Optimisation overview

An optimisation problem

$$\text{Min } f(x)$$

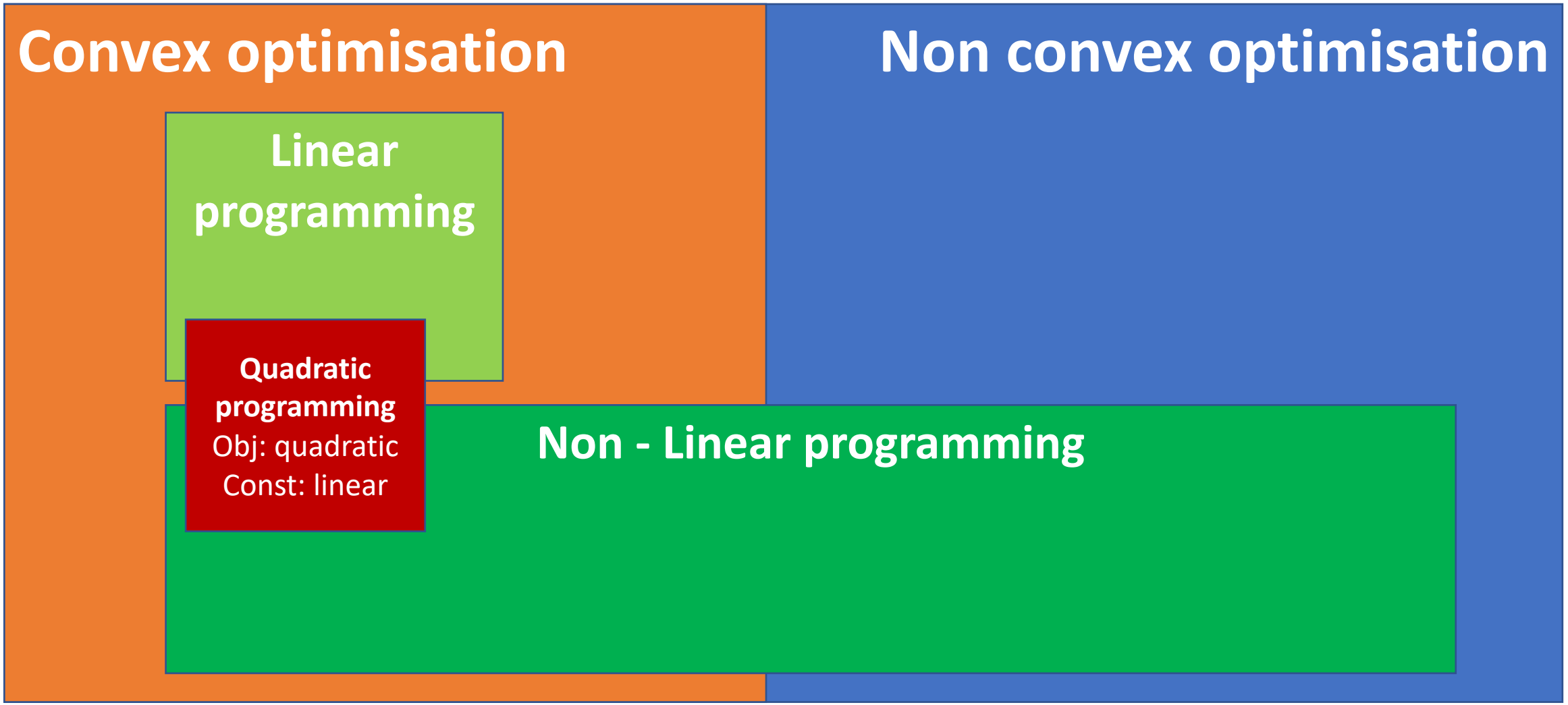
- If $x \in \mathbb{R}^n$ it is an **UNCONSTRAINTED** problem
- If $x \in S \subseteq \mathbb{R}^n$ it is a **CONSTRAINED** problem

Convex optimisation

- A convex programming problem is an optimisation problem with a convex objective function f(x) and a convex feasible region.

- Linear programming is a special case
  - The objective function is linear, hence it’s convex
  - The feasible region is a convex set (intersection of linear inequalities)

