

Important: there are several different variables to describe density and temperature in oceanography and meteorology:

Density σ

Density σ_t

Potential density σ_θ

Potential temperature θ

Densities σ_2 , σ_4 (2 refers to 2000 dbar, 4 refers to 4000 dbar, etc.)

Neutral density γ

1. In-situ density σ

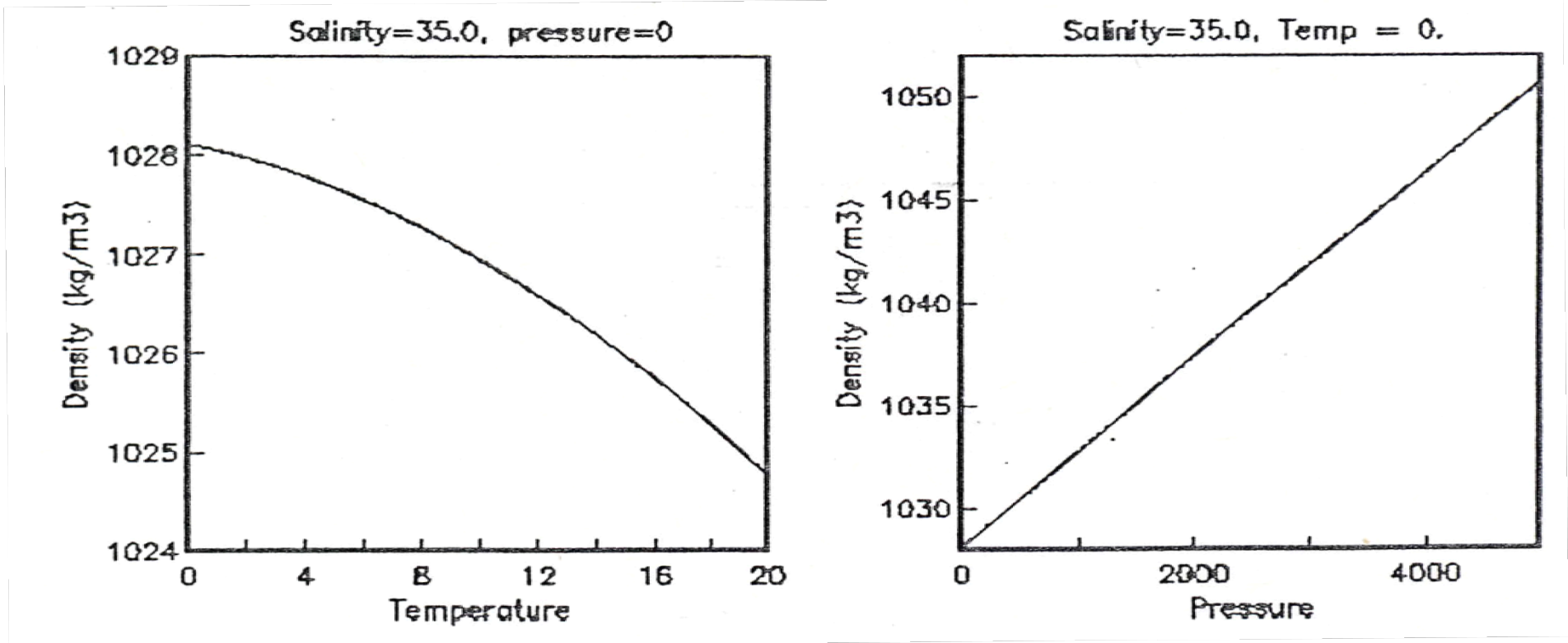
$\sigma = \rho - 1000 = \sigma(t, s, p)$, is given by the equation of state,
t - temperature

Definition: in-situ density – density we would measure if we could dive into the ocean and weigh a water parcel at its original location

However, for ocean dynamics other representations of density are more useful

Why do we need those different density representations?

Pressure and Temperature Effects on Density



The effect of temperature on density is relatively strong but often not important dynamically, so it is often desirable to eliminate this effect and consider only the effects of T and S. There are several ways to do so.

2. Density σ_t $\sigma_t = \sigma(t,s,0)$

Definition: this is a density of a water parcel for which we assume (in the equation of state for sea water) that pressure equals that of a standard atmosphere.

In other words, it is the density of a water parcel that we brought to the surface maintaining its temperature and salinity.

However, there is a problem with thermodynamics and energy conservation – if you moving a water parcel up the water column without supplying any heat it will cool by $\sim 0.12^\circ\text{C}$ per 1km because of expansion!

The first law of thermodynamics:

Heat supplied to a water parcel goes into changing the parcel's internal energy and/or into work done by the parcel on the surrounding fluid

$$\delta Q = dU + \delta W$$

or

$$dU = \delta Q - \delta W$$

where

$dU = c_v dT$ – internal energy of a water parcel

c_v – specific heat at constant volume

$\delta W = p dV$ – the work done on the surrounding fluid

δQ – the heat supplied to the parcel

$$c_v \delta T = \delta Q - p \delta V$$

δV – change in the parcel's volume

Let's raise a water parcel to the surface from depth. The parcel will expand.

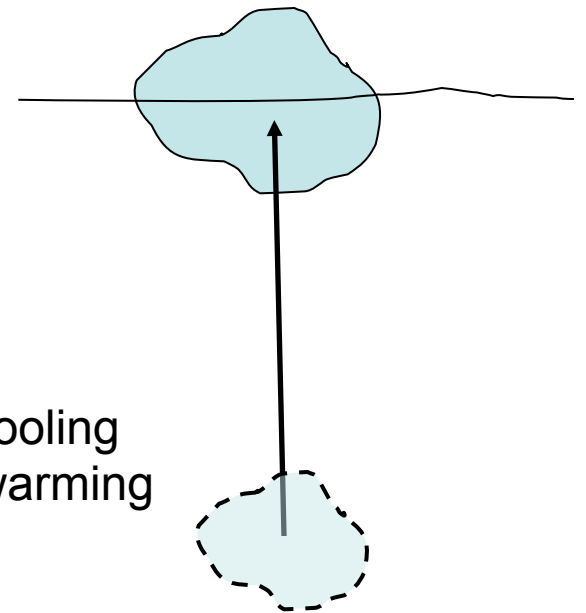
$$c_v \delta T = \delta Q - p \delta V$$

1) To maintain the temperature of the water parcel that you are bringing to the surface constant ($\delta T = 0$) you have to supply heat to this water parcel.

