Lecture 10

Geostrophy & Thermal wind

10.1 f and β planes

These are planes that are tangent to the earth (taken to be spherical) at a point of interest. The z axis is perpendicular to the plane (anti-parallel to gravity), the x-axis points toward the east, and the y axis points toward the north. The projection of the rotation vector along the z axis has magnitude $f = 2\Omega \sin\theta$, where Ω is the rotation rate of the earth, and θ is the latitude.

NEED A GOOD PICTURE HERE

If the basin is small compared to the radius of the earth, and if it is away from the equator, the we consider f, the Coriolis frequency to be constant, in which case the plane is called the f-plane. On the other hand, if the domain considered is large (for instance the North Atlantic), or near the equator, the effect of changing rotation is included in a linear fashion: $f = f_0 + \beta y$, where f_0 is the Coriolis frequency at the center of the plane, and β is the rate of change of f with y. Note that on the equator, $f = 0$, so rotation only acts through β.

10.2 Geostrophy

The mass conservation equation is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{10.1}$$

The linearized time-dependent momentum equations, in the hydrostatic limit are:

$$\rho\left[\frac{\partial u}{\partial t} - fv\right] = -\frac{\partial p}{\partial x} + \rho A_v \frac{\partial^2 u}{\partial z^2} \tag{10.2}$$

$$\rho\left[\frac{\partial v}{\partial t} + fu\right] = -\frac{\partial p}{\partial y} + \rho A_v \frac{\partial^2 v}{\partial z^2} \tag{10.3}$$

$$0 = -\frac{\partial p}{\partial z} - \rho g \tag{10.4}$$

where A_v is the vertical eddy diffusivity (the turbulent equivalent to the kinematic viscosity), and the lateral friction terms have been ignored on the grounds of the thinness of the ocean. So we have a system of four equations in four unknowns.

The geostrophic balance comes about when we look at steady flow with no friction:

$$-fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{10.5}$$

$$fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} \tag{10.6}$$

$$\frac{\partial p}{\partial z} = -\rho g \tag{10.7}$$

In meteorology, where pressure observations are available in near real time from around the globe, this is extremely useful. Once the pressure field is mapped, the direction of the geostrophic flow is known, since it must be along lines of constant pressure: In Fig. 10.1, a high pressure cell is illustrated, In three dimensions, the pressure is a dome, locate where the letter H is. The plots on the top and the left side show the pressure as a function of x and y through the center of the high. Note that $\partial p/\partial x$ is zero at $x = 0$ for all y, so $v = 0$ along the y axis, and for the same reason, $u = 0$ along the x axis: the flow is circular, and clockwise around the high in the northern hemisphere (opposite in the southern hemisphere where, since the latitude is negative, f is negative as well).

10.3 Thermal wind

In the ocean the situation is more difficult, because, even with altimeters carried on satellites, it is difficult to measure the height of the sea level. For

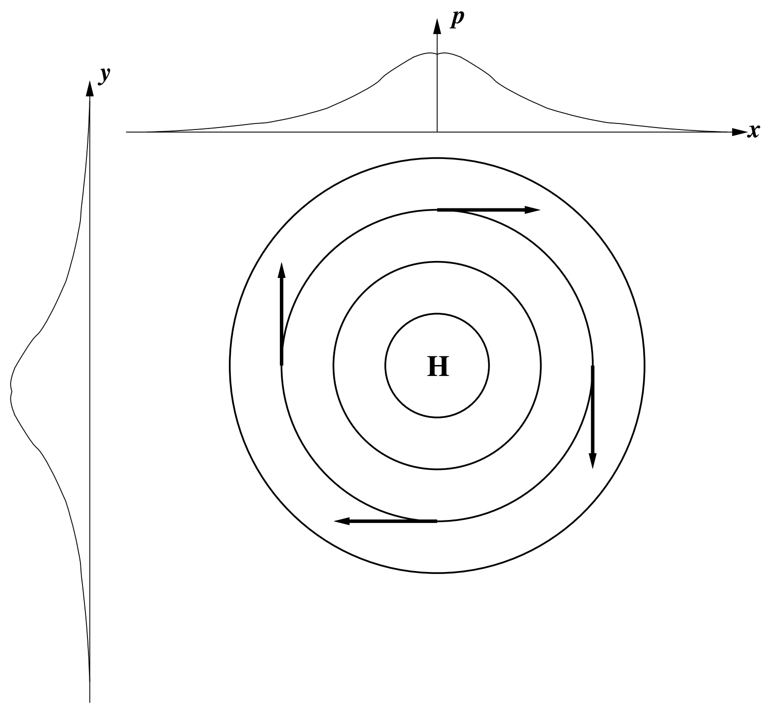

Figure 10.1: Circulation is clockwise around a High.

