

# Complete force (momentum) balance with rotation

Three equations:

Horizontal (x) (west-east)

acceleration + advection + Coriolis =  
pressure gradient force + viscous term

Horizontal (y) (south-north)

acceleration + advection + Coriolis =  
pressure gradient force + viscous term

Vertical (z) (down-up)

acceleration + advection (+ neglected very small Coriolis) =  
pressure gradient force + effective gravity  
(including centrifugal force) + viscous term

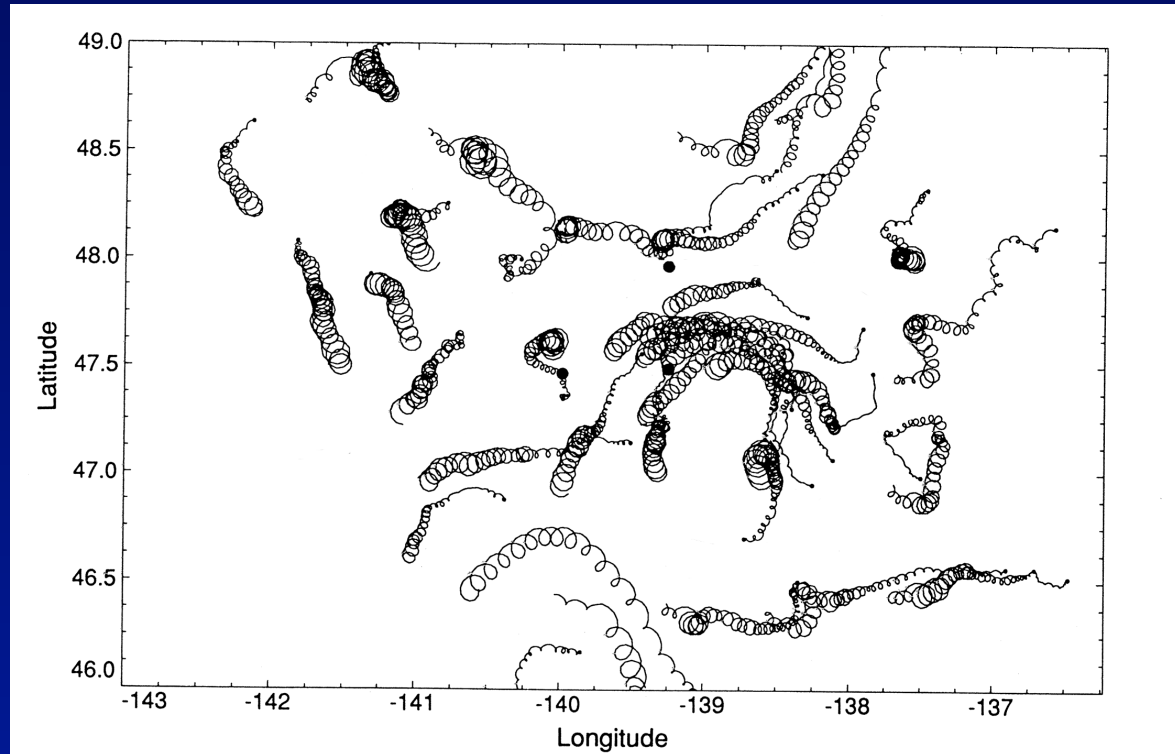
## Final equations of motion (momentum equations in Cartesian coordinates)

$$\begin{aligned} \text{x: } & \partial u / \partial t + u \partial u / \partial x + v \partial u / \partial y + w \partial u / \partial z - f v = \\ & - (1 / \rho) \partial p / \partial x + \partial / \partial x (A_H \partial u / \partial x) + \\ & \partial / \partial y (A_H \partial u / \partial y) + \partial / \partial z (A_V \partial u / \partial z) \quad (7.11a) \end{aligned}$$

$$\begin{aligned} \text{y: } & \partial v / \partial t + u \partial v / \partial x + v \partial v / \partial y + w \partial v / \partial z + f u = \\ & - (1 / \rho) \partial p / \partial y + \partial / \partial x (A_H \partial v / \partial x) + \\ & \partial / \partial y (A_H \partial v / \partial y) + \partial / \partial z (A_V \partial v / \partial z) \quad (7.11b) \end{aligned}$$

$$\begin{aligned} \text{z: } & \partial w / \partial t + u \partial w / \partial x + v \partial w / \partial y + w \partial w / \partial z \text{ (+} \\ & \text{neglected small Coriolis)} = \\ & - (1 / \rho) \partial p / \partial z - g + \partial / \partial x (A_H \partial w / \partial x) + \\ & \partial / \partial y (A_H \partial w / \partial y) + \partial / \partial z (A_V \partial w / \partial z) \quad (7.11c) \end{aligned}$$