If there is no heat added, water temperature will drop by

$$\delta T = - (p/c_v) \delta V$$

i.e. adiabatic expansion cools the water parcel = adiabatic cooling
adiabatic compression warms the water parcel = adiabatic warming



3. Potential temperature θ

Definition: θ is temperature that a water parcel would have if it were raised adiabatically from some depth to the sea surface without change in its salinity. Raising the parcel adiabatically means that the parcel is not allowed to exchange heat with its surroundings

$$\theta < T$$

Again, the idea behind this is to eliminate the effect of compressibility from consideration

*Adiabatic processes should be relatively fast.
In case of the ocean raising a parcel adiabatically
implies that you raise it sufficiently fast as compared
to turbulent mixing of heat (turbulent diffusion)*

4. Potential density σ_θ

Definition: Potential density σ_θ is density that a water parcel would have if it were raised adiabatically to the surface without change in its salinity.

By definition:

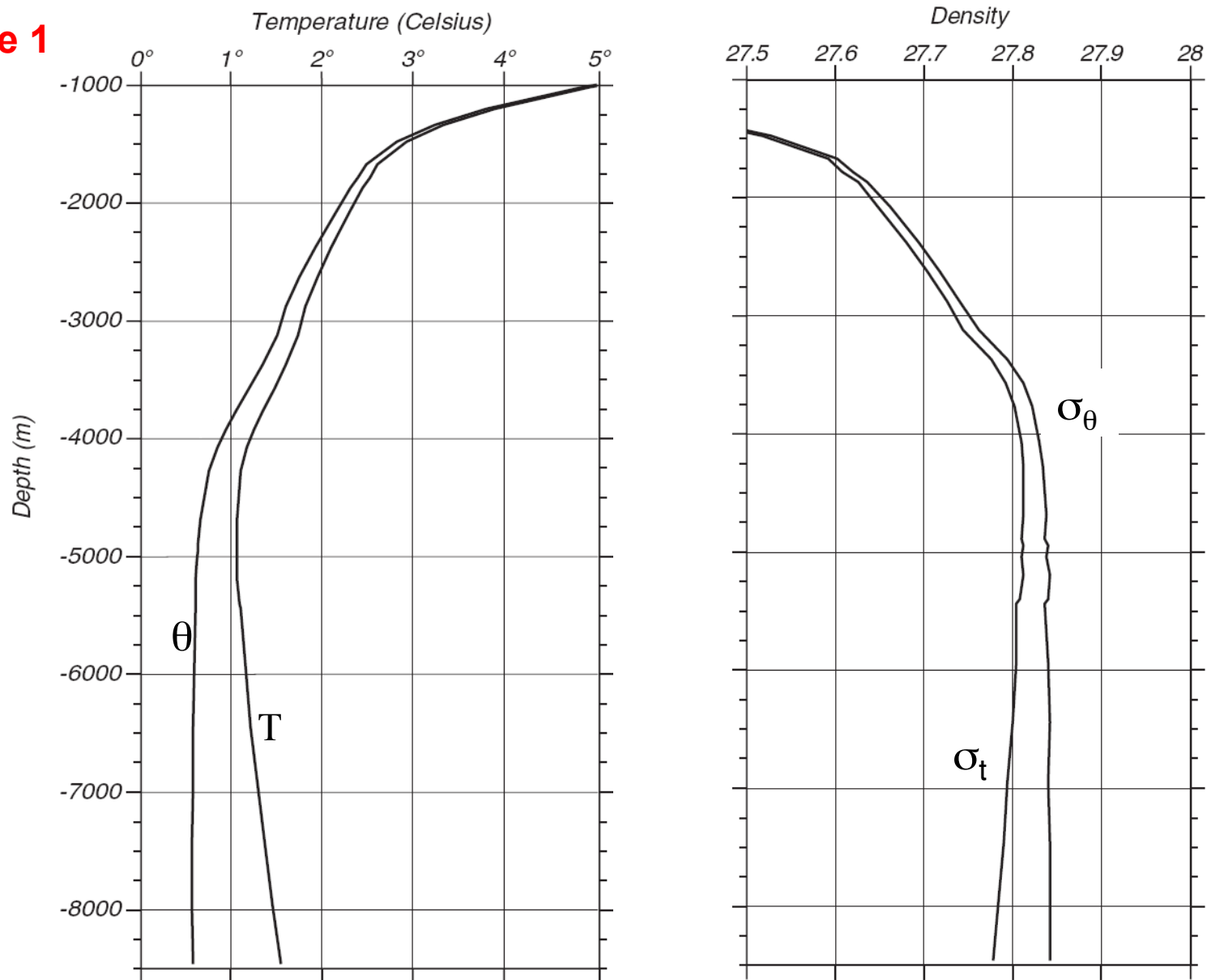
$$\sigma_\theta = \sigma(\theta, s, 0)$$

Again, the idea behind this is to eliminate the effect of compressibility from consideration

Why do you need potential and neutral densities, temperature (some to be discussed later in class)?

- 1) Comparing densities of two water parcels from different depths
- 2) Ocean vertical stratification – stable or unstable?
- 3) Calculating ocean currents from dynamical relationship (geostrophy)
- 4) If there is little mixing in the ocean, water parcels move along surfaces of constant potential (more accurately neutral) density

Example 1



Profiles of the potential and actual temperature (left) and σ_t and σ_θ in the Kermadec Trench in the Pacific (Warren 1973). Note that temperature T increases from about 5000m depth to the bottom (due to adiabatic compression), but not θ .

