Western Boundary Currents

Currents on the western side of basins:

-Gulf Stream- N. Atlantic

- Brazil Current- S. Atlantic
- Kuroshio- N. Pacific
- East Australian current- S. Pacific

Are narrow and strong currents, while eastern currents are weak and diffused.

Why?

This was first explained by the pioneering work of Stommel (1948)

Our understanding of the theory of ocean circulation is largely the results of 3 key studies

- *Harald Sverdrup* (1947): Showed that the circulation of the upper ocean is related to the curl of the wind stress
$$curl(\tau) = \frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y}$$
- *Henry Stommel* (1948): Showed that the circulation of ocean gyres is asymmetric because of the change of Coriolis with latitude (the beta effect)
$$\beta = \frac{\partial f}{\partial y}$$
- *Walter Munk* (1950): Add eddy viscosity and calculated the circulation in the Pacific using more realistic domain and winds

Sverdrup solution

→steady state →no horizontal friction →constant wind

→no deep motion for z<- D → $\boxed{f=f_o+\beta y}$ *(β=df/dy)*

vertically integrated Eqs.

M=transport from –D to 0

P=integrated pressure

τ=wind stress

continuity: $\left\{\dfrac{\partial M_x}{\partial x}+\dfrac{\partial M_y}{\partial y}=0\right.$

momentum:

$$\begin{cases} -fM_y = -\dfrac{\partial P}{\partial x}+\tau_x & \text{(a)} \\ fM_x = -\dfrac{\partial P}{\partial y}+\tau_y & \text{(b)} \end{cases}$$

$$\frac{\partial}{\partial x}(b)-\frac{\partial}{\partial y}(a):$$

$$\underbrace{f\left(\frac{\partial M_x}{\partial x}+\frac{\partial M_y}{\partial y}\right)}_{=0}+M_x\underbrace{\frac{\partial f}{\partial x}}_{=0}+M_y\underbrace{\frac{\partial f}{\partial y}}_{\beta}=-\underbrace{\left(\frac{\partial^2 P}{\partial x\partial y}-\frac{\partial^2 P}{\partial x\partial y}\right)}_{=0}+\underbrace{\left(\frac{\partial \tau_y}{\partial x}-\frac{\partial \tau_x}{\partial y}\right)}_{curl\tau}$$

Sverdrup's balance: $\boxed{\beta M_y = curl\,\tau,}$ $\beta=\dfrac{\partial f}{\partial y},\ M_y=\int \rho v dz,\ curl=\dfrac{\partial \tau_y}{\partial x}-\dfrac{\partial \tau_x}{\partial y}$

From continuity Eq: $M_x=-\int_x \dfrac{\partial M_y}{\partial y}dx$ (integrating from east boundary)

Sverdrup solution: calculate the transports in the upper ocean from observed winds

