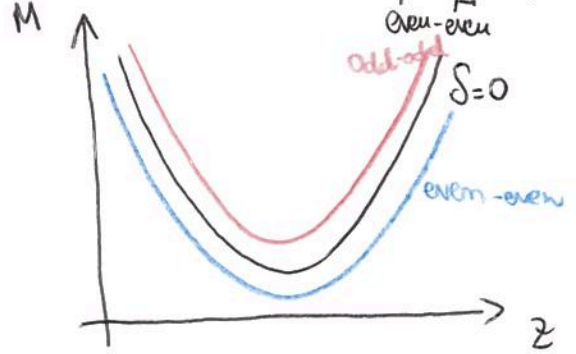
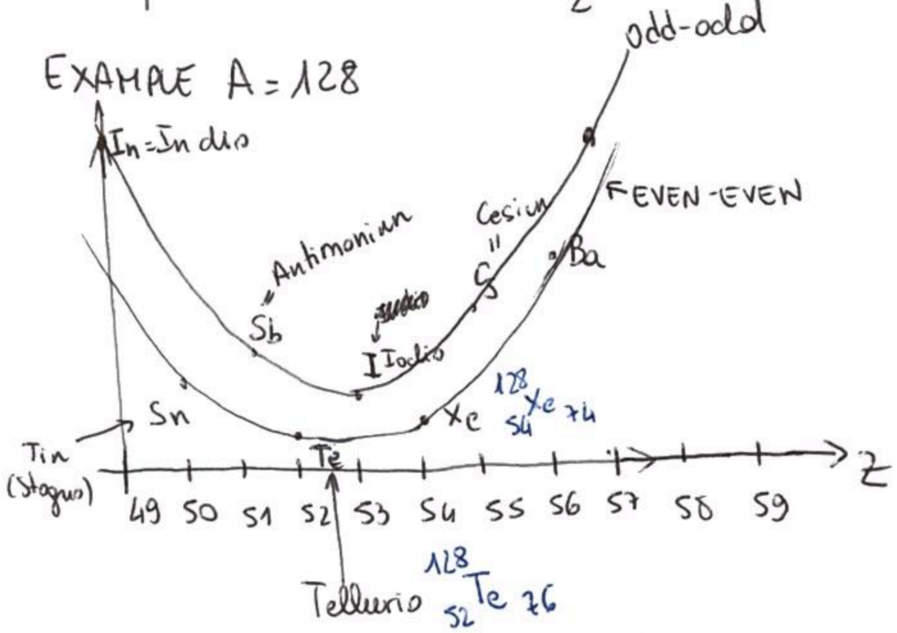
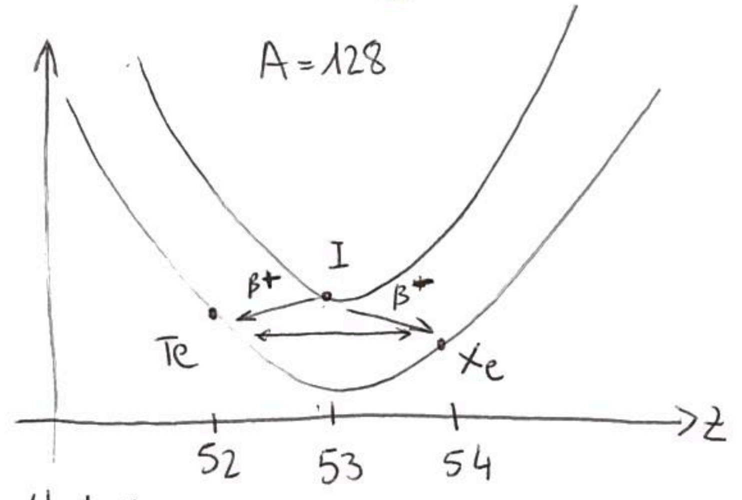
for even-even nuclei δ have to be taken into account. Since it does not depends on Z $\left(\delta = \underset{\text{even-even}}{\overset{\text{odd-odd}}{\pm}} \frac{a_p}{A^{3/4}}\right)$ it will acts as an additional term



