PROPERTIES OF NUCLEAR FORCE

1. At short distance it is stronger than Coulomb ⇒ it overcome the Coulomb repulsion of p
2. At long distance (i.e. atom size) is neglibgible, in fact molecule interactions are based on Coulomb only
3. Some particle are immune from nuclear force (e^- do not fell the nuclear force at all)

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4. N-N force is independent of wheater the N are p or n (⇒ charge independence)
5. N-N force depends on the spin of nucleons
6. N-N interaction include a repulsive term, which keep the nucleons at a certain distance
7. NN force has a non-central component which do no conserve the orbital angular momentum

THE FORCE BEETWEEN NUCLEONS

Nuclei have structure, we know that. How can we reveal this structure? More or less in the same way as Rutherford scattering, but with shorter wave lenght λ. To have a "short" probe, it must have an high energy. For example an e^- can be used: A 1 GeV electron has a $\lambda \leq 10^{-15}$ fm and with such probe we can look inside the nucleon.

Inside the nucleons we have QUARK

$$\begin{matrix} u & c & t & \frac{2}{3}e \\ d & s & b & -\frac{1}{3}e \end{matrix} \quad + \overline{q}$$

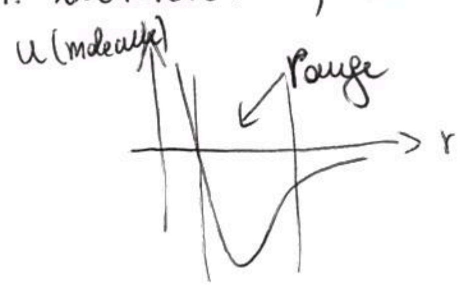
p is composed by 3 quark: $u\,u\,d$ with total e-charge $\left(\frac{2}{3}+\frac{2}{3}-\frac{1}{3}\right)e=$

n, similarly is composed by: $(u\,d\,d)$ wit total charge $\left(\frac{2}{3}-\frac{1}{3}-\frac{1}{3}\right)e$ so it's total charge is 0, but it is composed by "charged" components.

We know also that the force among quark is STRONG and the strong force is carried by gluons; additionally we know that the strong charge is called color and that the Σ color $=0$ in "compound" object.

$\Rightarrow$ Nucleons (p, n) are "color-charge" neutral but they fell the STRONG force.

This is somehow similar to molecules: they can interact through e.m. interaction, ~~but~~ only if they are close.



Nucleons can interact only if they are "close" i.e. only if the sepration is $\leq$ the size of the force lenght i.e. the "residual" color interaction

The nucleon-nucleon force originates from quark.

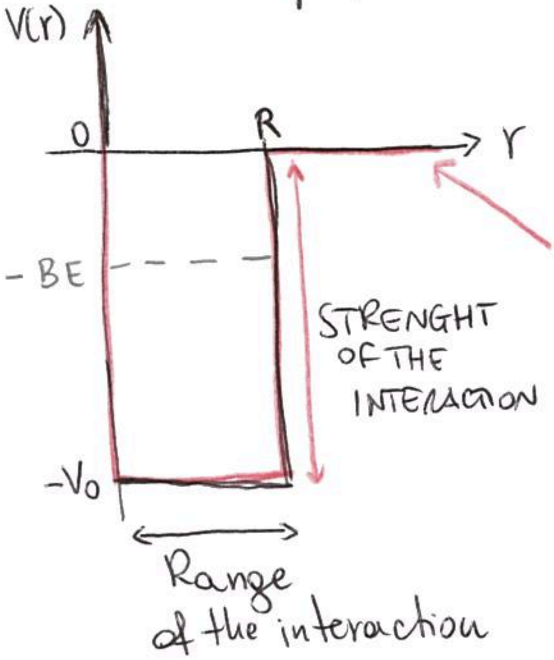
To study nuclear force we will start from the "simplest" case, i.e. the lightest bound system which exsist in nature: the DEUTERON.

Deuteron ${}^2_1H_1$ is a odd-odd nucleus formed by 1 p and 1 n with a Binding Energy BE = 2,225 MeV (*). This is weakly a Bound System $\frac{B}{A} \simeq 1$ MeV while "usually" the BE/A is $\simeq$ 7-8 MeV M_d = 1875 MeV

⊛ E_B is obtained using radiative capture of neutron by hydrgen.
In the reaction $H(n,\gamma)d$ a slow neutron is captured by Hydrogen atom followed by the emission of a γ-ray.
If the energy of the incident neutron is negligible, the energy of the γ-ray emitted gives the binding energy of the deuteron.
~~Usually~~ Since in much easier to determine γ-ray energies than the masses, B_E are better ~~[illegible]~~ known than absolute masses

The B.E can be studied to understand the property of the deuteron.

