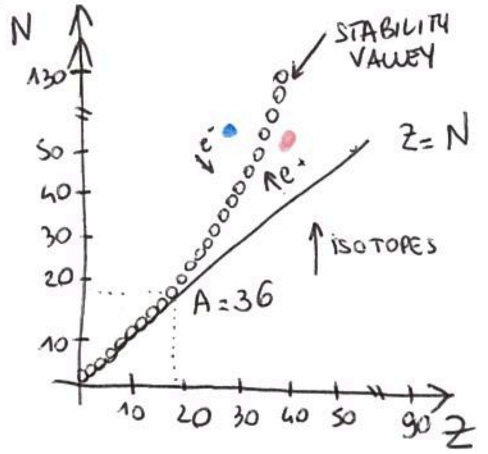
Obiviously if the correction is taken into account the radius can be extrapolated. Other methods to mesure the nucleon radius (obviusly) exist and involves the mesurement of radius using α decay or the mesurement of π-mesic X rays, but this methods are not described here.

NUCLEAR BINDING ENERGY

PAG. 11 NOT INCLUDED

If we look at the stable nuclei table what we can observe is



- Too many protons
- Too many neutrons

$$\left.\begin{array}{l} n \rightarrow p + e^- + \bar{\nu}_e \\ p \rightarrow n + e^+ + \nu_e \end{array}\right] \rightarrow \beta \text{ decays}$$

MAX $\simeq$ ~~[illegible]~~ $N \simeq 130$

$Z \simeq 80$

~~For nuclei which are not stable (i.e. lifetime < of Universe lifetime)~~

The so called <u>lifetime</u> gives us the "degree of unstability". Long life time nucl is more stable or better less unstable than nuclei with shorter lifetime.

If we have a nucleus, ${}^{A}_{Z}X_{N}$, its total ~~energy~~ mass is given by:

$$m\left({}^{A}_{Z}X_{N}\right)c^2 = Z \cdot m_p c^2 + N m_n \cdot c^2 - \text{Binding energy (B.E.)}$$

B.E. is the energy needed to "hold together" the nucleus. In atomic physics the easiest exaple is Hydrogen atom. In H, the atomic BE = 13,6 eV

Let's see how large is atomic B.E. in H.

$$\cancel{m^{atomic}\left({}^{1}_{1}H_{1}\right)c^2 = Z\ m_p c^2 - BE^{ATOMIC} + Z \cdot m_e c^2} \qquad Z = 1$$ Not needed

$$\Rightarrow \begin{cases} BE^{a} = 13,6\ eV \\ m_{N}^{ATOM} = 938\ MeV + 0{,}511\ MeV \end{cases} \qquad \frac{BE}{m} \simeq \cancel{[illegible]} = \frac{10}{10^9} \simeq 10^{-8}$$

, it is rather "small"

In ^{4}He, on the other hand, the NUCLEAR binding energy BE ≃ 28 MeV

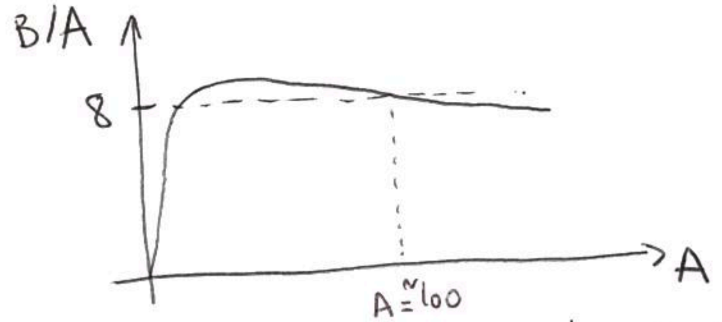
$$BE = 20\,\text{MeV}$$
$$m(^4He) \simeq 4 \cdot 1\,\text{GeV} \Rightarrow \frac{BE}{m} = \frac{20\,\cancel{\text{MeV}}}{4 \cdot 10^3\,\cancel{\text{MeV}}} \simeq 10^{-2}$$

⇒ Nuclear binding energy is 6 order of magnitude larger then atomic binding energy.