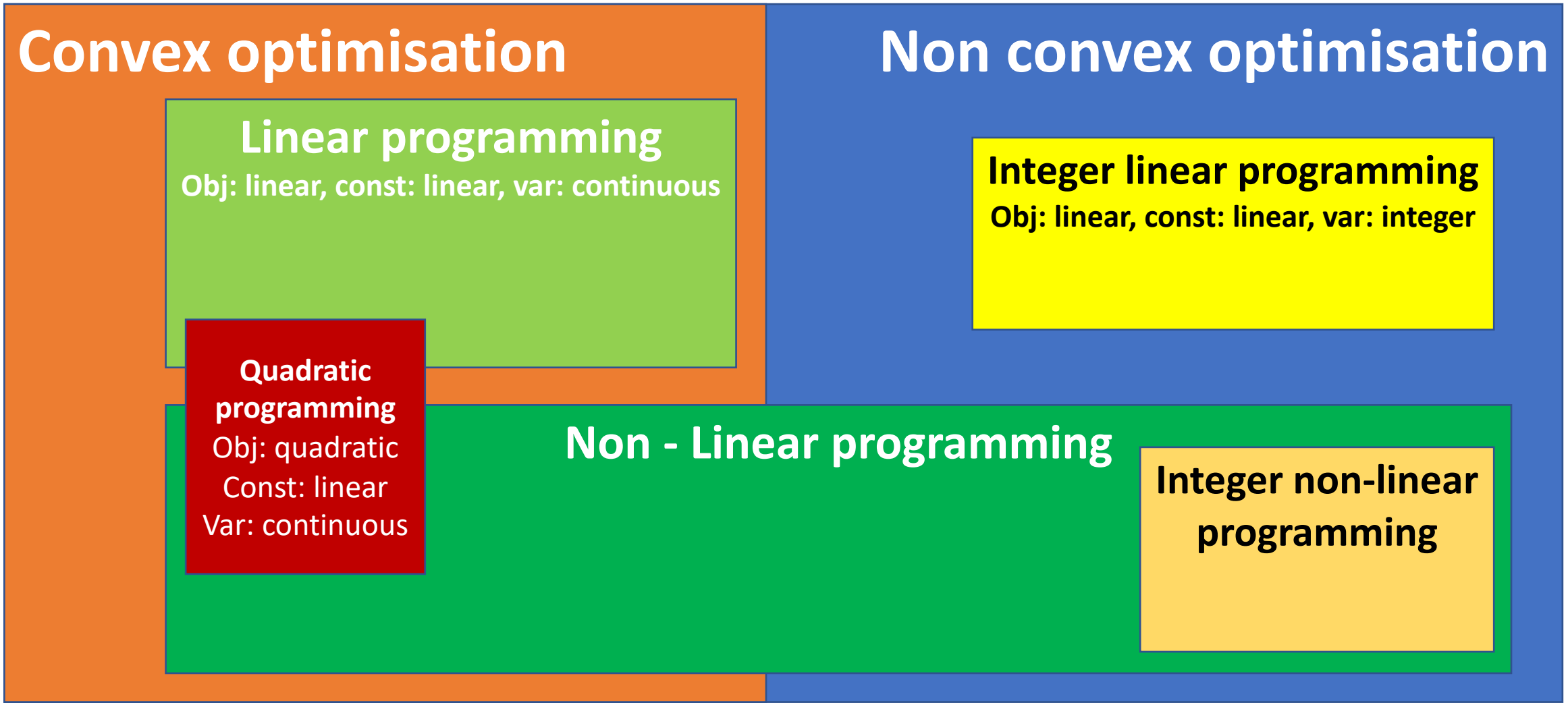
Constrained optimisation



What if variables are not continuous?

- Integer linear programming
  - Feasible region non convex
- Integer non-linear programming
  - Obj: non-linear and/or
  - Const: non linear

Constrained optimisation



You need algorithms to find the optimum

- Exact
  - If time and memory are enough, you always get the optimal solution
  - Simplex for continuous LP, B&B for integer LP
  - Could be slow
  - Mostly for LP
- Approximate
  - You don’t know whether you are at the optimum, but you know the maximum error
  - Could be fast (polynomial) for LP – iterative (NLP; local minima)
- Heuristics
  - You don’t even know which is the error w.r.t. the optimum
  - Could be fast (polynomial) for LP – several approaches (NLP; local minima)

Do we know everything at the onset?

- Deterministic optimisation (no uncertainty)
- Optimisation under uncertainty
  - Stochastic programming
  - Robust optimisation
  - *Optimisation & Machine learning*

"The interplay between optimization and machine learning is one of the most important developments in modern computational science. Optimization formulations and methods are proving to be vital in designing algorithms to extract essential knowledge from huge volumes of data."

(from Optimization for Machine Learning, Edited by Suvrit Sra, Sebastian Nowozin and Stephen J. Wright, 2011)

Is this all? NO!

- Single objective optimisation
- Multi-objective optimisation (2-4 objective functions)
- Many-objective optimisation (> 4 objective functions)

- Multi-objective Stochastic Programming for Mixed Integer Vendor Selection Problem (M. Ekhtiari and S. Poursafary, 2013)

Summary classification

- **Objective function**
  - Single, Multi, Many
  - Convex (linear) , non-convex
- **Feasible region**
  - NO: unconstrained
  - YES: convex (linear equations), non-convex
- **Variables**: continuous, integer (binary)
- **Parameters**: deterministic, uncertain (prob. distribution known, unknown)
- **For each problem, we need a bespoke algorithm**
  - Computational time
  - Quality of the solution: global vs local optimum, approx. vs heuristic