To simplify the analysis let's assume a square well potential. Athought this is oversimplified it can be used to make some analyses



How depth is the potential?
For sure we must have $V_0 > BE$, because a nucleous have to be formed

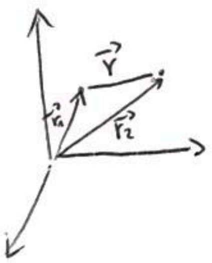
Deuteron has no exited state → i.e. only the ground state exists.

3-dim square well potential: this is a CENTRAL POTENTIAL i.e.

$$V(\vec{r}) = V(r)$$ no dependence on θ and φ

r is the relative distance/sepration beetween p and n.

We will work on the relative position vector coordinates:



$\vec{r} = \vec{r}_1 - \vec{r}_2$ RELATIVE separation vector; the origin is the first particle.

Note that this is a non-on shell frame (ptc 1 is accelerated) ⇒ correction are needed; BUT you can use the reduced mass m of the two ptc system to be able to work without any correction.

$$m = \frac{M_1 \cdot M_2}{M_1 + M_2}$$

The time idependent Schrödinger equation is:

$$H \psi(\vec{r}) = E \psi(\vec{r})$$

Nabla $= \sum_i \frac{\partial}{\partial x_i}$

$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r}) \right] \psi(\vec{r}) = E \psi(\vec{r})$$

In a spherical system:

$$\psi(\vec{r}) = \phi(r)\, Y_e^{m_e}(\theta, \varphi)$$

$\phi(r)$ depends on r not $\vec{r}$

$$\phi(r) = \frac{u(r)}{r}$$

$Y_e^{m_e}(\theta, \varphi)$ = Spherical harmonics

$l = 0, 1, 2, \dots$

$m_e = 0, \pm 1, \pm 2, \dots$

(have standard expression)

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \left[V(r) + \frac{l(l+1)\hbar^2}{2mr^2}\right]u(r) = E\,u(r)$$

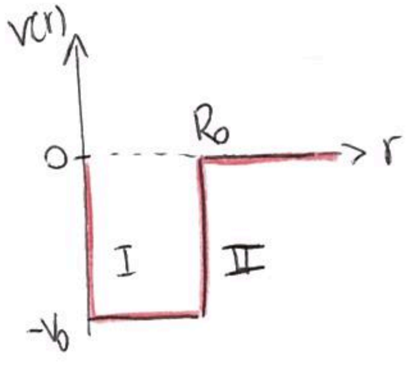
$\Rightarrow$ Only 1 variable equation

Deuteron has only one energy level, which must be the one with lower energy i.e. $l=0$. Note that this is true only because we assumed a central potential.

$l=0$, $m=0$ $\quad Y_0^0 = \sqrt{\frac{1}{4\pi}}$

the Schrodinger equation become:

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + V(r)\,u(r) = E\,u(r)$$



Boundary conditions:

- ψ must be continuos everywhere
- ψ " finite everywhere (i.e. cannot go to ∞ anywhere)
- ψ " square integrable; i.e. $\int |\psi^2| d\tau$ = FINITE
- $\frac{d\psi}{dr}$ " continuos (i.e. NO CUSP)

I) $r < R_0$ $\quad V = -V_0 \quad \Rightarrow \quad -\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} - V_0 u = E u$

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} = (V_0 + E)u \qquad \frac{d^2u}{dr^2} = -\frac{2m}{\hbar^2}(V_0+E$$

$$\frac{d^2u}{dr^2} = -k^2u \qquad k = \sqrt{\frac{2m}{\hbar^2}(V_0+E)}$$

V_0: POS, E: NEG == B.E

k is a real quantity

=> HARMONIC OSCILLATOR

$$u_I(r) = A \cdot \sin k \cdot r + B \cdot \cos k \cdot r$$

at $r=0$ $\quad u_I(0) = 0 \Rightarrow B=0 \qquad u_I(r) = A \sin kr$

(II) $r > R_0 \quad V=0$

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} = Eu$$

$$\frac{d^2u}{dr^2} = -\frac{2mE}{\hbar^2}u = \gamma^2 u \qquad \gamma = \sqrt{-\frac{2mE}{\hbar^2}}$$

$E<0 \Rightarrow$ γ is real

=> Exponential

$$u_{II}(r) = Ce^{\gamma r} + De^{-\gamma r}$$

$r \to \infty \quad u \to 0$ to be finite $\Rightarrow C=0 \qquad u_{II}(r) = De^{-\gamma r}$

