.707 \times 10^{-4}$ /sec

At the equator, R ~ 6380 km so $\Omega^2 R = .032$ m/s^2

Compare with gravity = 9.8 m/s^2

Centrifugal force should cause the equator to be deflected (0.032/9.8) x 6380 km = 21 km outward compared with the poles. (i.e. about 0.3%)

Effect of centrifugal force on ocean and earth

Radius:

Equatorial 6,378.135 km

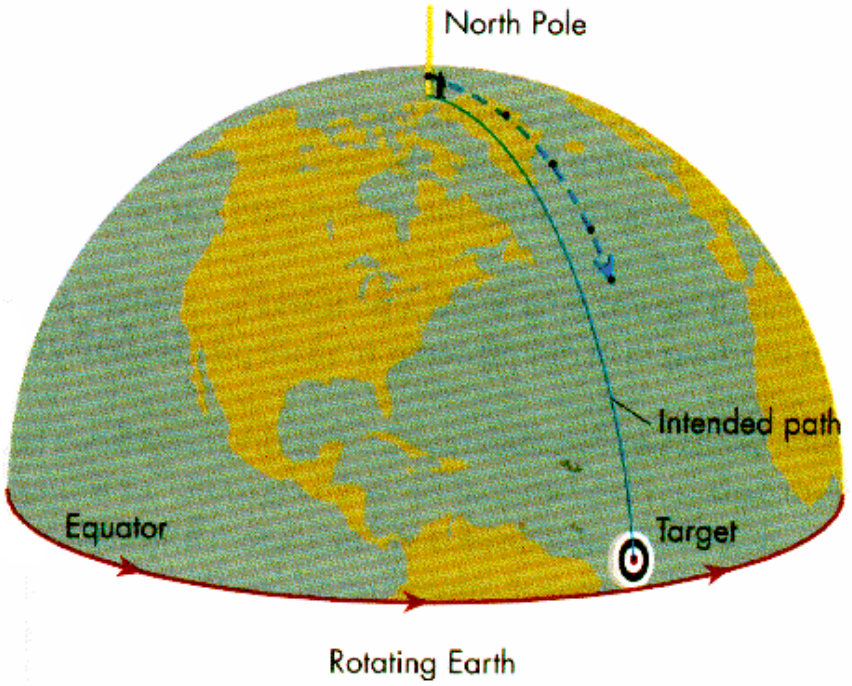
Polar 6,356.750 km

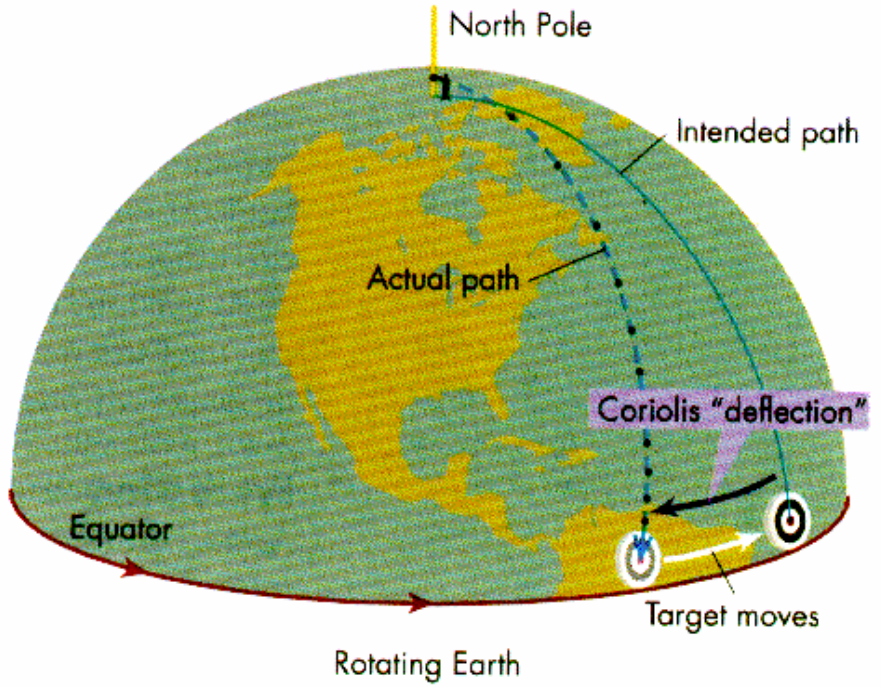
Mean 6,372.795 km

(From wikipedia entry on Earth)

The ocean is not 20 km deeper at the equator, rather the earth itself is deformed!