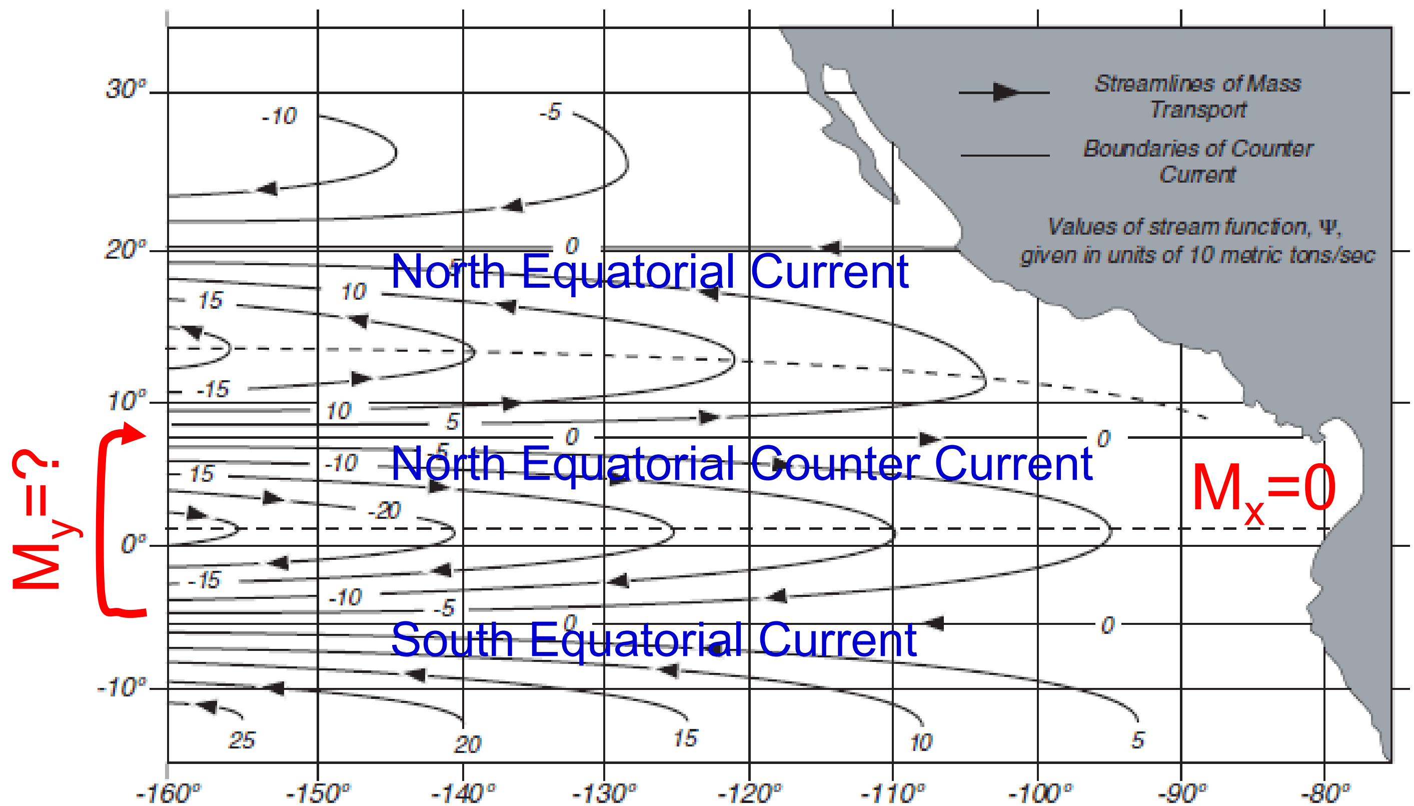
Sverdrup (1947) shows that :

- The flow is Geostrophic with level of no motion, obeys the Ekman transport
- The upper ocean circulation is directly related to the curl of the wind stress
- The change of Coriolis with latitudes is essential (“beta effect”)
- Sverdrup’s solution provides first order estimate of surface transports.
- Deficiency of theory: neglects lateral friction thus no boundary currents! (M_x=0 on east boundaries, ??? on west boundaries)

$$\beta M_y = curl\,\tau$$

$$M_x = -\int_x \frac{\partial M_y}{\partial y} dx$$

Circulation of the Pacific Ocean calculated from Sverdrup’s theory using mean wind stress (Reid, 1948)