Example 2

In-situ and potential temperatures in the Mindanao ocean trench off the Philippine Islands

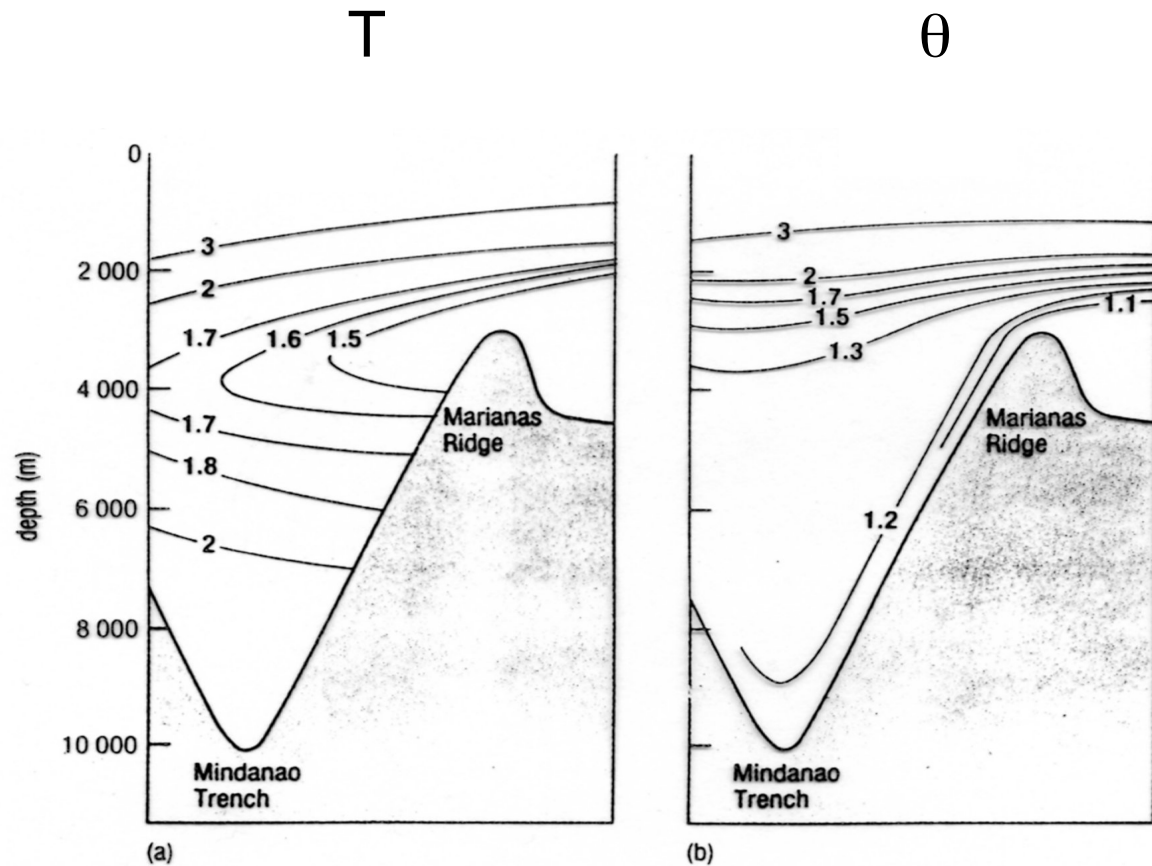
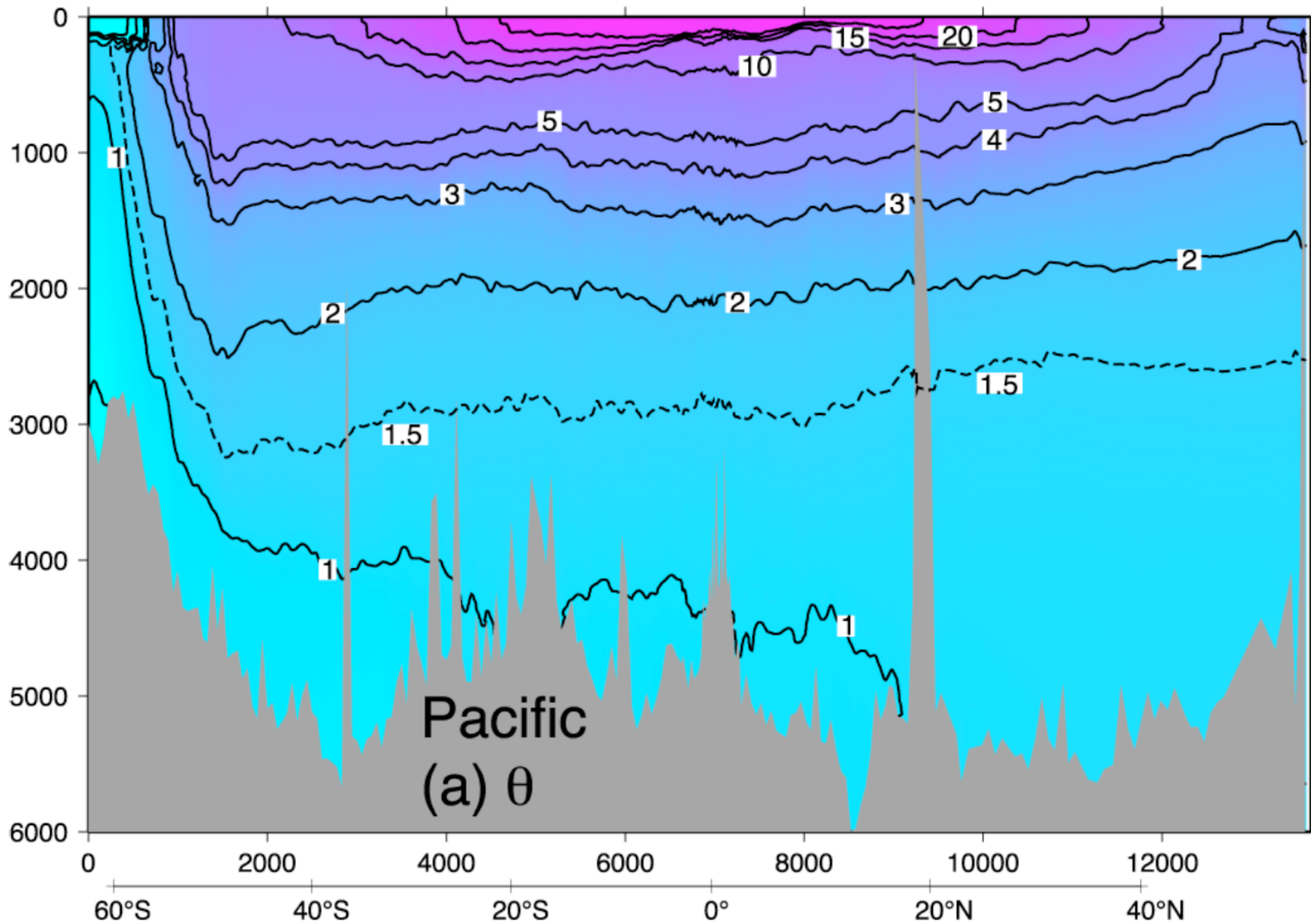
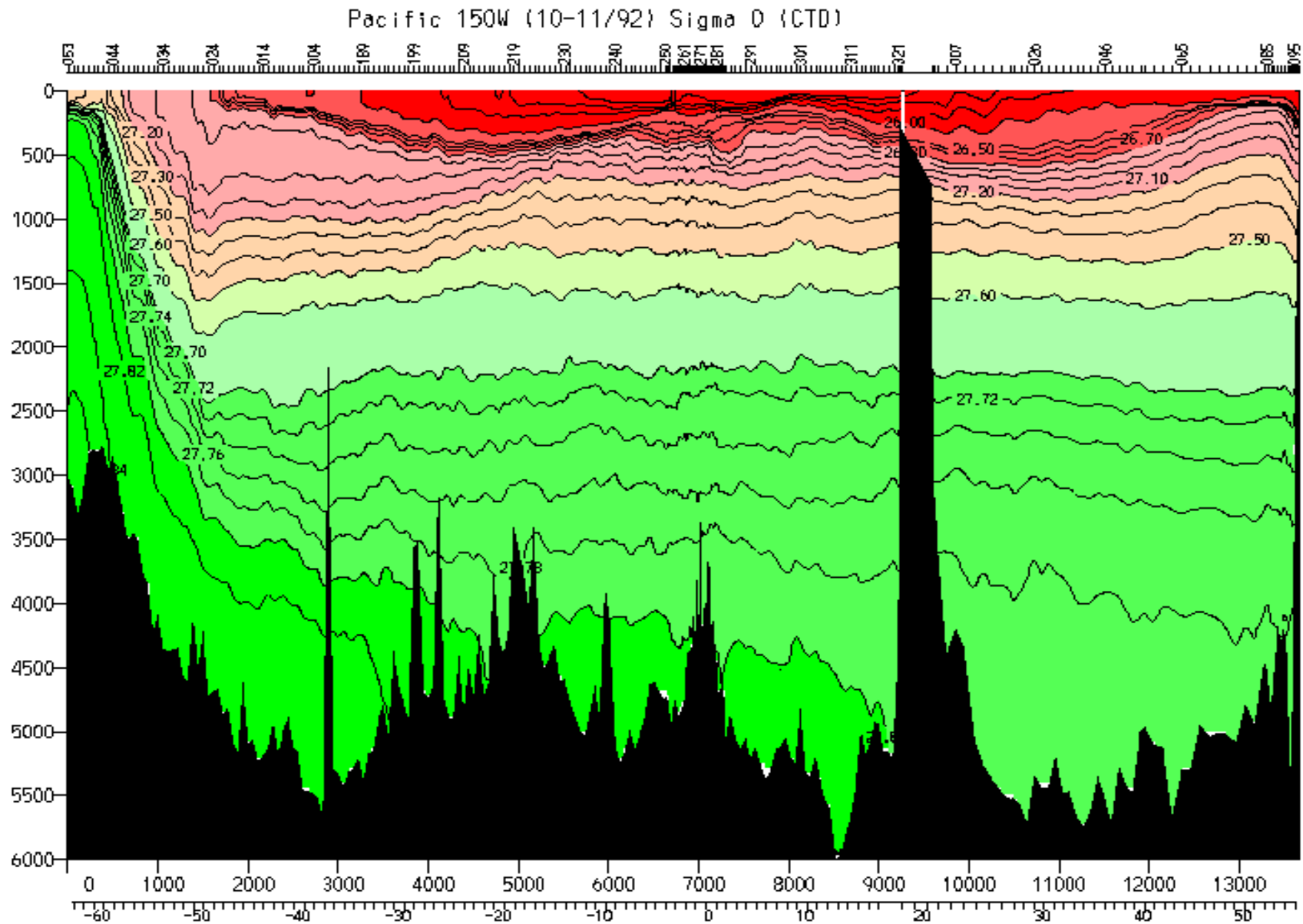