So, how could be "immaginated" the B.E.?

In a nucleus there are $\frac{A(A-1)}{2}$ pairs of N-N (nucleons) (pp, np or nn). If we immagine that each pair contributes to BE, then it should scale like A^2 ($BE \propto A^2$) and so that $\frac{BE}{A} \simeq A$.

What has been measured is (How? By precision spettroscopy measurement)



⇒ $B/A \not\propto A$!

⇒ So all pairs do not contribute to BE.

The behaviour of BE(A) can be "inferred" taking into account several "ingredients" (5 in total: 3 related to "classical physics" and 2 related to Quantum mechanics)

$$BE = \underbrace{a_v A}_{\text{VOLUME TERM}} - \underbrace{a_s A^{2/3}}_{\text{SURFACE TERM}} - \underbrace{a_c \frac{Z(Z-1)}{A^{1/3}}}_{\text{COULOMB TERM}} + 2\,\text{Q.M. terms}$$

VOLUME TERM: this term only overestimates the BE; in fact at the surface there are less nucleus that can interact ⇒ We have to subtract a contribution

SURFACE TERM: This is $\propto A^{2/3}$ because $R = R_0 A^{1/3}$, surface goes with R^2 ⇒ $\propto A^{2/3}$
↓
BUT inside nuclei protons can repel each other via Coul...

COULOMB TERM: $\propto$ number of protons. If we have Z protons $\frac{Z(Z-1)}{2}$ pairs can be formed + Coulomb is $\propto \frac{1}{r} \Rightarrow \propto \frac{1}{A^{1/3}}$

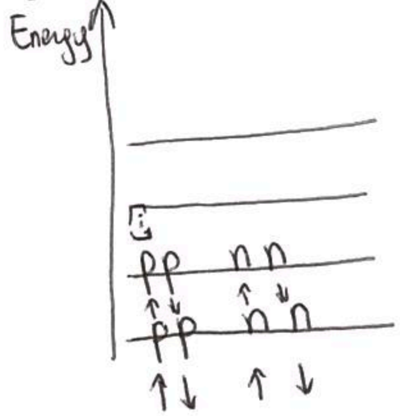
QUANTUM MECHANICS TERMS:

(1) ASYMMETRY

For light nuclei, stability is reached when $Z = \frac{A}{2}$. Heavier nuclei do not show this feature since, when the coulomb term decreases then the symmetry tends to be less "needed".

Why this happens? LIGHT NUCLEI: Strong force 'prefers' $Z=N$ → smaller nuclei → strong > coulomb
HEAVY NUCLEI: Larger → strong "less effective".

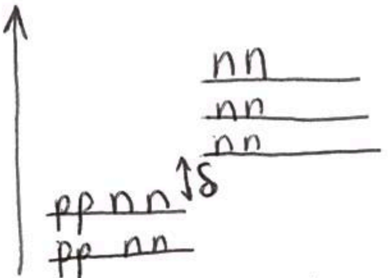
Let's introduce the idea of "shell model" which will be described with more details in the following lessons. Similarly to "atomic physics" we can assume that p and n in the nuclei lies on ≠ Energy levels regulated by Pauli exclusion principle. p and n ~~are~~ FERMIONS $S = \pm\frac{1}{2}$



On each energy level we can have 2p and 2n, not more

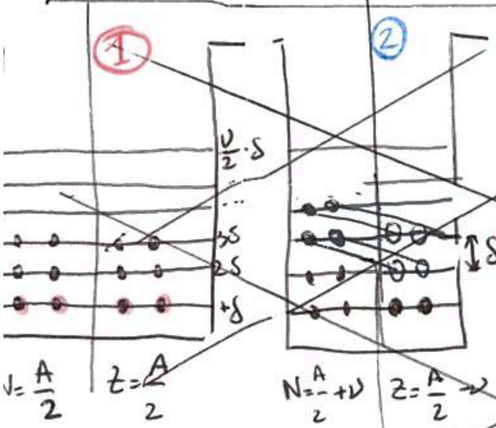
$$A = Z + N \rightarrow Z = N \Rightarrow N = Z = \frac{A}{2}$$

Nuclei with $N = Z = \frac{A}{2}$ will occupy all the energy levels in parallel for p and n. What happens for heavier nuclei where $N \neq Z$ ($N > Z$)



Let's Assume that the Energy gap is the same for each "JUMP" and let's call it δ. The B.E. will be maximum if $Z = N = A/2$

If $Z \neq N$ ν = difference between the number of p and n
⇒ the BE will decrease. How much?