Testing Sverdrup's theory against observations from the Pacific Ocean

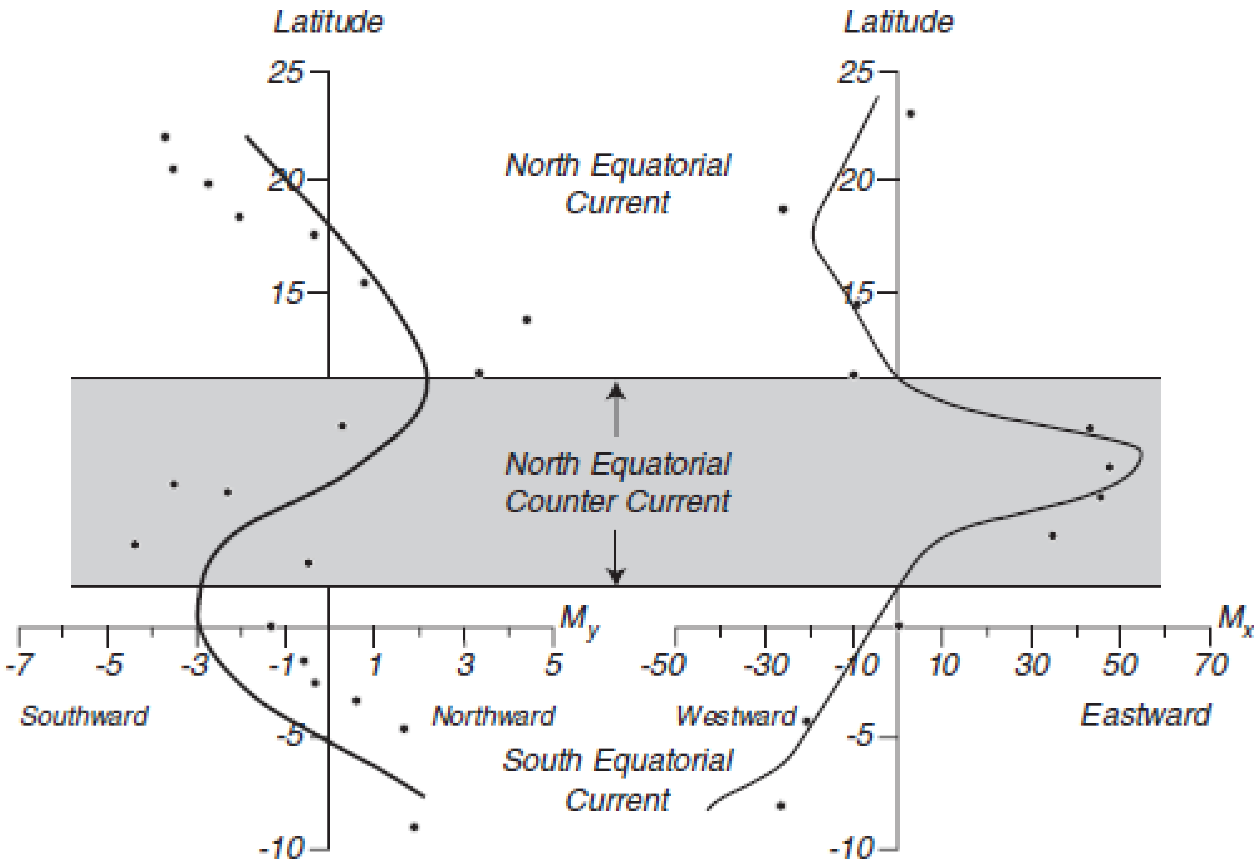


Figure 11.2 Mass transport in the eastern Pacific calculated from Sverdrup's theory using observed winds with 11.8 and 11.10 (solid lines) and pressure calculated from hydrographic data from ships with 11.4 (dots). Transport is in tons per second through a section one meter wide extending from the sea surface to a depth of one kilometer. Note the difference in scale between M_y and M_x. After Reid (1948).

Sverdrup's solution using more recent wind stress data

Transports are somewhat weak compared with observations, and western boundary currents are still not resolved