EXAMPLE A = 128



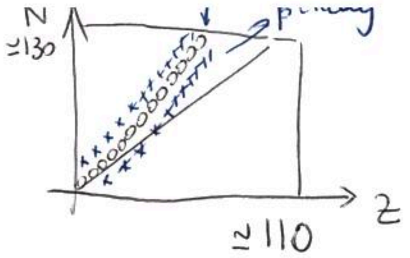


In the isobar A = 128 there are two stable isotopes Because going from Te → Xe you would need 2 β decays ⇒ very unlikely! $\Delta Z = 2$

Note that there are also cases where 3 "stable" nuclei can occur

Examples: A = 140 → 1 stable nucleus
A = 128 → 2 stable nuclei Te → Xe 2β decay
A = 130 → 3 stable nuclei Te, Xe, Ba

⇒ Semi-empirical mass formula is not perfect, but it gives us also some other insign of nuclear properties. For example, if we look at the distribution of nuclei as a function of Z and N we can understand why there are only a number of stable nuclei.



There seems that there re some sort of "limits" on the maximum number of Z and N. The semi-empirical mass formula gives us the range of A and Z which can gives us α decay.

For a spontaneous α-decay ($S_\alpha = 0$) we have:

$$BE\left({}^{A}_{Z}X_N\right) - \left[BE\left({}^{A-4}_{Z-2}Y_{N-2}\right) + BE\left({}^{4}_{2}He_2\right)\right] = 0$$

The limit to the region for spontaneous ~~α decay~~ α-emitters can be obtained from the B.E. equation. In general lifetime for emission become very short in the actinide region => Stable nuclei occour ~~f~~ $A \simeq 2$

$$m\left({}^{A}_{Z}X\right) \rightarrow m\left({}^{A-4}_{Z-2}Y\right) + m\left({}^{4}_{2}He\right)$$

A B C

If $m(A) < m(B) + m(C)$ => A is stable under α-decay! on the other hand if $m(A) > m(B) + m(C)$ -> A is unstable. Larger the mass, lower the lifetime => Heavy nuclei are instable under α decay.

Another thing that can be understand from the semi-empirical mass formula are the "DRIP-LINE". The condition $S_n = 0$ ($S_p = 0$) indicates the borderline where neutron (proton) is no longer bound in the nucleus.

Finally we can also understand something about spontaneous fission. (*) NEXT PAGE

The energy released in nuclear fission, in the "easy" case of fission into two elements of equal Z and A $\left({}^{A}_{Z}X \rightarrow 2\,{}^{A/2}_{Z/2}Y\right)$ is

$$E_{fission} = M\left({}^{A}_{Z}X_N\right)c^2 - 2M'\left({}^{A/2}_{Z/2}Y_{N/2}\right)c^2$$

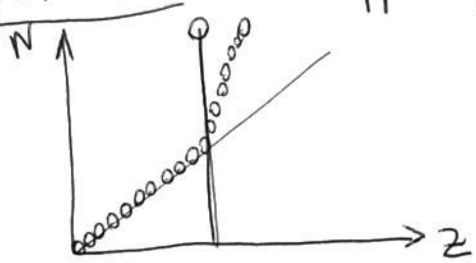
Using a simplified mass equation, replacing $Z(Z-1) \rightarrow Z^2$ and neglecting the pairing term

$$E_{fission} = \left[a_S A^{2/3}\left(1 - 2^{1/3}\right) + a_C Z^2 A^{-1/3}\left(1 - 2^{-2/3}\right)\right]c^2$$

$$= \left[-5{,}12\, A^{2/3} + 0{,}28\, Z^2 A^{-1/3}\right]c^2$$

is value becomes positive for $A \gtrsim 90$ and reaches a value of

⊛ $\boxed{\text{Drip - lines}}$: let's suppose that we fix Z and we force the N to be "outside" the β valley. What will happen is that the nucleus won't be stable and it will decay emitting a n



$$\underbrace{{}^{A}_{Z}X}_{m_i} \longrightarrow \underbrace{{}^{A+1}_{Z}X + n}_{M_f}$$

If $M_i > M_f \Rightarrow$ the Neutron emission is favored w.r.t. "β" and we don't see the ucleus in the table. The same is valid also if we fix N and we ush protons: the nucleus so created will prefer to emit a p, instead f undergo into β^+ decay. The limit will be reach when the reaction

$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + p$$

will be energetically favored.