Qui ~~Bla~~ $\nu = 3$
$A = 10$
$N = 8$
$Z = 2$

$$N = \frac{A}{2} + \nu \qquad \left(\frac{A}{2} - \nu\right) p$$
$$Z = \frac{A}{2} - \nu \qquad \left(\frac{A}{2} + \nu\right) n \qquad N - Z = \frac{\nu}{2}$$

If we have $N = Z$ then the Energy can be written as:

$$E_1 = 4\delta + 4\cdot 2\delta + 4\cdot 3\delta + \dots + 4\cdot\frac{\nu}{2}\delta \text{ Terms} = 4\delta\,\frac{\frac{\nu}{2}\left(\frac{\nu}{2}-1\right)}{2}$$

N° of nucleons in each energy level

$$E_2 = \cancel{\ } + 2\cdot\delta' + 2\cdot 2\delta' + \dots + \nu \text{ terms} = 2\delta\,\frac{\nu(\nu+1)}{2}$$

δ' is for "extra neutrons"

The "pairing" term is related to fact that nuclei prefer to be formed (into pairs in order to lower the energy (This is related to the angular momentum; in fact if we have 2 p they should have angular momentum J, J_1 and J_2 which will have to sum up and goes to $J=0$)

It is possible to define (and measure) a Neutron/proton separation energy (S_n, S_p). $S_{n,p}$ is defined as the energy needed to take a particle out of the nucleus, and so the separation energy becomes equal to the energy with which a particular particle is bound in the nucleu

The B.E. can be expressed as:

$$BE(A,Z) = Z M_p c^2 + N M_n c^2 - M'({}^A_Z X_N)c^2$$

The B.E. with wich a cluster ${}^{A'}_{Z'}Y_{N'}$ is bound in the nucleus ${}^A_Z X_N$ in general with $A' \ll A$, $Z' \ll Z$, $N' \ll N$ becomes ($A'' = A - A'$; $Z'' = Z - Z'$, $N'' = N$

$$S_y = M'\left({}^{A'}_{Z'}Y_{N'}\right)c^2 + M'\left({}^{A''}_{Z''}U_{N''}\right)c^2 - M'\left({}^A_Z X_N\right)c^2$$

USING THE B.E. of X, Y, U nuclei

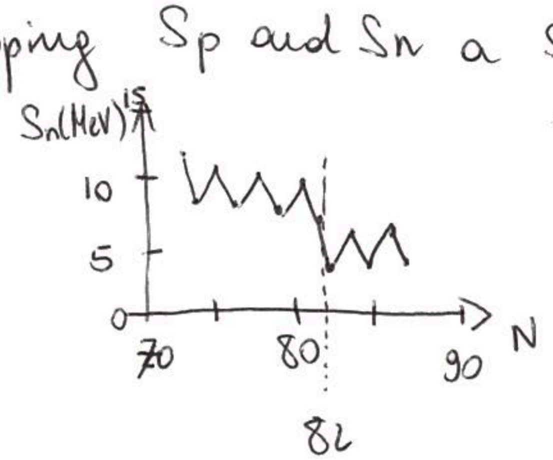
$$S_y = BE(A,Z) - \left[BE(A',Z') + B.E.(A'',Z'')\right]$$

