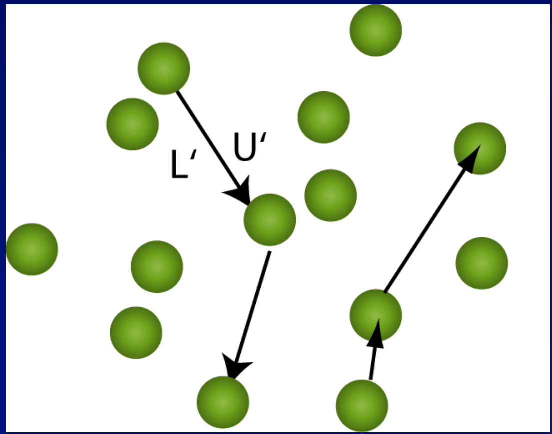
Mixing and diffusion

- Random motion of molecules carries "stuff" around



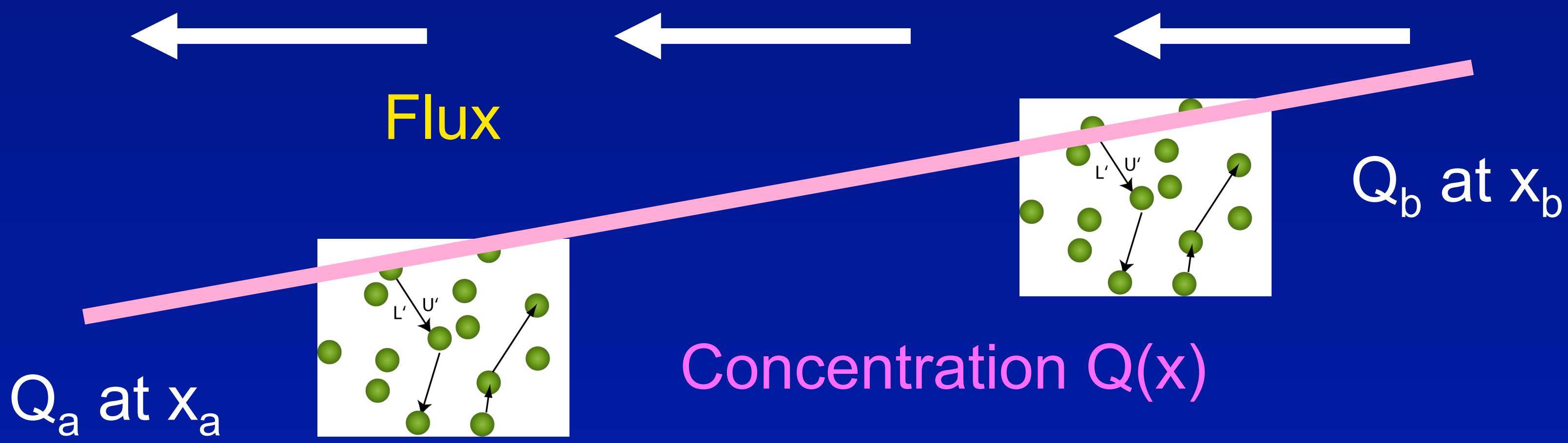
- Fick's Law: net flux of "stuff" is proportional to its gradient
  - Flux = $-\kappa(Q_a - Q_b)/(x_a - x_b) = -\kappa\nabla Q$
    - where κ is the diffusivity

Units: [Flux] = [velocity][stuff], so
$[\kappa]$ = [velocity][stuff][L]/[stuff] =
$[L^2/\text{time}] = m^2/\text{sec}$