Figure 4.6 Two patterns of temperature distribution in the Mindanao Trench (for use with Question 4.6). Contours are in °C and represent either *in situ* temperature, or potential temperature, θ .



*Differences between θ and T are small,
smaller than 1°C in the deep ocean*

Potential density (σ_θ) transect in the Pacific Ocean



Latitude

*Variations in σ_θ are much smaller than in σ
(the latter values could reach 70 kg/m³)*

5. Potential densities σ_4 , σ_5 , σ_1 , etc.

Definition: Potential density σ_4 is the density that a parcel of water would have if it were raised (or lowered) adiabatically to the isobaric surface 4000dbar (~4000m).

i.e. potential density with respect to 4000dbar surface

If you want to compare water masses that occur, say, between 6000 m and 4000 m, it is not productive to use surface as the reference level. Rather, you would use σ_5

Example 3

Note the tiny salinity and larger temperature variations

Depth (m)	Salinity (psu)	Temperature (°C)	θ (°C)	σ_{θ} (kg m ⁻³)	σ_4 (kg m ⁻³)	σ_{10} (kg m ⁻³)
1487	34.597	2.800	2.695	27.591	45.514	69.495
2590	34.660	1.730	1.544	27.734	45.777	69.903
3488	34.680	1.500	1.230	27.773	45.849	70.015
4685	34.697	1.431	1.028	27.800	45.898	70.090
5585	34.699	1.526	1.004	27.803	45.904	70.099
6484	34.599	1.658	1.005	27.803	45.904	70.099
9940	34.700	2.266	1.007	27.804	45.904	70.099

Temperature inversion due to adiabatic compression

?

Values close to in-situ density

Comparison of in situ and potential temperatures and potential densities relative to the sea surface (σ_{θ}), 4,000 dbar (σ_4), and 10,000 dbar (σ_{10}) in the Mariana Trench in the western North Pacific. (Data from R/V T. Washington, 1976.)

Q: Why is θ increasing with depth near the bottom (very very slightly)?
Why is σ_{θ} stratification stable with θ increasing?