At boundary $u_I \equiv u_{II}$ to be continuos

$$u_I(R_0) = u_{II}(R_0)$$

Plus the derivative has to be continuos

$$\left.\frac{d\,u_I(r)}{dr} = \frac{d\,u_{II}(r)}{dr}\right|_{r=R_0}$$

$$\begin{cases} A \cdot \sin kR_0 = De^{-\gamma R_0} \\ kA \cos kR_0 = -\gamma De^{-\gamma R_0} \end{cases}$$

$$k \cot kR_0 = -\gamma \qquad \sqrt{\frac{2m}{\hbar^2}(V_0+E)} \cdot \cot\left[R_0\sqrt{\frac{2m}{\hbar^2}(V_0+E)}\right] = \sqrt{+\frac{2mE}{\hbar^2}}$$

V_0 can be extracted ! Using numerical solution

$$\boxed{V_0 = 36\ \text{MeV}}$$

Strenght of the potential <=> strenght of the strong force

SUMMARY

$r_0 = 2{,}1$ fm

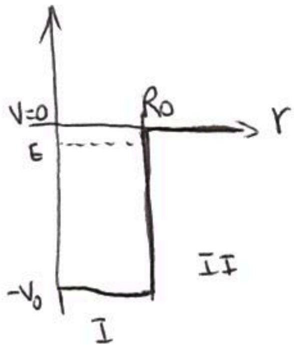
$V = 0 \quad r > r_0$

$V = -V_0 \quad r < r_0$

Energy level ~ 0 considering that $V_0 = 36$ MeV

-> Deuteron is weakly bound

Only one state $\rightarrow \ell = 0$



$$k = \sqrt{\frac{2m(V_0+E)}{\hbar^2}} \qquad \gamma = \sqrt{-\frac{2mE}{\hbar^2}}$$

$$E = -2{,}225\ \text{MeV}$$
$$V_0 = 36\ \text{MeV}$$

$$k = \sqrt{\frac{2m(V_0+E)}{\hbar^2}} \qquad m = \frac{m_p}{2} \qquad \frac{\hbar^2}{2m} = \frac{\hbar^2}{m_p} = \frac{\hbar^2 c^2}{m_p c^2} = \frac{(197\ \text{MeV}\cdot\text{fm})^2}{938\ \text{MeV}} = \frac{197^2\ \text{fm}^2\text{MeV}}{938} = 41{,}3\ \text{MeV}\cdot\text{fm}$$

$$k = \sqrt{\frac{(36-2{,}225)\text{MeV}}{41{,}3\ \text{MeV}}\ \text{fm}^{-2}} = 0{,}90\ \text{fm}^{-1}$$

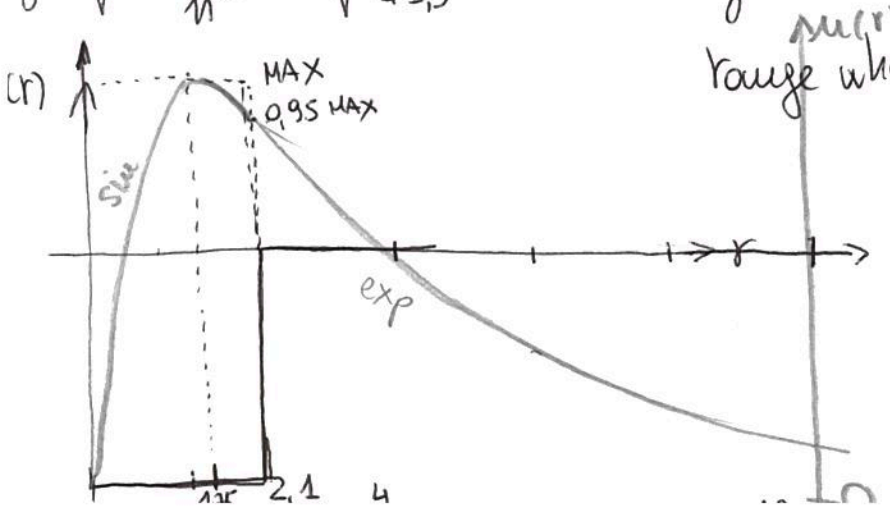
$$\sin k \cdot r_0 = \sin(0{,}90 \cdot 2{,}21) = 0{,}95$$

The maximum would be at $\sin kr = 1 \rightarrow kr = 1{,}57 \quad r = \frac{1{,}57}{k} = 1{,}744$ fm

where the ψ is maximum

$$\gamma = \sqrt{-\frac{2Em}{\hbar^2}} = \sqrt{\frac{2{,}225}{41{,}3}}\ \text{fm}^{-1} \qquad \frac{1}{\gamma} = 4{,}31\ \text{fm}$$

range where the expo decreases by $1/e$



Most of the time p and n are outside the 2,1 fm range => deuteron is very weakly bound !