Mixing and diffusion

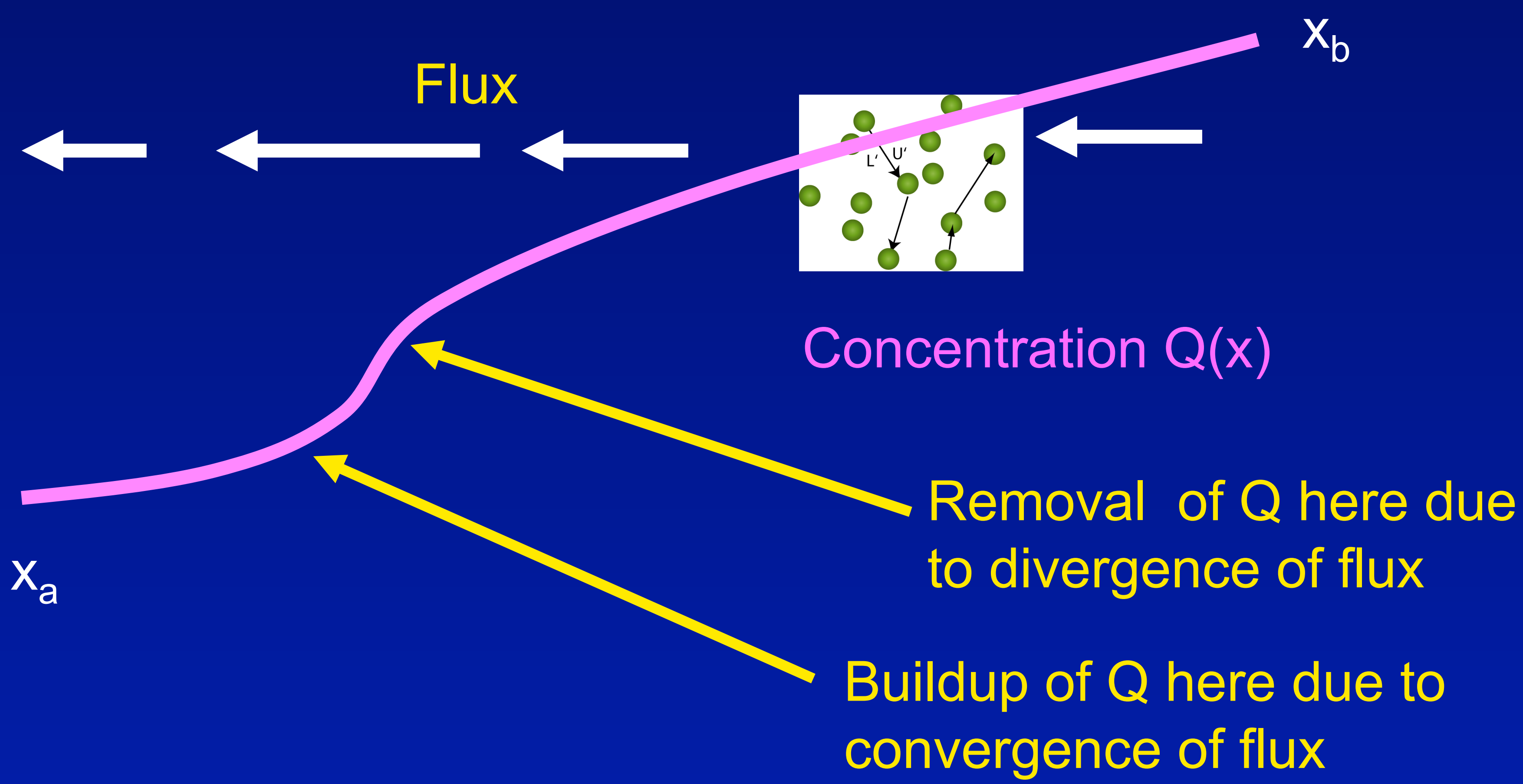
Fick's Law: flux of "stuff" is proportional to its gradient $\quad \text{Flux} = -\kappa \nabla Q$

- If the concentration is exactly linear, with the amount of stuff at both ends maintained at an exact amount, then the flux of stuff is the same at every point between the ends, and there is no change in concentration of stuff at any point in between.