We bury the centrifugal force term in the gravity term (which we can call "effective gravity"), and ignore it henceforth. Calculations that require a precise gravity term should use subroutines that account for its latitudinal dependence.





Coriolis effect

Inertial motion:

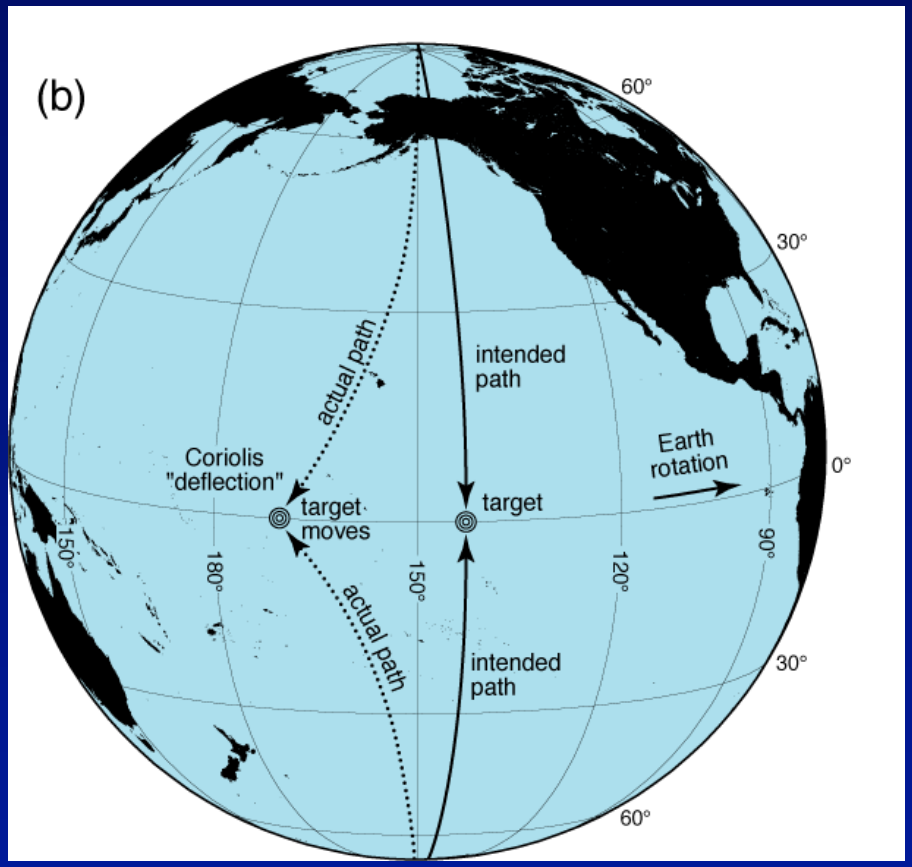
motion in a straight line relative to the fixed stars

Coriolis effect:

apparent deflection of that inertially moving body just due to the rotation of you, the observer.

Coriolis effect deflects bodies (water parcels, air parcels) to the right in the northern hemisphere and to the left in the southern hemisphere

Coriolis force



Additional terms in x, y momentum equations, at latitude φ
(horizontal motion is much greater than vertical)

x-momentum equation:

$-2\,\Omega \sin\varphi\, v = -f\, v$

y-momentum equation:

$2\,\Omega \sin\varphi\, u = f\, u$

$f = 2\,\Omega \sin\varphi$ is the "Coriolis parameter". It depends on latitude (projection of total Earth rotation on local vertical)