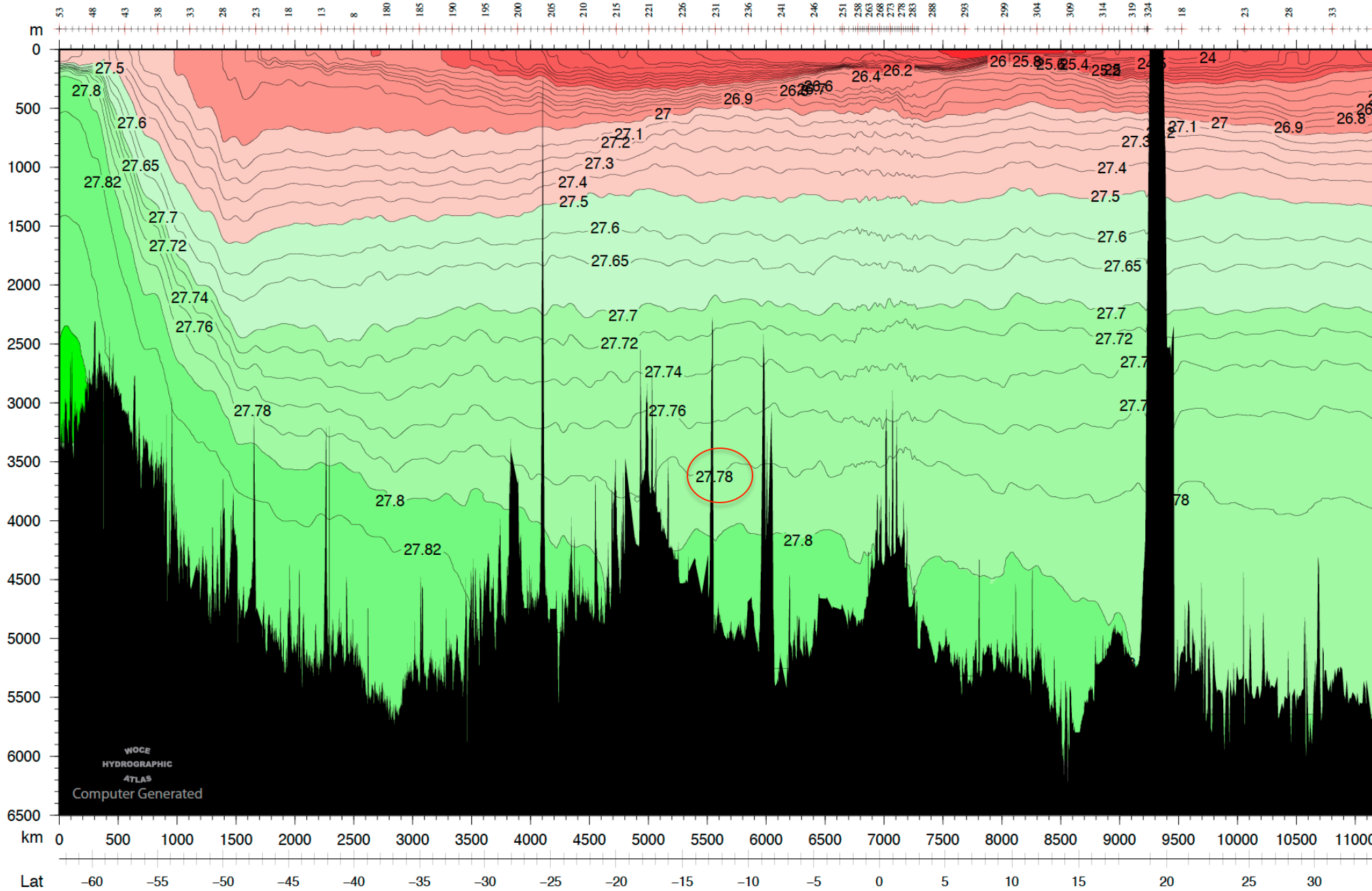
6. Neutral density γ

Very sketchy

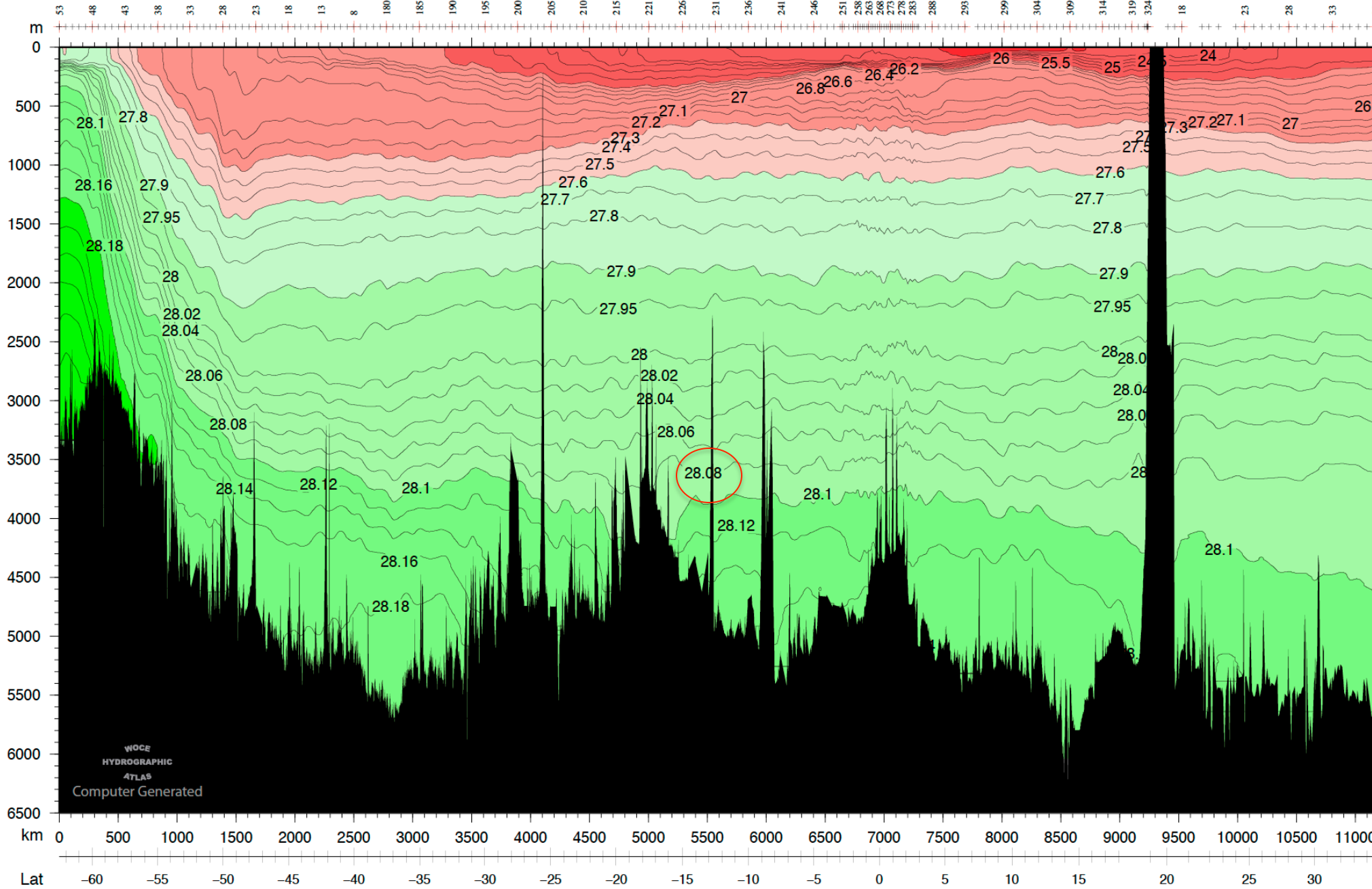
Definition (not very accurate but gives an idea): Neutral density γ , a function of T, S, P and latitude and longitude, gives neutral surfaces – surfaces along which a parcel would travel in the absence of mixing