Mixing and diffusion

Diffusion: if there is a convergence or divergence of flux then the “stuff” concentration can change



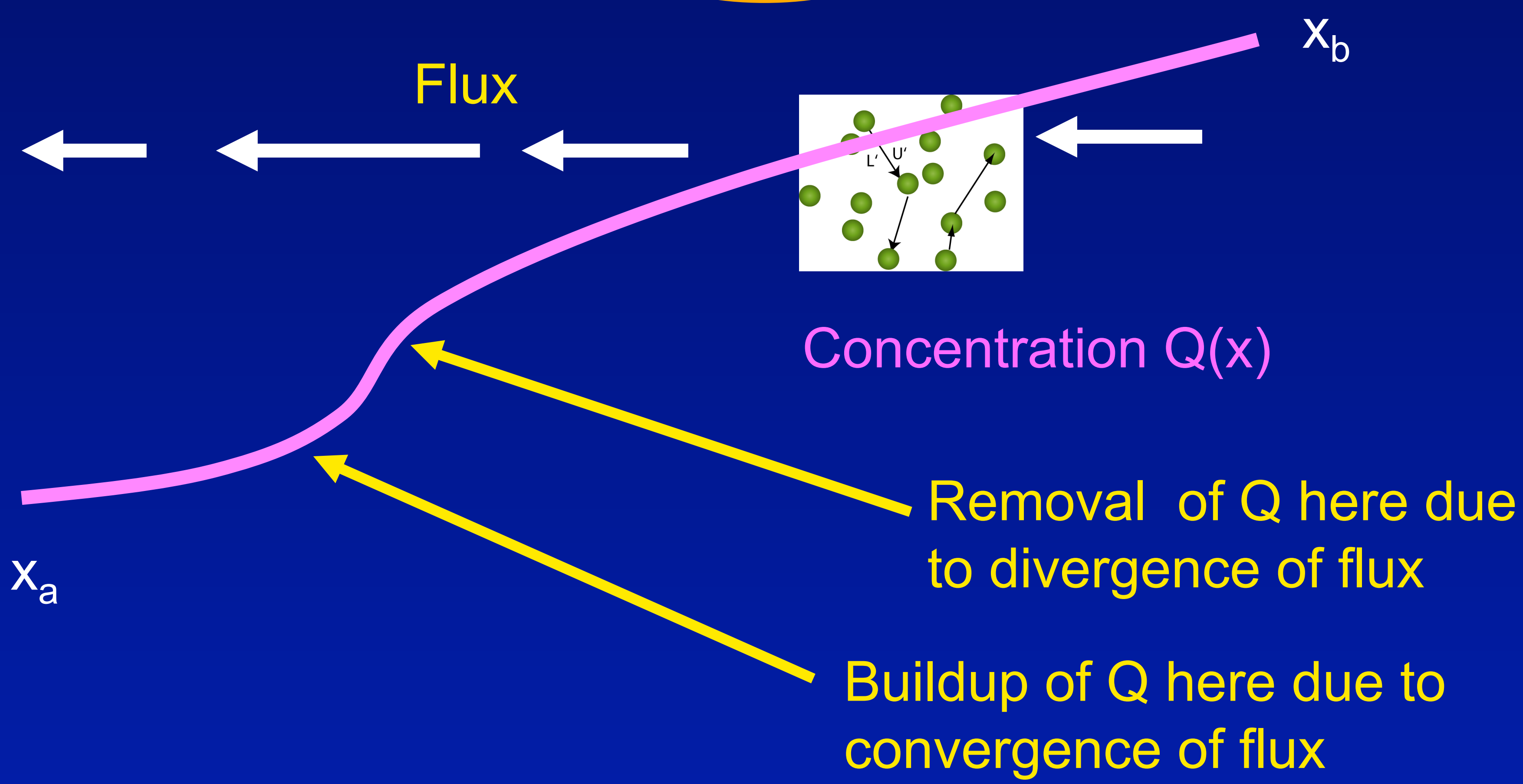
Mixing and diffusion

Change in Q with time = $\Delta Q/\Delta t$ =

change in Q flux with space = $-\Delta \text{Flux}/\Delta x$ =

$$\partial Q/\partial t = \kappa\, \partial^2 Q/\partial x^2$$