many years oceanographers had to be content with observations of density profiles ($\rho(z)$) at various locations in the ocean. For instance Fig. 10.2 shows a sketch of a map of density in an $x,,z$ section taken off Jacksonville Florida, and proceeding toward North Africa might look like: Since the observations are in terms of density, it is useful to take the vertical derivative of equations 10.5 and 10.6, and substitute density for pressure using the hydrostatic equation **??**:

$$-f\frac{\partial v}{\partial z} = g\frac{\partial \rho}{\partial x} \tag{10.8}$$

$$f\frac{\partial u}{\partial z} = g\frac{\partial \rho}{\partial y} \tag{10.9}$$

These are called the *Thermal wind* equations (because they were first derived in a meteorological explanation for desert winds). They relate the *vertical* gradients of velocity to *horizontal* gradients in density, that are given by sections. If, in addition, it is assumed that the velocity is known at some depth (often it is simply assumed to be zero, in which case the depth is

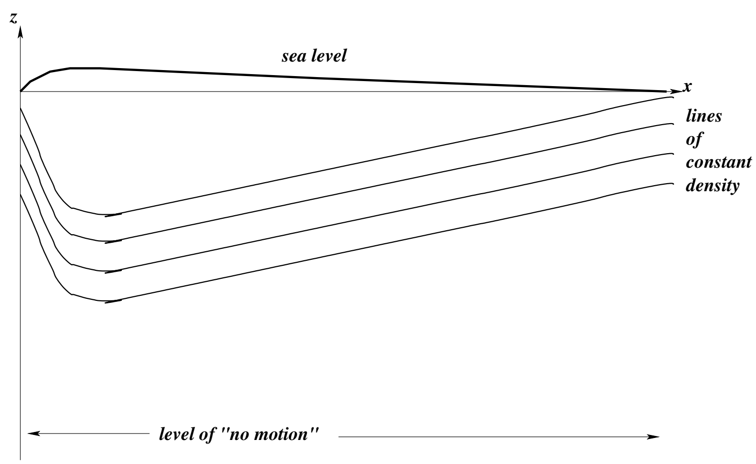

Figure 10.2: Circulation is clockwise around a High.

called the “level of no motion”), then given the thermal wind relations, the horizontal velocities can be estimated by integrating up from the depth where the velocity is known.

So for the section illustrated in Fig.10.1, the v velocity (positive into the page) increases rapidly with z on the left side of the plot, where the density decreases with x, leading to large meridional (to the north) velocities off the coast of Florida. This is of course the Gulf stream. Further off shore, where line of constant density are flat, so that $\partial\rho/\partial x = 0$, the meridional velocity is zero, and further offshore, over the central Atlantic, surface water are weakly toward the south.

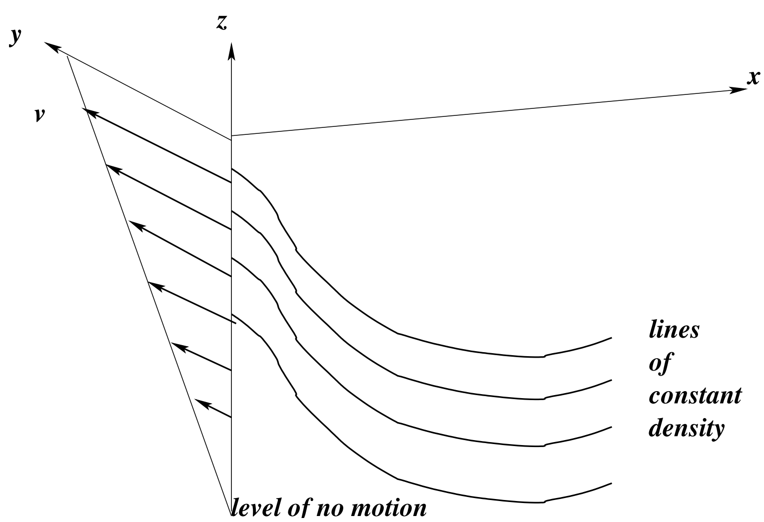

Figure 10.3: In the $x - z$ plane lines of constant density are falling, meaning that at some depth z, the density is decreasing with increasing x, or that $\partial\rho/\partial x < 0$. According to equation 10.8, the v velocity must therefore increase with z ($\partial v/\partial z > 0$) from the level of no motion. The result is a northward currents at the surface.