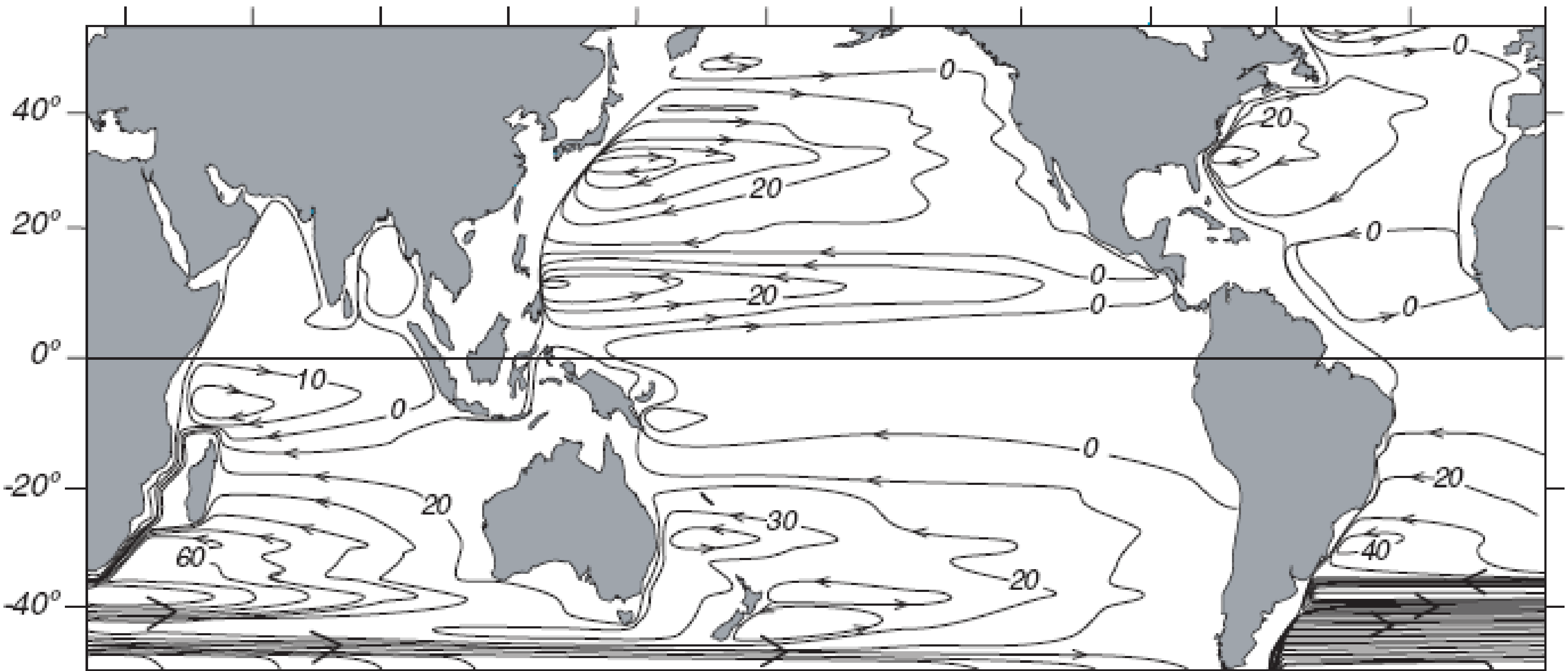
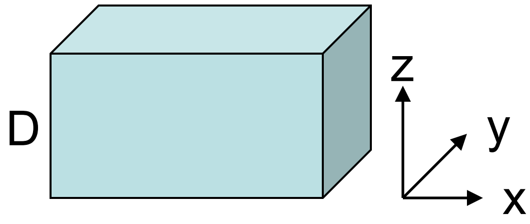


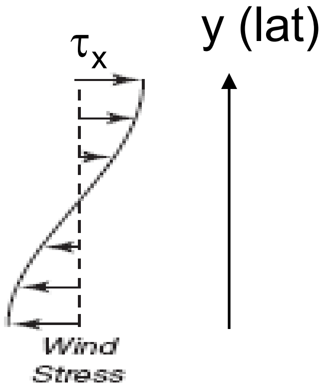
Figure 11.3 Depth-integrated Sverdrup transport applied globally using the wind stress from Hellerman and Rosenstein (1983). Contour interval is 10 Sverdrups. After Tomczak and Godfrey (1994: 46).

Stommel's Solution

assumptions:



- "Box model": rectangular basin with constant depth, D, and constant density, ρ.
- Steady state solution (neglect acceleration)
- Simple wind forcing, $\tau_x(y) = A\cos\left(\frac{\pi}{b}y\right)$
- Sverdrup's balance plus **simple bottom friction, F=Ju, (J=friction coefficient)**



$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv + \frac{1}{\rho}\frac{\partial \tau_x}{\partial z} - Ju$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu + \frac{1}{\rho}\frac{\partial \tau_y}{\partial z} - Jv$$

Made experiments with constant *f* and with *f* changes with latitude

Vorticity balance equation:
(derived by taking derivatives of the u & v equations)