Diffusion term

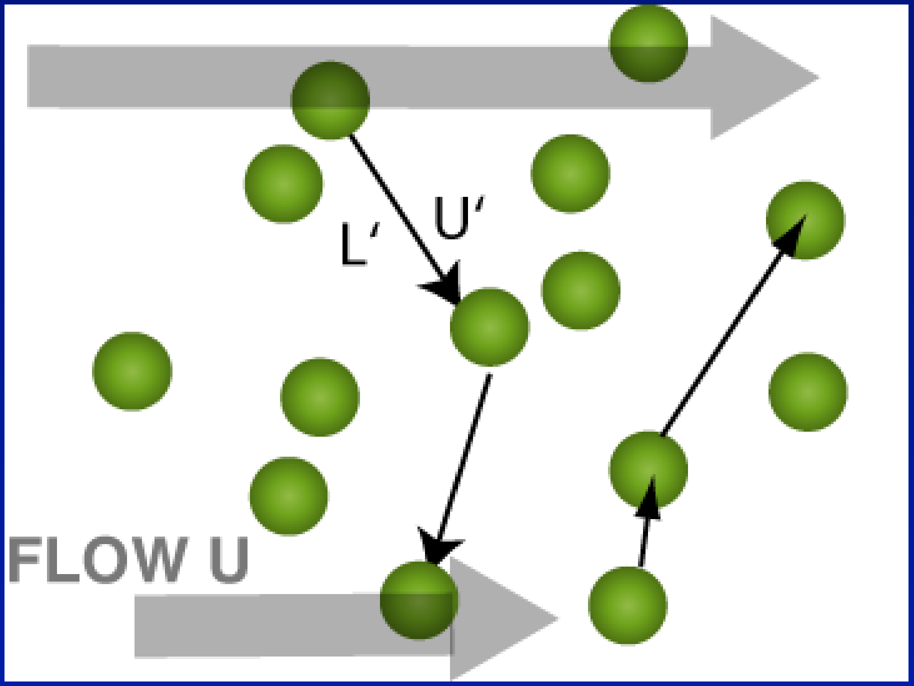


Viscosity

- Viscosity: apply same Fick's Law concept to velocity. So viscosity affects flow if there is a convergence of flux of momentum.
- Stress ("flux of momentum") on flow is

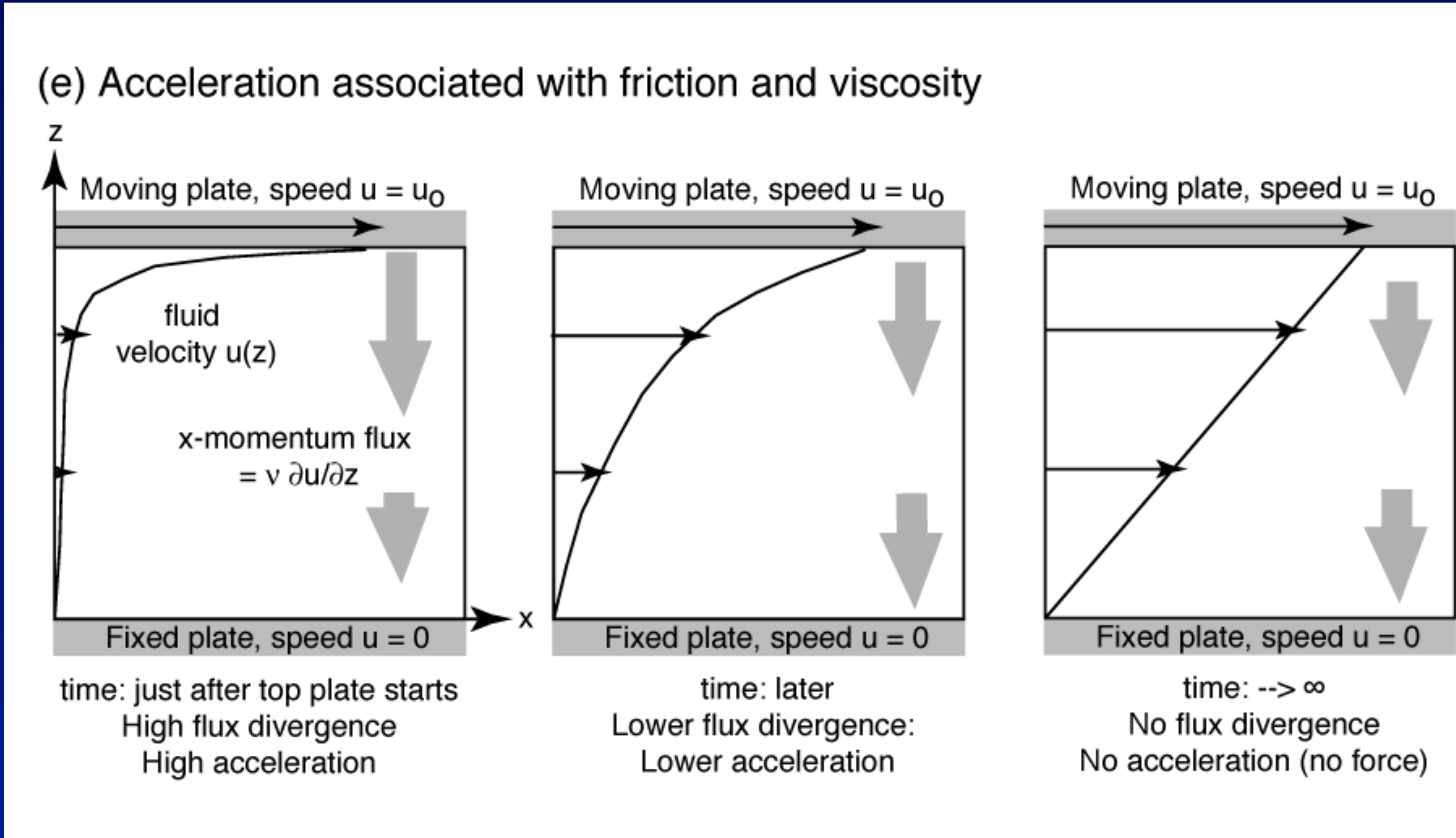
$$\tau \text{ (= "flux") } = -\rho\upsilon\nabla u$$