ANGULAR MOMENTUM AND PARITY OF DEUTERON

In general,

$$\vec{J} = \vec{\ell} + \vec{S}$$

Final total angular momentum of a nucleons

Angular momentum of nucleon

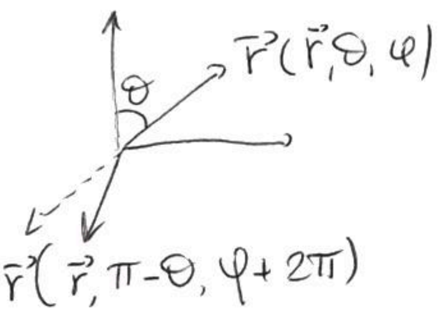
Spin of nucleons

I.e. what can we learn from the fact that $J^{\pi}_{deuteron} = 1^{+}$??
Let's start from angular momentum

The NUCLEAR SPIN $\vec{I}$ is the total angular momentum and can be measured

$$\vec{I} = \vec{J}_1 + \vec{J}_2 + \dots$$

The parity is definided in this way: The wave function ψ can be symmetric or anti-symmetric: this defines the parity.
If we take into consideration the space part of $\psi(\vec{r}, \theta, \varphi) \to \phi(\vec{r})$
then, if $\phi(\vec{r}) = \phi(-\vec{r})$ the parity π is even or $+$
if $\phi(\vec{r}) = -\phi(-\vec{r})$ π is odd or $-$



Parity is related to $\vec{\ell}$ (orbital angular momentum)

$\ell = 0, 2, 4, \quad \Rightarrow \pi = +$
$\ell = 1, 3, 5, \dots \quad \Rightarrow \pi = -$

For deuteron one can measure parity π (di parity) and spin $\vec{J}$ and isospin. What can we learn from that?

$J = 1 \quad \pi = + \qquad J^{\pi} = 1^{+} \qquad T = 0$ (Isospin)

If we have a deuteron, the possible spin configurations are:
$|\uparrow\uparrow\rangle_{pn}$, $|\downarrow\downarrow\rangle_{pn}$, $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$: In this case spin is not coupled

If pn couple to make the final spin S,

$\vec{S} = \vec{S}_p + \vec{S}_n$ $S_p = S_n = \frac{1}{2}$

$S = 1, 0$ Why? Because S can range from $S_1 - S_2$ to $S_1 + S_2$ with steps of 1

If $S = 0$, $S_z = S_{p_z} + S_{n_z} = 0 \Rightarrow S_{p_z} = -S_{n_z} \Rightarrow |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle$

$\Rightarrow S = 0, S_z = 0 \Rightarrow \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$

if $S = 1$, $S_z = \hbar, 0, -\hbar$ where $\hbar$ means $S_{n_z} = S_{p_z} = \uparrow\uparrow$
$-\hbar \Leftrightarrow \downarrow\downarrow$

So 3 possible combinations are possible:

$S = 1, S_z = \hbar$ $|\uparrow\uparrow\rangle$

$S = 1; S_z = -\hbar$ $|\downarrow\downarrow\rangle$

$S = 1; S_z = 0$ $\frac{|\uparrow\downarrow\rangle + |\uparrow\downarrow\rangle}{\sqrt{2}}$

At the end, in total we could have 4 states:

75% couple to $S = 1$

25% couple to $S = 0$

$\boxed{JT}$ the data tell us that we measure $J = 1$. *

(7) Since $\vec{J} = \vec{l} + \vec{S}$, and we measure $J = 1$ and we know that l should be even $\Rightarrow$ $S = 0$ should ~~not~~ be possible in fact:

because the ground state ∃ and so $l = 0$ ∃ and the parity is (+)!

$J = 1$, $\vec{J} = \vec{l} + \vec{S}$

The possible states are

$S = 0$; $l = 1$ $\Rightarrow$ $J = 1$ $\pi = -$

$\cancel{S = 0 \; ; \; l = 0 \Rightarrow J = 0}$

$\cancel{S = 0 \; ; \; l = 2 \Rightarrow J = 2}$

$S = 1$; $l = 0$ $\Rightarrow$ $J = 1$ $\pi = +$

$S = 1$; $\cancel{l = 1}$ $\Rightarrow$ $J = 2, 1, 0$ $\pi = -$

$S = 1$; $l = 2$ $\Rightarrow$ $J = 3, 2, 1$ $\pi = +$

For $l > 2$ it is not possible to obtain a state with $J = 1$.

From the measurement, it is possible to obtain the the only allowed values are: $\begin{cases} l = 0 & S = 1 \\ l = 2 & S = 1 \end{cases}$ The dominant part will be the "ground state" $l = 0$