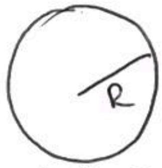
For the fission products, neutron rich nuclei are obtained which will decay by the emission of $n \to p + e^- + \bar{\nu}_e$, so will be a source of $\bar{\nu}_e$.

APPLICATION OF SEMI EMPIRICAL MASS FORMULA: NEUTRON STARS

What are neutron stars? As a first approximation the neutron stars can be considered as a compact object of neutrons (What there is inside we do not consider now). This pose a problem, in fact we know that few neutron cannot "stay close" and form a bound state.
So why are the neutron stars bound? without any protons S_n would be too large ... BUT neutron stars are object with a radius of $\simeq 20$ km not 4 fm $\Rightarrow$ GRAVITATION plays a role!

$m \simeq 2 m_\odot$
$r \simeq 10\text{-}20$ km

Can the semi-empirical mass formula be applied for "gravitational" object?
Let's see: If we have an object of mass ~~M~~ and radius R the density ρ ~~is~~ can be evaluated



M, R, ρ

$$\rho = \frac{M}{\frac{4}{3}\pi R^3}$$

If we want to evaluate how large is the contribution for gravitat
We start from a sphere of radius r and we ask ourself how much work is needed to go from r to $r + dr$

$$\rho = \frac{M}{\frac{4}{3}\pi r^3}$$

$$dm = \left(\frac{4}{3}\pi r^2 dr\right)\rho \qquad m = \frac{4}{3}\pi r^3 \rho$$

$$V = \frac{Gm}{r} = G\frac{4}{3}\pi r \rho \quad = \text{gravitational potential}$$

$$\text{gravitational potential} \to V = \frac{U}{m} = \frac{\text{gravitational potential energy}}{m}$$

$$V = \frac{U}{m}$$
$$U = G\frac{mM}{r}$$
$$V = \frac{GM}{r}$$

$$\Rightarrow dU = V \cdot dm = \left(G \cdot \frac{4}{3}\pi r^2 \rho\right)\left(4\pi r^2 dr \rho\right)$$

$$= G\frac{(4\pi)^2}{3}\rho^2 r^4 dr = G\frac{(4\pi)^2}{3}\left[\frac{M^2}{\left(\frac{4}{3}\pi R^3\right)^2}\right] r^4 dr$$

$$U = \int_0^R du = \int_0^R 3G \frac{M^2}{R^6} r^4 dr = \frac{3GM^2}{R^6} \frac{R^5}{5}\Big|_0^R = \frac{3GM^2}{5R}$$ ← This acts as gravitational "binding energy"

And that term has to be added to the B.E.

Assuming that the B.E. formula can be extended we have:

$$B.E. = a_v A - \cancel{a_s A^{2/3}} - a_c \cancel{\frac{Z(Z-1)}{A^{1/3}}} - a_{sym} \frac{(N-Z)^2}{A} + \cancel{\delta} \oplus \frac{3GM^2}{5R}$$

In a (neutron star : i.e. "large neutral object :

- surface term : When the volume increases, also the surface increases, here the volume increases much more than the surface. The surface has a smaller role than the volume => we can neclect $a_s A^{2/3}$ terms.
- In a neutron star (in first approximation) there are no protons => coulomb term can be neclected, too
- $Z = 0$ => The symmetry terms become: $a_{sym} \cdot A$
- Pairing term can be neclected since ~~its~~ there are a lot of neutrons
- Coulomb become : $\frac{3G \cdot m_n^2 A^2}{5 \quad R_0 A^{1/3}}$ with $R_0 = 1{,}2\,fm$ (from ~~neutron~~ radius on Earth)

NOTE that this is a rought estimation

$$B.E. = a_v \cdot A - a_{sym} \cdot A + \frac{3}{5} G \frac{m_n^2}{R_0} A^{5/3}$$