where υ is the viscosity coefficient



DPO Fig. 7.3

Acceleration due to viscosity



DPO Fig. 7.1

Acceleration due to viscosity

- $\partial u/\partial t = \rho \upsilon \, \partial^2 u/\partial x^2 \underline{\equiv} \mu \, \partial^2 u/\partial x^2$

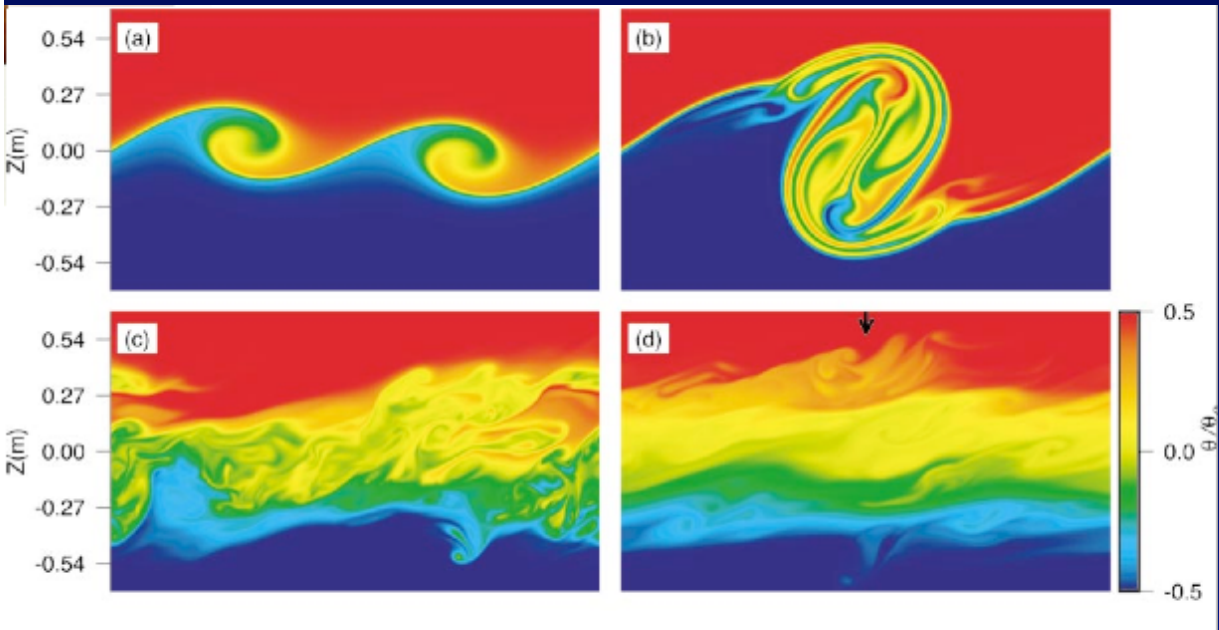
[Fine print: υ is the kinematic viscosity and μ is the absolute (dynamic) viscosity)