It is a potential density (it is close but not identical to σ_θ) where the reference level is slightly adjusted (because of nonlinearity of the equation of state) at each point. Calculated numerically <http://www.marine.csiro.au/~jackett/NeutralDensity/>

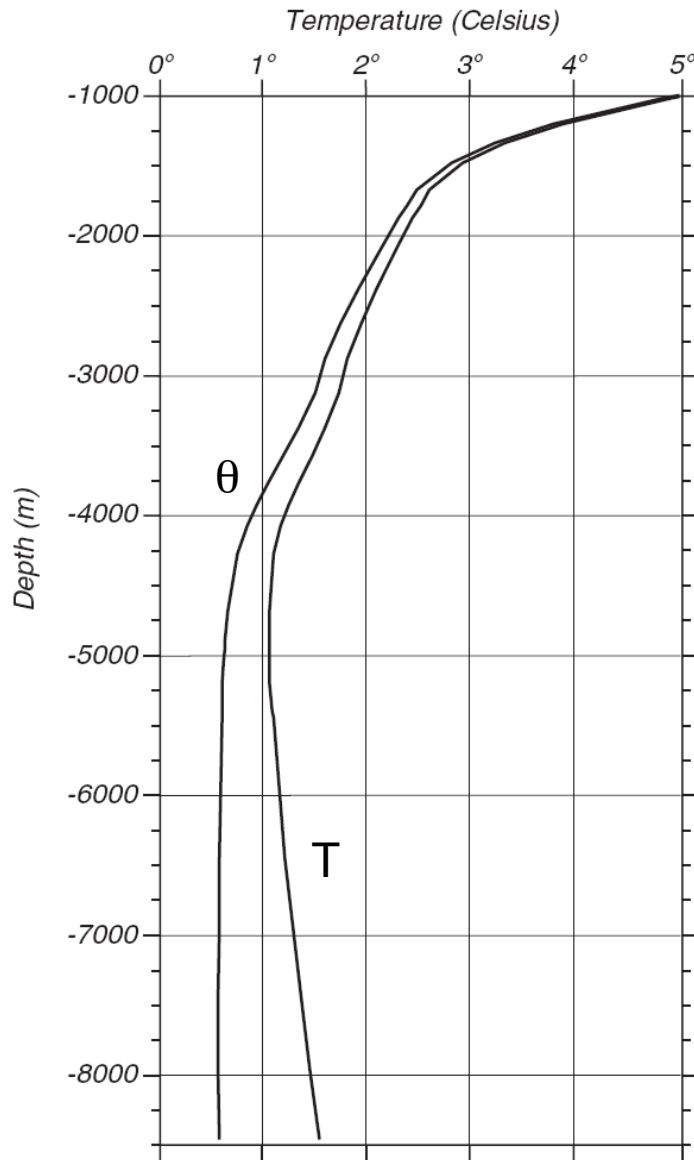
σ_0 (kg/m³) for P16 150°W (1000:1)



Neutral Density (kg/m^3) for P16 150°W (1000:1)