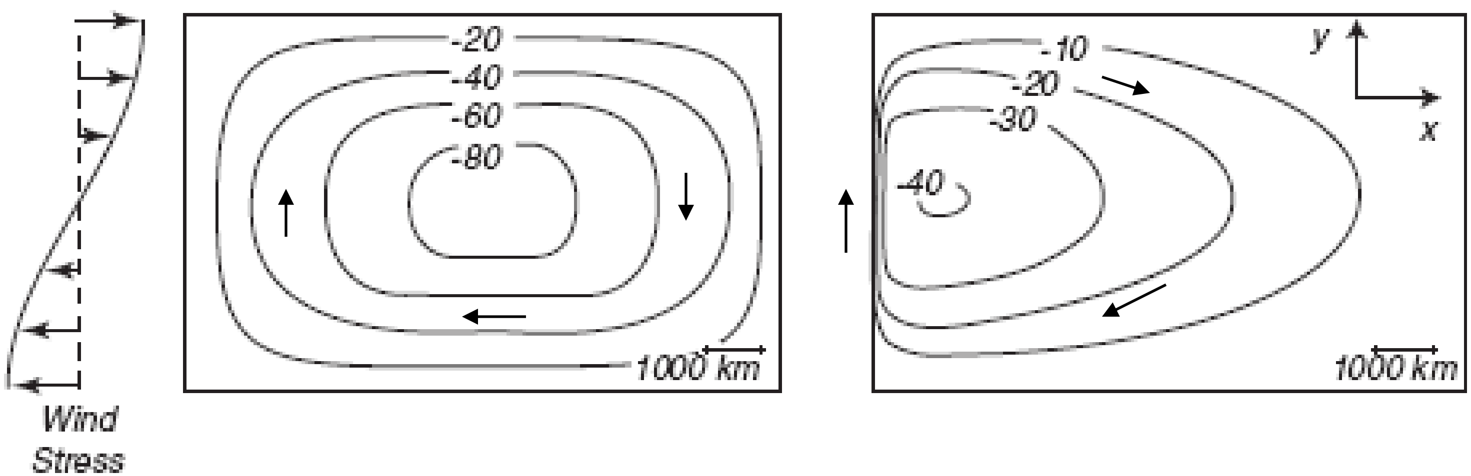
$$\boxed{\begin{array}{c}\textit{Vorticity}\\ \xi = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y}\end{array}}$$

$$0 = \overbrace{\frac{1}{\rho}\frac{\partial}{\partial z}\left(\frac{\partial \tau_x}{\partial y} - \frac{\partial \tau_y}{\partial x}\right)}^{wind-stress\ curl} - \overbrace{J\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)}^{friction} + \overbrace{v\frac{\partial f}{\partial y}}^{Coriolis\ change\ with\ latitude\ (\beta v)}$$

Beta Term

Without Beta term

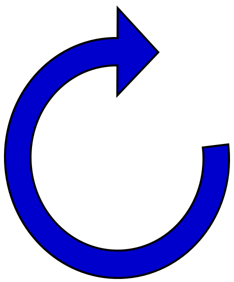
With Beta term



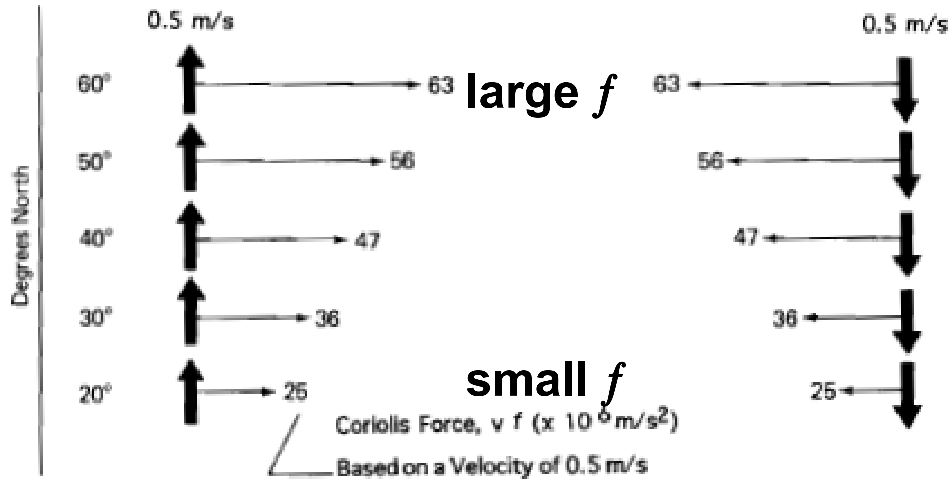
Stommel (1948) **solution**

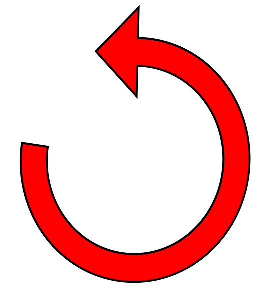
How can the change of Coriolis (*f*) with latitude cause the western boundary current? It creates a *planetary vorticity*.