For a proton and neutron this become

$$S_p = BE(A,Z) - BE(A-1, Z-1)$$

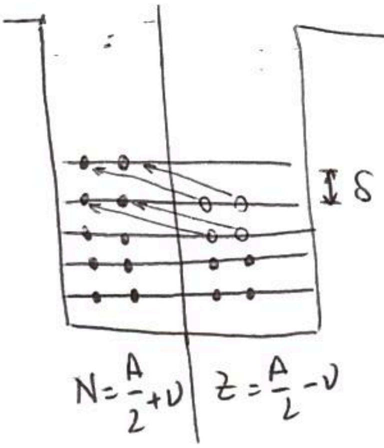
$$S_n = BE(A,Z) - BE(A-1, Z)$$

Mapping Sp and Sn a saw-tooth figure results:



→ Separation energy (Single neutron) S_n for even-even and even-odd Ce ($Z=58$). A part from a sudden lowering at $N=82$ a odd-even staggering is experimentally observed. I.e. e.e are more "bound" then e.o.

An extra δ term has to be added to the BE formula



Due to Pauli exclusion principle, if we start from

$N=\left(\frac{A}{2}\right)+\nu$ and $Z\left(\frac{A}{2}\right)-\nu$ we have to occupy only free "orbitals" If the averaged energy beetwen orbitals is δ then replacing ~~p~~ with n will have a cost.

The "cost" is $\Delta E_{BINDING} = \nu\left(\delta \cdot \frac{\nu}{2}\right)$

~~ASYMMETRY~~ ASymmetry in # GAP only 2 n $\forall$ orbitals

If $\nu = \frac{(N-Z)}{2}$ $\qquad \Delta E = \frac{N-Z}{2}\left(\delta \cdot \frac{N-Z}{4}\right) = \frac{1}{8}\delta(N-Z)^2$

The potential depth which describe nuclear well do not change much going from ^{16}O up to ^{208}Pb ($\simeq 10\%$) so the δ has to vary $\propto$ to A

$$\delta \propto A^{-1}$$

So, the binding energy become

$$BE = a_{clonic} - \underbrace{a_{asym}\frac{(N-Z)^2}{A}}_{\text{Related to}} + \delta$$

Related to
- Pauli Principle
- Discrete energy level

PAIRING ENERGY TERM

δ is the pairing energy term and is related to the pairing of p and n.

$$\delta = \begin{cases} 0 & \text{odd-A nucleons} \\ +\frac{a_p}{A^{3/4}} & \text{even-even nuclei} \\ -\frac{a_p}{A^{3/4}} & \text{odd-odd nuclei} \end{cases}$$

EASY EXP.

O<->O have BE > ●
p p
(E.E)

E.O

O O have BE < Becuse they are both n or p

E.O => NO CONTRIBUTION

$$\Delta E_{pair} = \begin{cases} +\delta & \text{even-even} \\ 0 & \text{even-odd} \\ -\delta & \text{odd-odd} \end{cases}$$

↑ increase the B.E.

↓ decrease the B.E.

How many nuclei are even-even, even-odd or odd-odd?
(Z = even, odd; N = even, odd)

e.e ≃ 165
e.o ≃ 105
o.o = 4 $\quad {}^{2}_{1}H_{1} \quad {}^{6}_{3}Li_{3} \quad {}^{10}_{5}B_{5} \quad {}^{14}_{7}N_{7}$ (Nitrogen/Azoto)

$$B.E. = a_v \cdot A - a_s A^{1/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{asym} \frac{(N-Z)^2}{A} + \delta \qquad \delta = \begin{cases} +\frac{a_p}{A^{3/4}} \\ 0 \\ -\frac{a_p}{A^{3/4}} \end{cases}$$

If we measure BE, a_v, a_s, a_c, a_{sym} and δ are <u>FIXED</u> from experi[ment]
(Via precise mass measurements) * $a_v = 15{,}8$ MeV
$a_s = 18{,}3$ MeV
$a_c = 0{,}714$ MeV
$a_{sym} = 23{,}3$ MeV
$\delta = \pm 33{,}5$ MeV

EXAMPLE:
Determine the B.E. of ${}^{4}_{2}He_{2}$ starting from the following data:

$m({}^{1}_{1}H) = 1{,}007825\ u$

$m_n = 1{,}008665\ u$

$m({}^{4}_{2}He) = 4{,}00260\ u$

u = atomic mass $\quad 1u = \frac{m({}^{12}_{6}C)}{12}$

$1u = 1{,}6605402 \cdot 10^{-27}$ kg

$1u(c^2) = kg \cdot \frac{m^2}{s^2} = J$ or $eV = 931{,}478$ MeV

$1u = 931{,}478\ MeV/c^2$

$$B.E. = \left[Z \cdot m({}^{1}_{1}H) + N \cdot m_n\right] c^2 - m({}^{4}_{2}He_2) c^2$$

$$= 2 \cdot 1{,}007825\ u \cdot c^2 + 2 \cdot 1{,}008665\ u \cdot c^2 - 4{,}00260\ u c^2$$

$$= u \cdot c^2 (0{,}03038) = 931{,}478\ MeV \cdot 0{,}03038 = 28{,}3\ MeV$$

NUCLEAR STABILITY : Why are there only "few" stable nuclei? Because E_{MIN} is always favored? (1)

From the Semi-empirical mass formula it is possible to obtain a "mass equation" that will be usefull to understand nuclear stability.

For doing so we start from mass equation:

$$m\left({}^{A}_{Z}X_{N}\right)c^2 = \left[Z \cdot m\left({}^{1}_{1}H\right) + N \cdot m_n\right]c^2 - B.E. =$$

$$= \left[Z \cdot m\left({}^{1}_{1}H\right) + \underset{A-Z}{\underset{\parallel}{N}} m_n\right]c^2 - a_V \cdot A + a_S A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + a_{sym} \frac{(A-2Z)^2}{A} - \delta \qquad (N-Z)^2$$

For a given $\boxed{A}$ which is the lowest mass (i.e. largest B.E)?

We have to differentiate vs Z and asign 0 to the derivative:

$$\frac{d(mc^2)}{dZ} = 0 \qquad \boxed{\text{A IS FIXEX}} \text{ i.e. constant}$$

$$\frac{d(mc^2)}{dZ} = m\left({}^{1}_{1}H\right)c^2 - m_n c^2 + \frac{a_c}{A^{1/3}}(2Z-1) + \frac{a_{sym}}{A} 2(A-2Z)(-2) = 0$$

$$m\left({}^{1}_{1}H\right)c^2 - m_n c^2 + \frac{2a_c Z}{A^{1/3}} - \frac{a_c}{A^{1/3}} - a_{SYM} \cdot 4 + \frac{8 a_{sym} Z}{A} = 0$$

$$Z\left[\frac{2a_c}{A^{1/3}} + \frac{8a_{sym}}{A}\right] = m_n c^2 - m\left({}^{1}_{1}H\right)c^2 + \frac{a_c}{A^{1/3}} + 4a_{sym}$$

$$Z = \frac{\overbrace{\cancel{\left[m_n - m\left({}^{1}_{1}H\right)\right]c^2}}^{0{,}8\ \text{MeV}} + \overbrace{\cancel{\frac{a_c}{A^{1/3}}}}^{\frac{0{,}72}{A^{1/3}} < 0{,}72\ \text{MeV}} + \overbrace{4a_{sym}}^{\to 4 \cdot 23\ \text{MeV}}}{\underbrace{\frac{2a_c}{A^{1/3}}}_{\lesssim 0{,}72\ \text{MeV}} + \underbrace{\frac{8a_{sym}}{A}}_{\to \frac{8 \cdot 23\ \text{MeV}}{100}}} \qquad \text{If } A = 100 \to 0{,}23\ \text{MeV}$$

knowing that:

$a_V = 15{,}5$ MeV

$a_S = 16{,}8$ MeV

$a_c = 0{,}72$ MeV

$a_{sym} = 23$ MeV

$a_P = 34$ MeV

$$Z \cong \frac{4a_{sym}}{\frac{2a_c}{A^{1/3}} + \frac{8a_{sym}}{A}} = \frac{4 \cdot A \cdot a_{sym}}{2a_c A^{2/3} + 8a_{sym}} = \frac{4A}{\frac{2a_c A^{2/3}}{a_{sym}} + 8}$$

$$Z_0 = \frac{A}{2}\left(\frac{1}{1+\frac{1}{4}\frac{a_c}{a_{sym}}A^{2/3}}\right) = \frac{A}{2}\left[\frac{1}{1+\frac{1}{4}\frac{0{,}72}{4\cdot 23}A^{2/3}}\right] = \frac{A}{2}\left[\frac{1}{1+0{,}0078}\right.$$