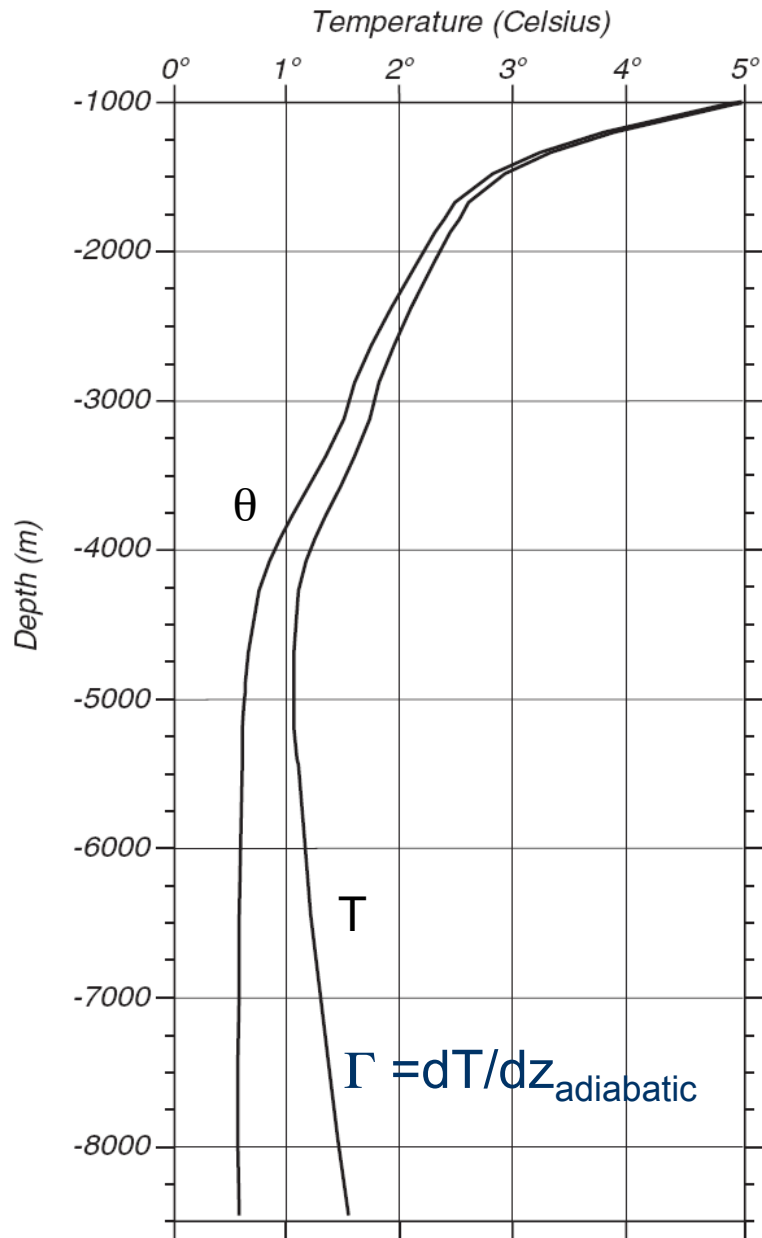


How to calculate potential temperature?



We need to know adiabatic lapse rate $\Gamma = (dT/dz)_{\text{adiabatic}}$

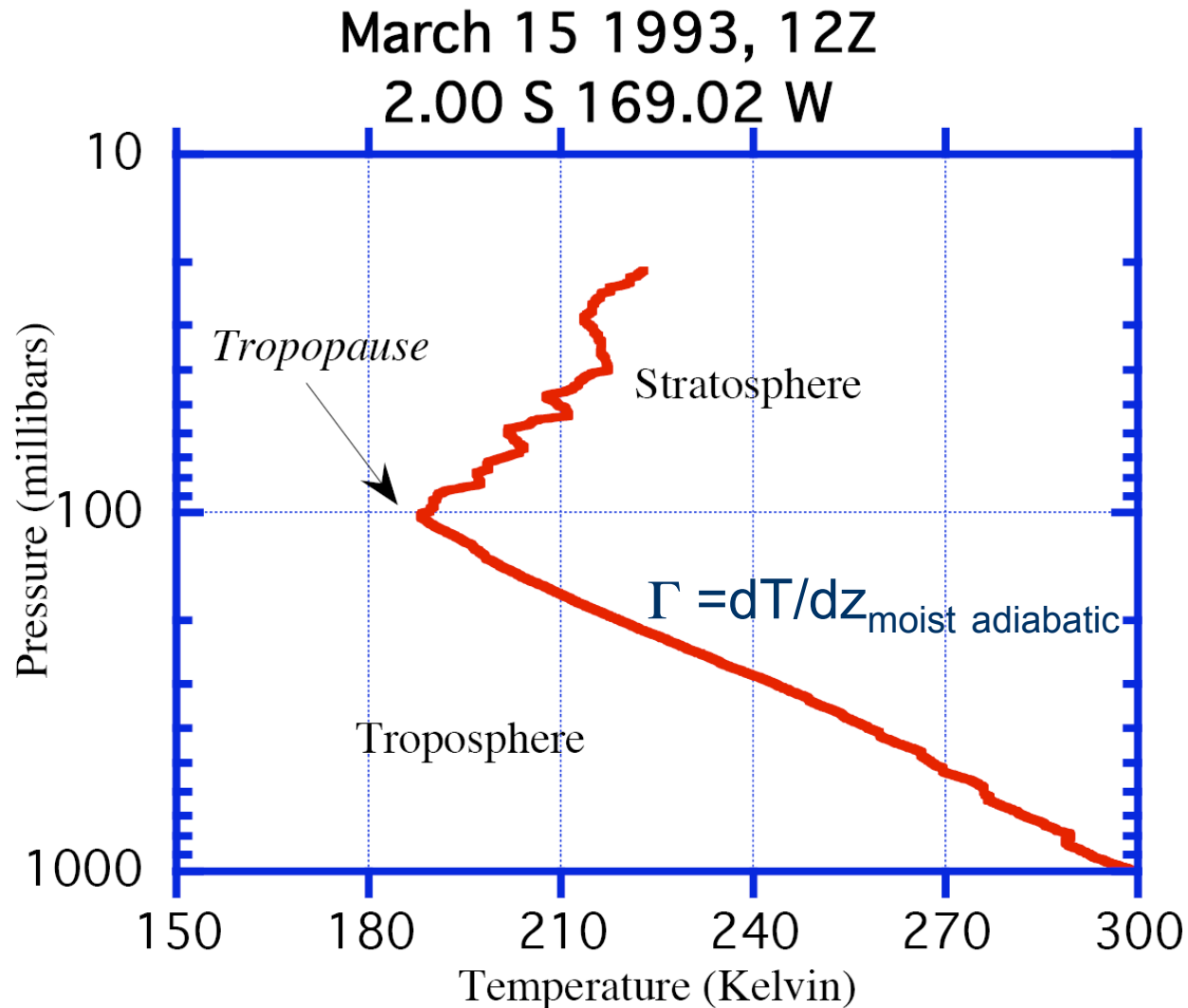
This is the lapse rate that the ocean would have if the heat exchange were determined solely by adiabatic compression /expansion (no irreversible mixing)



The slope of the temperature inversion between 8000 and 5000m is very close to adiabatic lapse rates

$$\Gamma = dT/dz_{\text{adiabatic}} = 0.12-0.13^{\circ}\text{C}/1000\text{m}$$

This is because adiabatic compression and expansion are the main mechanisms of heat exchange at these depths.