[terms: wind-stress (WS) + beta, planetary vorticity (PV) + friction (Fr)]



anticyclonic vorticity

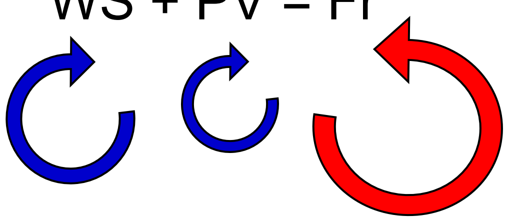


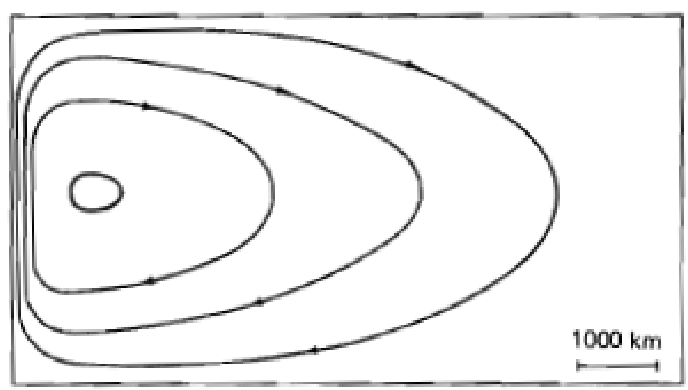


cyclonic vorticity

West boundary balance:

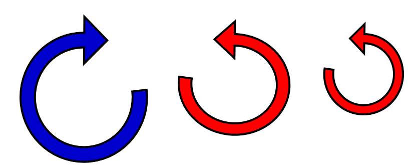
WS + PV = Fr





East boundary balance:

WS = PV + Fr



Friction must be larger in western oceans!

Vorticity

$$\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

Munk's Solution

Munk (1950) build on Sverdrup's and Stommel's theories:

- Active upper ocean layer (~1000m) over motionless deep ocean
- Added **lateral eddy friction with constant eddy coefficients** to consider more realistic ocean basins
- He integrated the equations of the upper ocean (similar to Sverdrup's) and wrote them in terms of a stream function

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = f\,v + \frac{\partial}{\partial z}\left(A_z\frac{\partial u}{\partial z}\right) + A_H\,\frac{\partial^2 u}{\partial x^2} + A_H\,\frac{\partial^2 u}{\partial y^2}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y} = -f\,u + \frac{\partial}{\partial z}\left(A_z\frac{\partial v}{\partial z}\right) + A_H\,\frac{\partial^2 v}{\partial x^2} + A_H\,\frac{\partial^2 v}{\partial y^2}$$

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\,\frac{\partial^4}{\partial x^2\,\partial y^2} + \frac{\partial^4}{\partial y^4}$$

$$\boxed{\underbrace{A_H\nabla^4\Psi}_{\text{Friction}} - \underbrace{\beta\,\frac{\partial\Psi}{\partial x} = -\,\mathrm{curl}_z T}_{\text{Sverdrup Balance}}}$$