→ SMALL A ⇒ $Z = \frac{A}{2}$!
LARGE A ⇒ $Z < N$

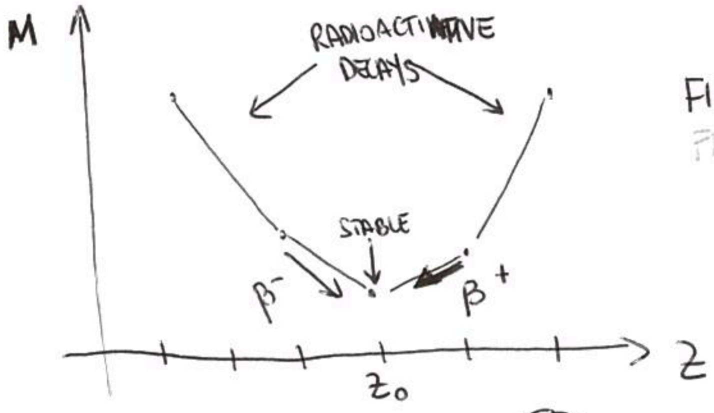
Now, since the mass can be expressed as

$$m(^A_Z X_N)c^2 = [Z(m(^1_1H)) + N m_N]c^2 - a_V A + a_S A^{2/3} + a_C\frac{Z(Z-1)}{A^{1/3}} + a_{sym}\frac{(A-2Z)^2}{A} + \delta$$

$m(A,Z) \propto Z, Z^2 \to$ pol 2

$$m(A,Z) = a + bZ + cZ^2 \qquad \frac{dM}{dZ} = 0 \qquad b + 2cZ = 0$$

$$Z_0 = -\frac{b}{2c} = \frac{A/2}{1+0{,}0078A^{2/3}}$$



FIXED A : ISOBAR
FIXED Z : ISOTOPES

This is valid for even-odd nuclei.

$p \to n + e^+ + \nu_e$ (β^+) $m_n > m_p \to$ Impossible in "free space" but allowed in the nucleus since we have to look at the total mass

$n \to p + e^- + \nu_e$ (β^-)

But also $p + e^- \to n + \nu_e$ process (Electron capture (*)) is possible

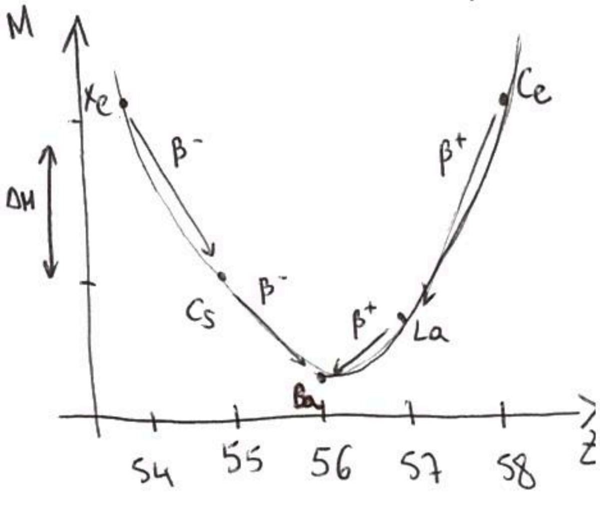
Example: $A = 137$

Lantanium (57→71) (lantanides serie); Cerio

Z =	54	55	56	57	58
	Xe	Cs	Ba	La	Ce
Lifetime	3,82m	30y	STABLE	60ky	9,8h

less unstable

More unstable



Lifetime is inversely proportional to ΔM.

(*) Electron capture $n + e^- \to p + \nu_e$ is more likely to happen in $\boxed{1S}$ because S-electron are inside the nucleus for most part of the time!