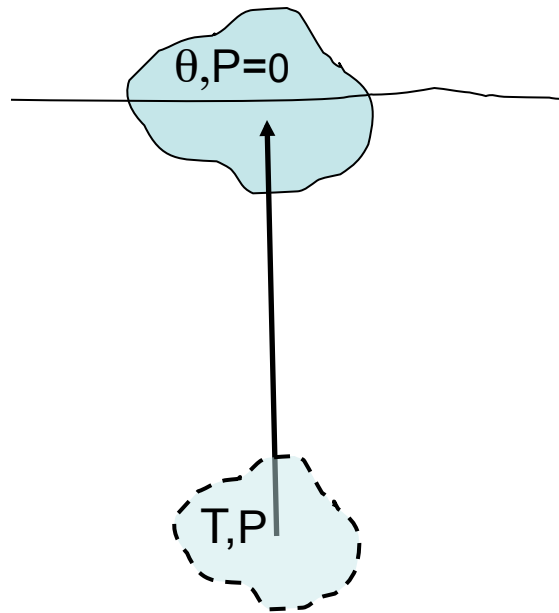


An atmospheric analogue: Temperature profile in the atmosphere measured at a point in the tropical Pacific. The measurements were obtained from a radiosonde ("weather balloon") on Mar 15, 1993. Temperature distribution in the atmosphere is best described by the moist adiabatic lapse rates.

Calculations of potential temperature

If we know Γ then we can integrate $(dT/dz)_{\text{adiabatic}} = \Gamma$ from some depth to the surface

$$\theta(S, T, p) = T + \int_p^0 \Gamma(S, \theta(S, T, p)) dp$$



Here, Γ is defined as

$$\Gamma = (dT/dp)_{\text{adiabatic}}$$

The two definitions are analogous because of the hydrostatic law

Often Γ is defined as

$$\Gamma = (dT/dz)_{\text{adiabatic}}$$

$$\frac{dP}{dz} = -g\rho$$

Calculations of of adiabatic lapse rate for the ocean

$$\Gamma = (dT/dz)_{\text{adiabatic}} \quad - \text{adiabatic lapse rate?}$$

We will use several ideas from the thermodynamics, including

- (1) The first law of thermodynamics
- (2) Entropy
- (3) Maxwell's relations

The first law of thermodynamics and entropy:

$$\delta U = \delta Q - \delta W$$

Where

$\delta U = c_v \delta T$ – internal energy of a water parcel

c_v – specific heat at constant volume

$\delta W = p \delta V$ – the work done on the surrounding fluid

δQ – the heat supplied to the parcel

$$c_v \delta T = \delta Q - p \delta V$$

Entropy η .

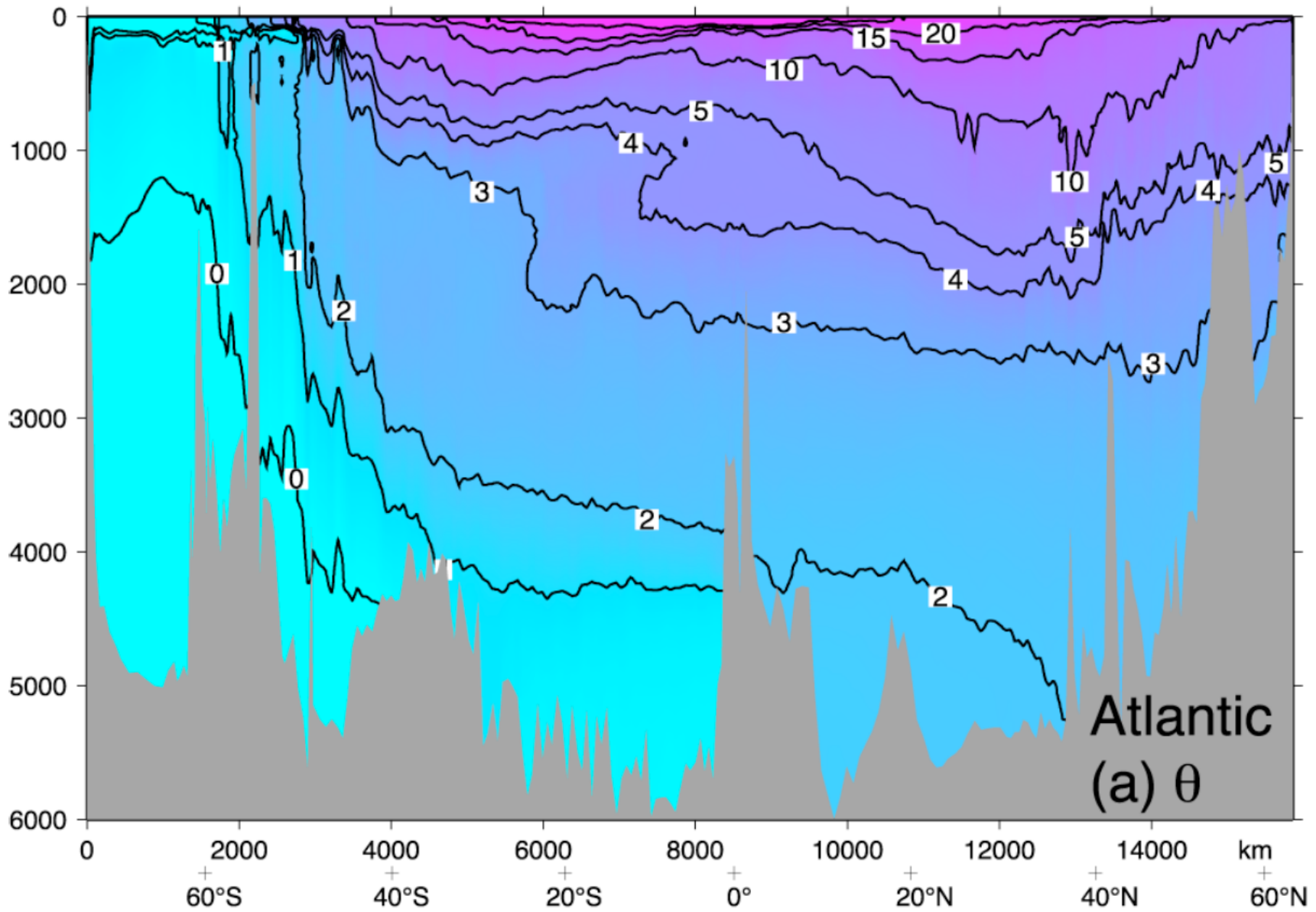
Heat supplied to the system (to a water parcel) can be represented as $T\delta\eta$ in reversible (relatively slow) processes where η is entropy:

$$\delta Q = T\delta\eta$$

T – absolute temperature, K

Important, heat is not a variable describing the state of the system, but η and T are.

For further calculations see paper notes



Differences between θ and T are small, smaller than 1°C in the deep ocean