To be bound, ~~the~~ the neution stron must have a $BE \geq 0$ =>

$$(a_v - a_{sym}) A + \frac{3}{5} \frac{G}{R_0} m_n^2 A^{5/3} \geq 0$$

$$(a_v - a_{sym}) + \frac{3}{5} \frac{G}{R_0} m_n^2 A^{2/3} \geq 0$$

$$(15{,}5 - 23)\,MeV + \frac{3}{5} \frac{G}{R_0} m_n^2 A^{2/3} \geq 0$$

This is negative: without gravitation you cannot bound any neutron

$$\frac{3}{5}\frac{G}{R_0} M_n^2 A^{2/3} \geq 7{,}5 \text{ MeV}$$

$F = G\frac{M_1 M_2}{r^2}$ $G = \frac{N}{kg^2} m^2$

$$A^{2/3} \geq \frac{7.5\,\text{MeV}\cdot 5\cdot R_0}{3G\cdot m_n^2} = \frac{7{,}5\cdot 10^6\cdot 1{,}6\cdot 10^{-19}\cdot 5\cdot 1{,}2\cdot 10^{-15}\ J\cdot m}{3\cdot 6{,}67\cdot 10^{-11}\cdot (1{,}67\cdot 10^{-27})^2\ \frac{Nm^2}{kg^2}\,kg^2} =$$

$N\cdot m^2$

$$= \frac{7{,}5\times \overset{10^2}{1{,}6}\times 5\cdot 10^{-28}}{3\times 6{,}67\times 1{,}67\times 1{,}67\times 10^{-65}} \cong 10^{37}$$

$$A \gtrsim (10^{37})^{3/2} = 10^{37}\times 10^{19} = \boxed{10^{56}}$$

You cannot have a nucleas of 100 neutrons... but you can have nucleus of 10^{56} neutrons

$$M = 10^{56}\times 1{,}67\cdot 10^{-27}\ kg = 1{,}67\cdot 10^{29}\ kg$$

$$M_\odot \cong 10^{30}\ kg$$

SAME ORDER OF MAGNITUDE of The OBSERVED NEUTRON STARS

The semi empirical mass formula can be extended to very large objects with large mass. calculations are not wrong, ~~though~~ thought not perfect. The order of magnitude are match.

EXERCISE ON BINDING ENERGY

Among the $A = 197$ isobars, the nucleus ${}^{197}_{79}Au$ is stable. Which are the expected radioactive decay types for ${}^{197}_{78}Pt$ and ${}^{197}_{80}Hg$ to ${}^{197}_{79}Au$?

$A = 197$ is odd so there is only 1 stable nucleus. The stable nucleus has $Z_s = 79$, the nucleus with $Z = Z_s - 1 = 78$ can transmutate to it via β^- decay whereas the nucleus $Z = Z_s + 1 = 80$ can do it via β^+ decay or electron capture.

$$M(197, Z) \doteq Z m_p + (197 - Z) m_n - B(197, Z)/c^2 + Z m_e$$

$$M(197, Z)\, c^2 = \text{const} + Z (m_p - m_n + m_e) c^2 + a_C \frac{Z^2}{197^{1/3}} + a_{ASY} \frac{(197 - 2Z)}{197}$$

$$\simeq \text{const} - 0{,}782\, Z + 0{,}697 \frac{Z^2}{5{,}82} + 23{,}3 \frac{(197 - 2Z)^2}{197}$$

For β^- transition: ${}^{197}_{78}Pt$ where $M(197, 78) - M(197, 79) \simeq 0{,}90$ MeV

$\Rightarrow$ it is allowed (β^- is allowed is ΔM is > 0 because $m_n > m_p$)

β^+ is allowed if $M(197, 80) - M(197, 79) > 2 m_e$

EC " if $\Delta M > 0$

In this case we have $\Delta M \cong 0{,}3$ MeV/c^2 $\Rightarrow$ Only EC is allowed

$$\beta^- : \quad {}^{197}_{78}Pt \rightarrow {}^{197}_{79}Au + e^- + \bar{\nu}_e$$

$$EC : \quad e^- + {}^{197}_{80}Hg \rightarrow {}^{197}_{79}Au + \nu_e$$