If the viscosity itself depends on space, then it of course needs to be INSIDE the space derivative: $\partial_x (\mu \, \partial u/\partial x)$]

Eddy diffusivity and eddy viscosity

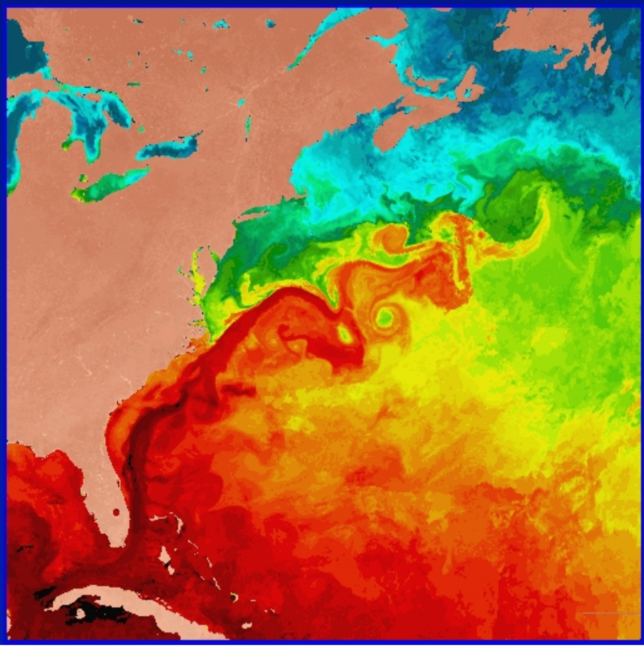
- Molecular viscosity and diffusivity are extremely small (values given on later slide)
- We know from observations that the ocean behaves as if diffusivity and viscosity are much larger than molecular (I.e. ocean is much more diffusive than this)
- The ocean has lots of turbulent motion (like any fluid)
- Turbulence acts on larger scales of motion like a viscosity - think of each random eddy or packet of waves acting like a randomly moving molecule carrying its property/mean velocity/information

Stirring and mixing



Vertical stirring and ultimately mixing:

Internal waves on an interface stir fluid, break and mix

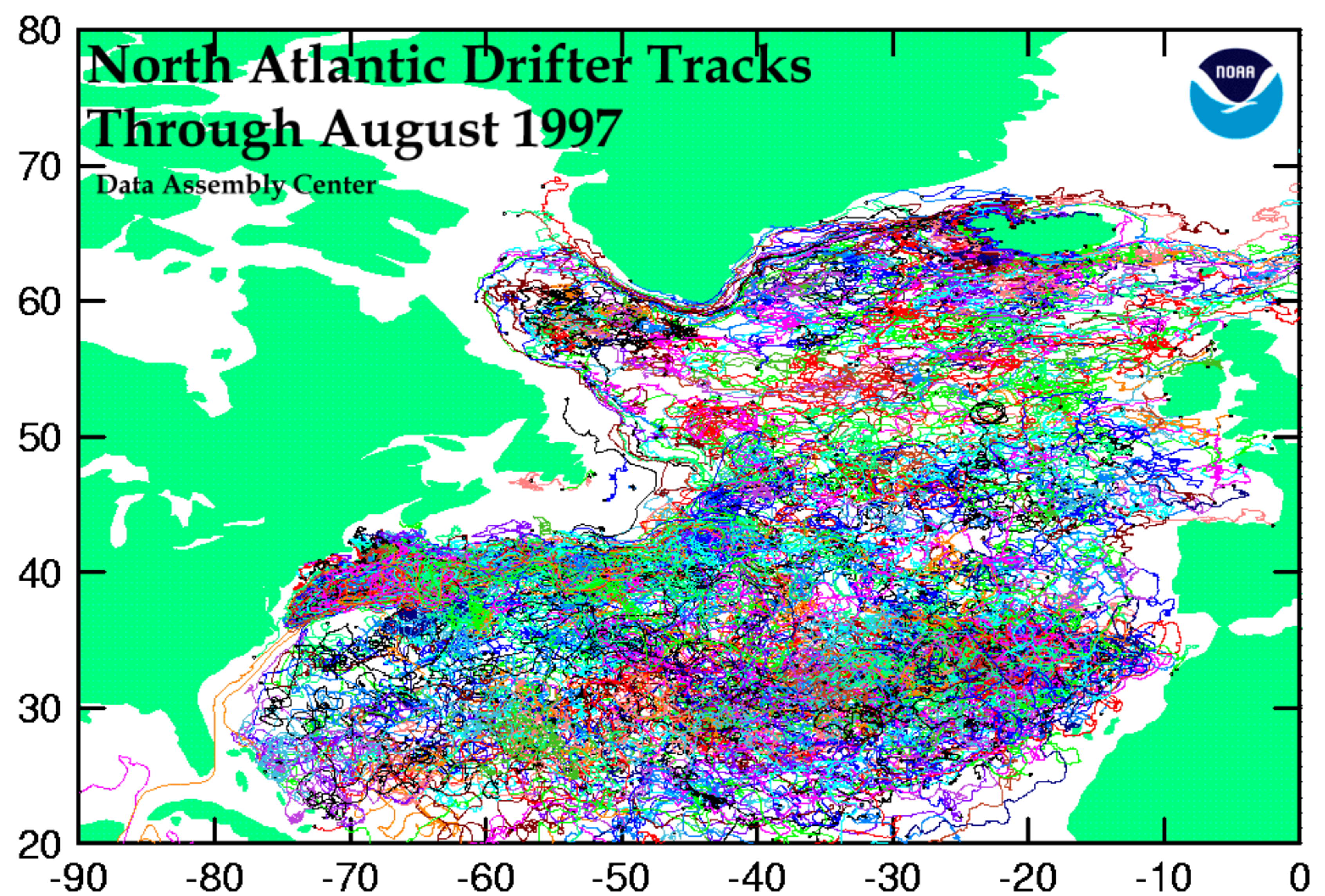


Horizontal stirring and ultimately mixing:

Gulf Stream (top): meanders and makes rings (closed eddies) that transport properties to a new location

Eddy diffusivity and viscosity

Example of surface drifter tracks: dominated to the eye by variability (they can be averaged to make a very useful mean circulation)



