-plane

forming one class are the planetary waves, in which the time evolution is prompted by the weak but important *planetary effect*.

As we may recall from Section 2-6, on a spherical earth (or planet or star, in general), the Coriolis parameter, f, is proportional to the rotation rate, Ω, times the sine of the latitude, φ:

$$f = 2\Omega \sin \varphi.$$

Large wave formations such as alternating cyclones and anticyclones contributing to our daily weather and, to a lesser extent, the Gulf Stream meanders span several degrees of latitude; for them, it is necessary to consider the meridional change in the Coriolis parameter. If the coordinate y is oriented northward and is measured from a reference latitude φ_0 (say, a latitude somewhere in the middle of the wave under consideration), then $\varphi = \varphi_0 + y/a$, where a is the earth's radius (6371 km). Considering y/a as a small departure, the Coriolis parameter can be expanded in a Taylor series:

$$f = 2\Omega \sin \varphi_0 + 2\Omega \frac{y}{a} \cos \varphi_0 + \cdots \tag{6-15}$$

Retaining only the first two terms, we write in traditional notation

$$f = f_0 + \beta_0 y, \tag{6-16}$$

where $f_0 = 2\Omega \sin \varphi_0$ is the reference Coriolis parameter and $\beta_0 = 2(\Omega/a) \cos \varphi_0$ is the *beta parameter*. Typical midlatitude values on Earth are $f_0 = 8 \times 10^{-5} \cdot \mathrm{s}^{-1}$ and $\beta_0 = 2 \times 10^{-11}\ \mathrm{m}^{-1} \cdot \mathrm{s}^{-1}$. The Cartesian framework where the beta term is not retained is called the *f-plane*, and that where it is retained is called the *beta plane*. The next step in order of accuracy is to retain the full spherical geometry (which we will avoid throughout this book). Rigorous justifications of the beta-plane approximation can be found in Veronis (1963, 1981), Pedlosky (1987), and Verkley (1990).

Note that the beta-plane representation is validated at mid latitudes only if the $\beta_0 y$ term is small compared to the leading f_0 term. In terms of the motion's meridional length scale L, this implies

$$\beta = \frac{\beta_0 L}{f_0} \ll 1, \tag{6-17}$$

where the dimensionless ratio can be called the *planetary number*.

The governing equations, having become

$$\frac{\partial u}{\partial t} - (f_0 + \beta_0 y) v = -g \frac{\partial \eta}{\partial x}, \tag{6-18a}$$

$$\frac{\partial v}{\partial t} + (f_0 + \beta_0 y) u = -g \frac{\partial \eta}{\partial y}, \tag{6-18b}$$

$$\frac{\partial \eta}{\partial t} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0, \tag{6-18c}$$