# Coriolis in action in the ocean: Observations of Inertial Currents



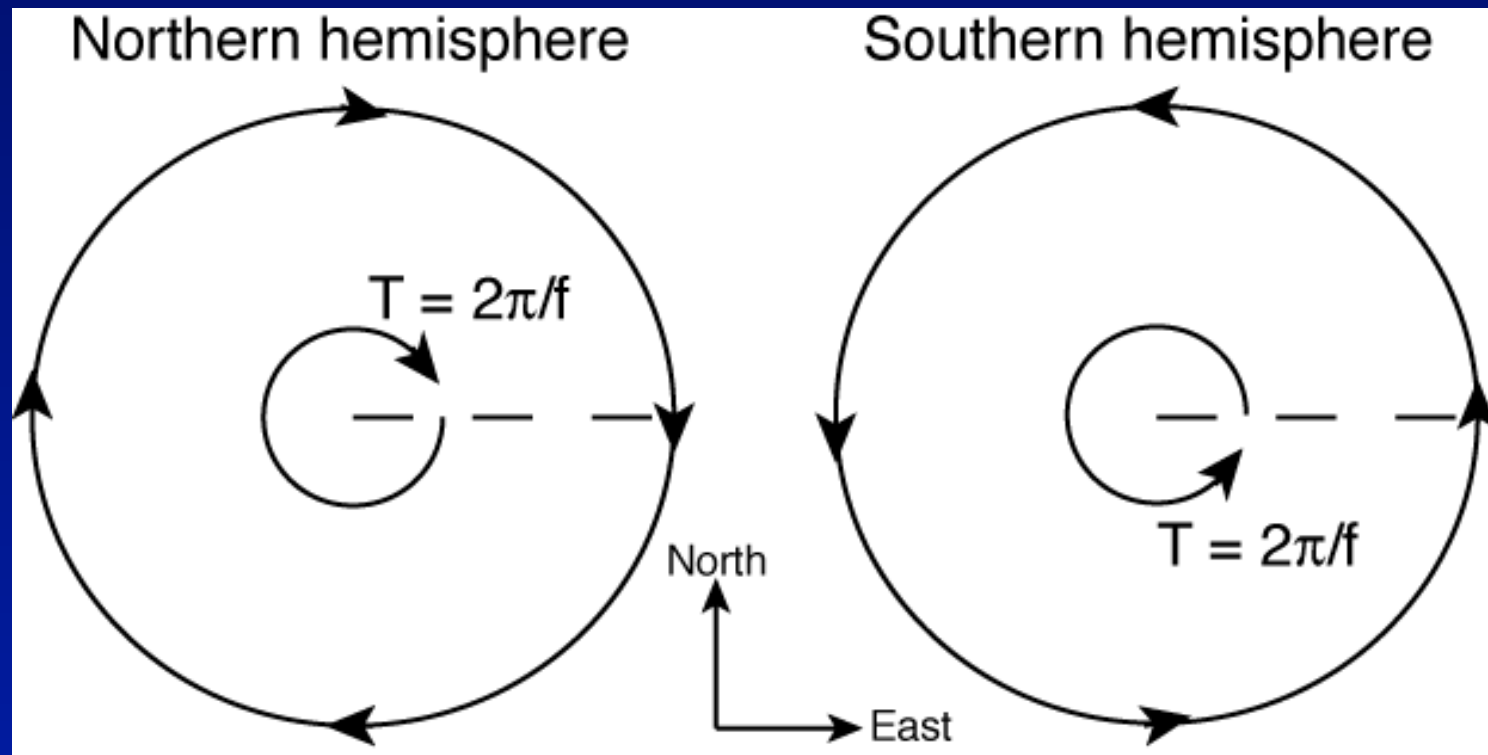
D'Asaro et  
al. (1995)

DPO Fig. 7.4

- Surface drifters in the Gulf of Alaska during and after a storm.
- Note the corkscrews - drifters start off with clockwise motion, which gets weaker as the motion is damped (friction)

# Inertial currents

Balance of Coriolis and acceleration terms: push the water and it turns to the right (NH), in circles



# Inertial currents: force balance

Three APPROXIMATE equations:

Horizontal (x) (west-east)

$$\text{acceleration} + \text{Coriolis} = 0$$

Horizontal (y) (south-north)

$$\text{acceleration} + \text{Coriolis} = 0$$

Vertical (z) (down-up) (NOT IMPORTANT) (hydrostatic)

$$0 = \text{pressure gradient force} + \text{effective gravity}$$

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That is:

$$x: \partial u / \partial t - fv = 0$$

$$y: \partial v / \partial t + fu = 0$$

$$1/2 d/dt(u^*u + v^*v) = R$$