Eddy field in a numerical model of the ocean

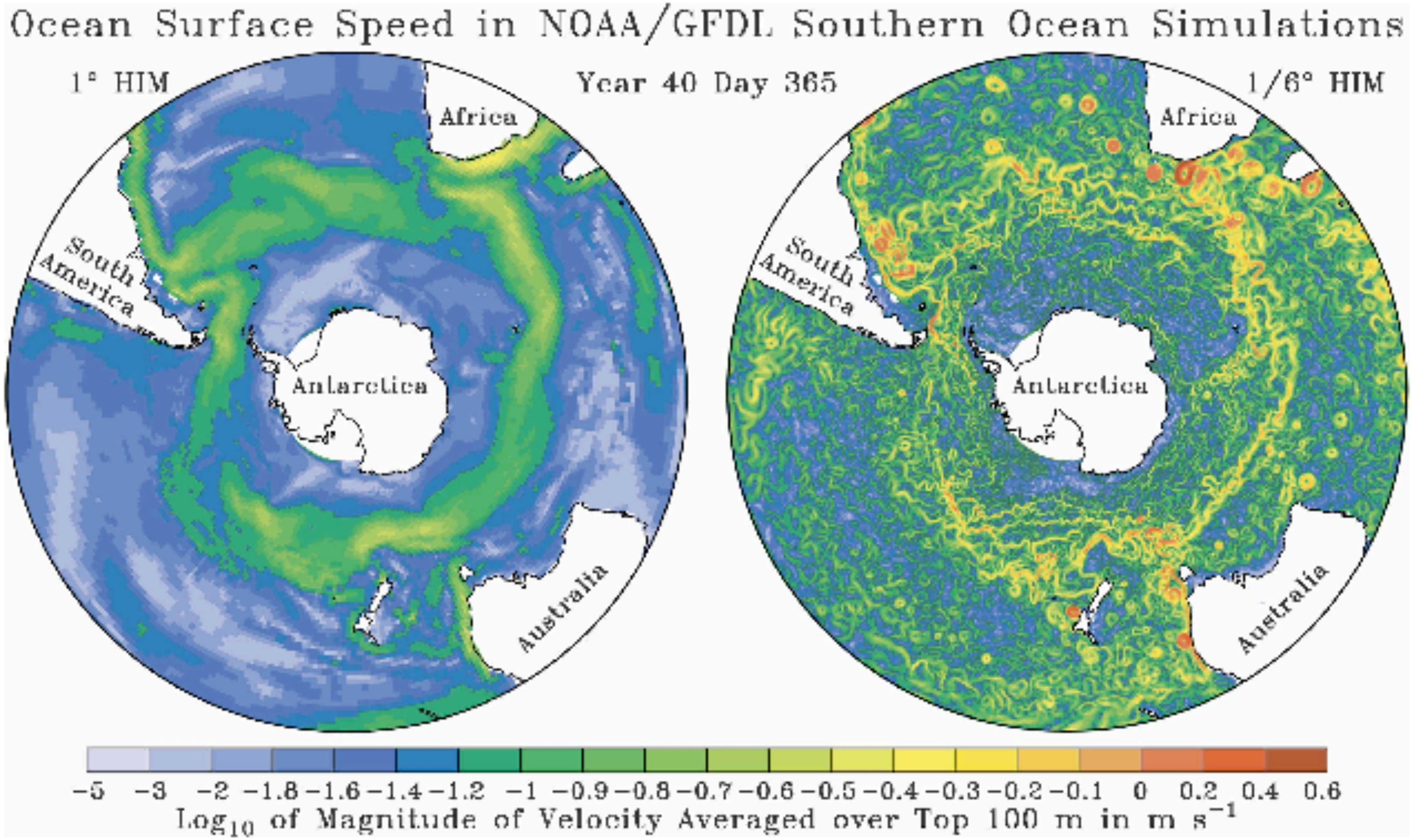


FIG. 6. Instantaneous surface speed in 1° and ⅙° models after 40 yr. Note that the large-scale structure of the 1° model is quite similar to the ⅙° model (the currents have similar locations and have similar horizontal extents). The main difference is in the presence of intense jets and eddies in the ⅙° model.

Values of molecular and eddy diffusivity and viscosity

- Molecular diffusivity and viscosity

  κ_T = 0.0014 cm^2/sec (temperature)

  κ_S = 0.000013 cm^2/sec (salinity)

  ν = 0.018 cm^2/sec at 0°C (0.010 at 20°C)

- Eddy diffusivity and viscosity values for heat, salt, properties are the same size (same eddies carry momentum as carry heat and salt, etc)

  But eddy diffusivities and viscosities differ in the horizontal and vertical

- Eddy diffusivity and viscosity

  A_H = 10^4 to 10^8 cm^2/sec (horizontal) = 1 to 10^4 m^2/sec

  A_V = 0.1 to 1 cm^2/sec (vertical) = 10^{-5} to 10^{-4} m^2/sec

Some scaling arguments

- Full set of equations governs all scales of motion. How do we simplify?
- We can use the size of the terms to figure out something about time and length scales, then determine relative size of terms, then find the approximate force balance for the specific motion.

- Introduce a non-dimensional term that helps us decide if the viscous terms are important

Acceleration	Advection	……	Viscosity
U/T	U^2/L	……	$\nu U/L^2$

Reynolds number: $Re = UL/\nu$ is the ratio of advective to viscous terms

Large Reynolds number: flow nearly inviscid (quite turbulent)

Small Reynolds number: flow viscous (nearly laminar)