Taylor -Proudman theorem and Vorticity in inviscid rotating fluids

We first show that in a steady rotating flow of inviscidand homogeneous fluid, if the Rossby number is smal, then the flow is essentially two diemensional. This is known as the Taylor-Proudman theorm.

Under these conditions, the momentum equation reads,

$$2\vec{\Omega} \times \vec{q} = -\frac{\nabla p}{\rho} \tag{7.2.1}$$

Taking the curl of both sides we get

$$\nabla \times (\vec{\Omega} \times \vec{q}) = 0 \tag{7.2.2}$$

Using the identity

$$\nabla \times \left(\vec{A} \times \vec{B}\right) = \vec{A}\nabla \cdot \vec{B} - \vec{B}\nabla \cdot \vec{A} + \vec{B} \cdot \nabla \vec{A} - \vec{A} \cdot \nabla \vec{B} \tag{7.2.3}$$

we get

$$\vec{\Omega}\nabla \cdot \vec{q} - \vec{q}\nabla \cdot \vec{\Omega} + \vec{q} \cdot \nabla \vec{\Omega} - \vec{\Omega} \cdot \nabla \vec{q} = 0$$

Invoking continuity and the constancy of Ω we obtain

$$\vec{\Omega} \cdot \nabla \vec{q} = 0 \tag{7.2.4}$$

Thus the velocity field does not vary in the direction of Ω, say z. Note that $\vec{q}$ can still have three components, but they must all be independent of z. This is the

Theorem 1 *Taylor-Proudman theorem : A steady and slow flow in a rotating fluid is two-dimensional in the plane perpendicular to the vector of angular velocity.*

Laboratory verification has been demonstrated in a setup shown in figure 7.2.1.

More generally, let us consider the vortity transport in a rotating and invoscid fluid. Let $\vec{\zeta} = \nabla \times \vec{q}$ and use the identity

$$\vec{\zeta} \times \vec{q} = \vec{q} \cdot \nabla \vec{q} - \nabla \frac{|\vec{q}|^2}{2}$$