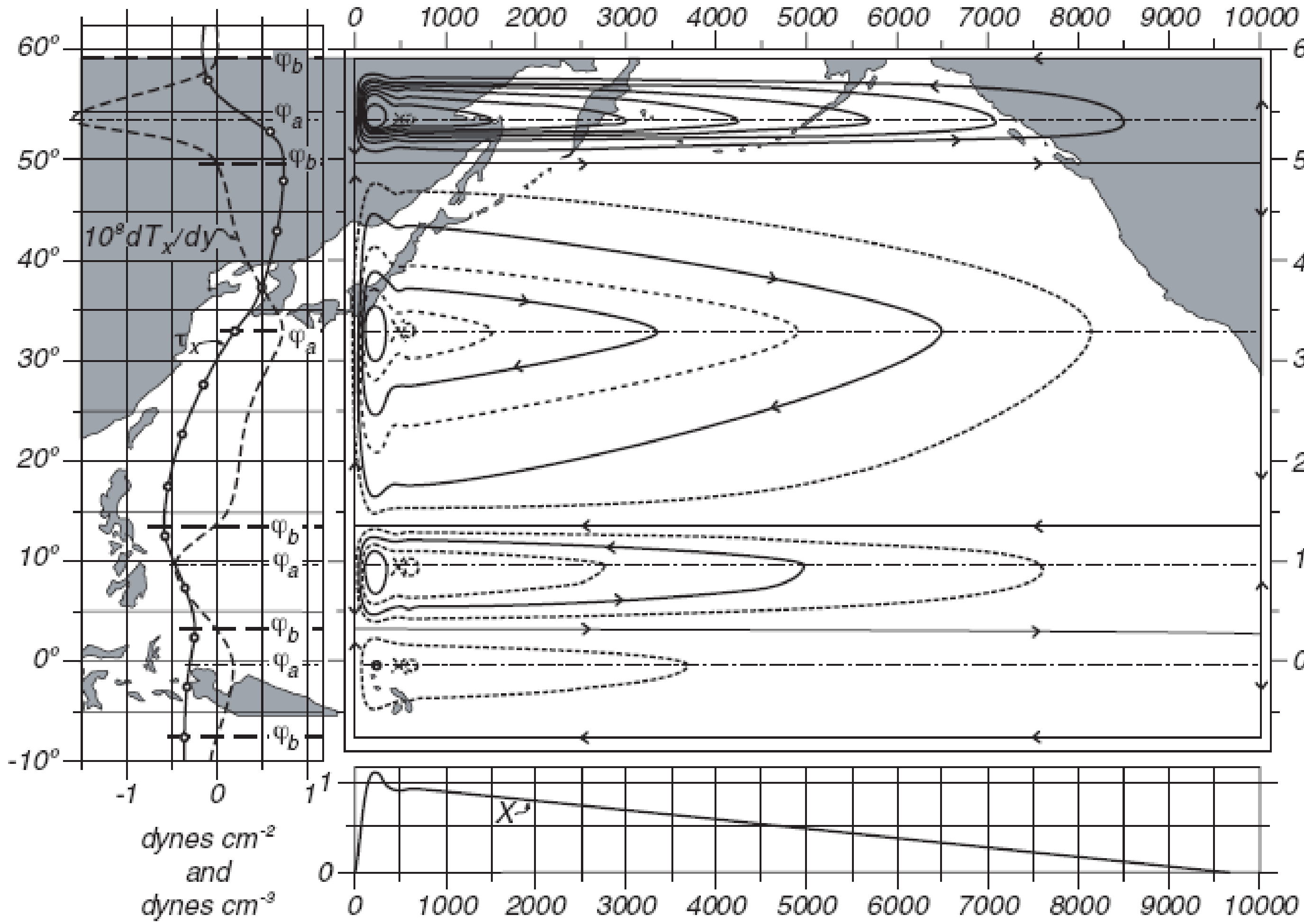
$$v = -\frac{\partial\psi}{\partial x},\quad u = \frac{\partial\psi}{\partial y}$$

Munk Solution

He solve the equation for a stream function using observed winds and estimated the Gulf Stream transport at 36Sv and the Kuroshio at 39Sv (~1/2 the observed). Note that the western boundary currents are not too realistic.

Wind stress & wind-stress Curl

Resulted Munk's circulation



Estimated ocean transports based on observations: including recirculation gyres that are not taken into account in the simple theories of ocean circulation

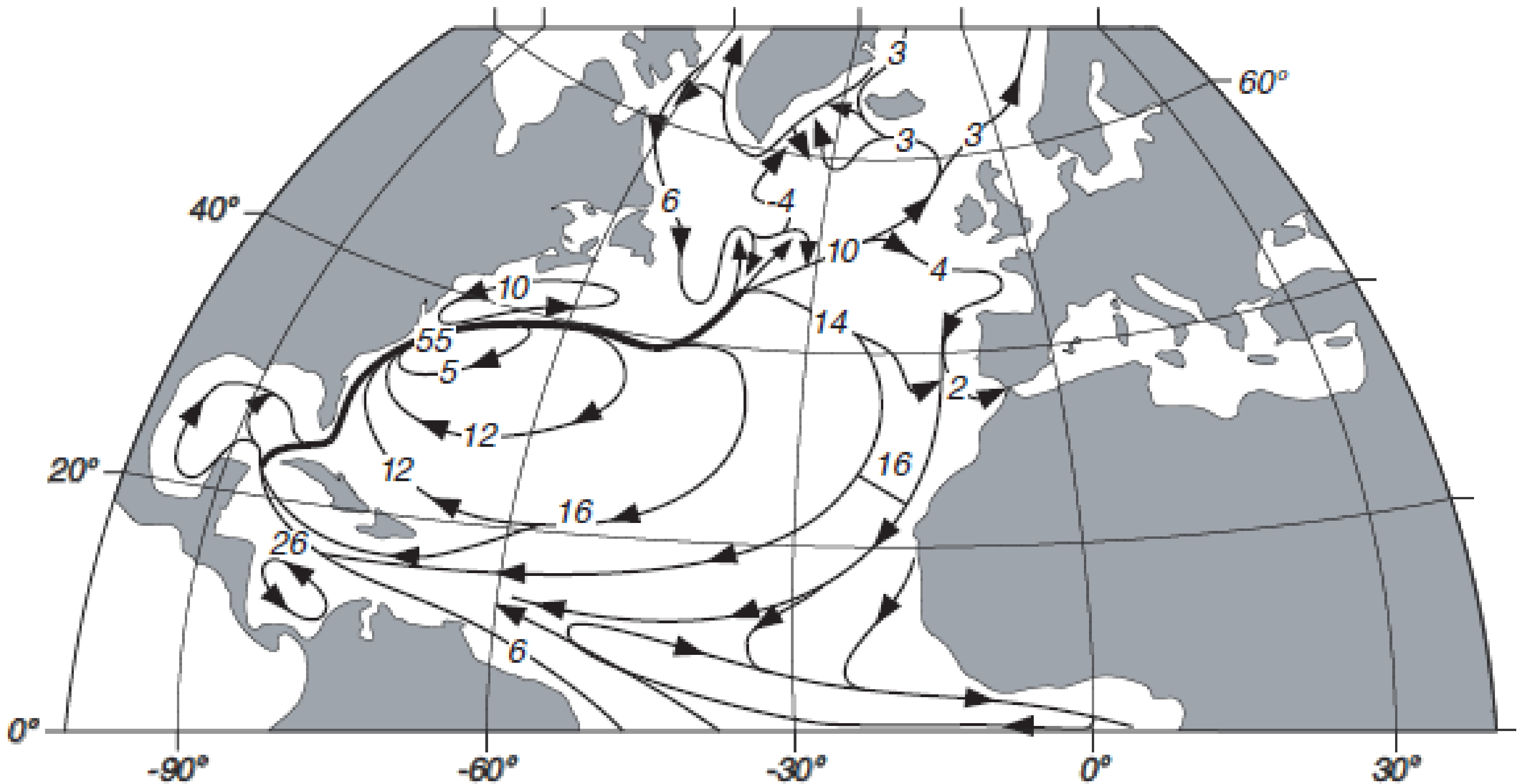


Figure 11.7 Sketch of the major surface currents in the North Atlantic. Values are transport in units of 10^6 m^3/s. After Sverdrup, Johnson, and Fleming (1942: fig. 187).

• Summary:
Simple theories (Sverdrup/Stommel/Munk) set the stage for the basic understanding of ocean general circulation, but these theories can not describe the details of particular ocean currents.

• What's next?
Particular ocean currents and their dynamics are affected by additional parameters not included in the theories above.

> Local topography and coastline
> stratification (heat fluxes, rivers, etc..)
> local wind pattern
> location (high or low latitudes)
> seasonal changes >more…

- Wed Oct 24 & Mon Oct 26: Ocean currents & circulation
- Wed Oct 30: No Class (Sea Level conference)
- Mon Nov 4: Review for exam
- Wed Nov 6: Exam #2