PARITY | The parity of the ground state is positive. If we separate the wave function of d into 3 parts: Intrinsic ψ of p; intrinsic ψ of n and the orbital wave function for the relative motion, since a p and a n are just 2 different states of a nucleon, their intrinsic ψ has the same ~~positive~~ parity. => The product of their intrinsic wave function has positive parity, regardless of the parity of the nucleon.
This leaves the parity of deuteron to be determined solely by the relative motion between the two nucleons.
For states with a given orbital momentum $\vec{L}$, the angular dependence is given by $Y_L^{m_L}(\theta,\phi)$, spherical harmonics of order L.
Under an inversion of coordinate system, SH transform as:

$$Y_\ell^m(\theta,\phi) \longrightarrow Y_\ell^m(\pi-\theta,\pi+\phi) = (-1)^\ell\, Y_\ell^m(\theta,\phi)$$

=> The parity of $Y_\ell^m(\theta,\varphi)$ is therefore $(-1)^\ell$.
Since the parity of deuteron is $\oplus$ => ℓ must be even $(0,2)$

Similarly, the isospin I of deuteron can be deduced from L and S. Since the projection of isospin on the quantization axis is $T_{o_p} = +\frac{1}{2}$ and $T_{o_n} = -\frac{1}{2}$ (for neutron) the deuteron is a state with the sum of the projections $T_0 = 0$. The isospin of such system can be coupled to $T=0$ and $T=1$. For light nuclei, the isospin is expected to be a good quantum number and the ground state can take only one of the two values.

If we consider a p and an n as two different isospin states of a nucleon, p and n in a deuteron might be treated as identical particles and the total wave-function must be antisymmetric under a permutation of indices of 2 Fermi-Dirac particles:

$$P_{12}\ \psi(1,2) = \psi(2,1) = -\psi(1,2)$$

The wave function $\psi(1,2)$ may be decomposed into a product of spatial, spin and isospin parts.

For the spatial part:

$$\vec{r} \equiv \vec{r}_1 - \vec{r}_2 \xrightarrow[P_{12}]{} -\vec{r}$$

permutation

In spherical polar coordinates system:

$$(r, \theta, \phi) \xrightarrow[P_{12}]{} (r, \pi - \theta, \phi + \pi)$$

Since r is unchanged by this transformation, the symmetry is given by angular dependence => SH.

For $\ell = 0, 2$, the spatial w.f. is symmetric under permutation.

It is also easy to see that the intrinsic spin part of the deuteron is even in $S=1$ state.

If we consider the $m_S = 1$ state the triplet $m_S = 0, \pm 1$, the spin wave function of 2 nucleons must be written as:

$$|S=1, m_S=1\rangle = \underbrace{\left|S=\tfrac{1}{2}, m_S=\tfrac{1}{2}\right\rangle_1 \left|S=\tfrac{1}{2}, m_S=\tfrac{1}{2}\right\rangle_2}\quad (\star)$$

The function on the right is even under permutation. Since there is no other way to construct $(S, m_S) = (1, 1)$ state, then Ⓐ is the intrinsic spin wave function of this state.

The other 2 w.f. of $S = 1$ can be generated using the Angular momentum lowering operator, which is simmetric w.r.t. the 2 nucleons.

=> The intrinsic spin part of the deuteron wave function is <u>EVEN</u> under a permutation of two nucleons.

With both spatial and spin part of deuteron w.f. being symmetric the isospin part <u>MUST BE</u> <u>antisymmetric</u> as required by Pauli principle. From the discussion of spin, we can conclude that $T=1$ is symmetric under permutation.

In case of $T=0$, it is possible to obtain an antisymmetric linear combination:

$$|T=0, T_0=0\rangle = \frac{1}{\sqrt{2}} \left\{ \left|t=\frac{1}{2}, t_0=+\frac{1}{2}\right\rangle_1 \left|t=\frac{1}{2}, t_0=-\frac{1}{2}\right\rangle_2 + \right.$$

$$\left. - \left|t=\frac{1}{2}, t_0=-\frac{1}{2}\right\rangle_1 \left|t=\frac{1}{2}, t_0=+\frac{1}{2}\right\rangle_2 \right\}$$

The requiremet that the isospin part of 2-nucleon system is antisymm implies that the ground state of deuteron is in $T=0$ state.

The same conclusion could have been drawn also by a different set of arguments.

If $T=1$ would have been the ground state of deuteron, similar bound state would have been expected for 2p and 2n bound states. Which have never been observed. We can also conclude that the force is attractive only if $T=0$.

In conclusion, deuteron has $S=1$ and $T=0$ and 2 possible l states $l=0$ and $l=2$.

If l and s are "good quantum nubers" i.e. if the nuclear Hamiltonian H commutes with $\vec{L}^2$ and $\vec{S}^2$ the ground state of deuteron would have be in either one of the 2 states. But there is no fundamental reason for that. In fact we will see that both $\cancel{\ell} = 0$ and $\ell = 2$ states MUST be present in deuteron ground state. ⇒ NUCLEAR FORCE mixes different L components in an eigen state.

$\boxed{\text{MAGNETIC DIPOLE MOMENT}}$

The magnetic dipole moment arises from the combination of 2 sources.

① Eeach nucleon carries its intrinsic moment due to quark
② Each protons carries a net positive charge and its orbital motion constitutes an electric current loop.

Assuming that the proton charge is distributed along its orbit, the magnetic dipole moment can be obtained using classical e.m. theory

$$\mu_i^{(orbital)} = \frac{e\hbar}{2m_p} \, l_i$$

→ orbital angular momentum of the i^{th} protons in units of $\hbar$ and its mom.

It is more convenient to express the contributions to the nuclear magnetic dipole moment from individual nuclions in terms of gyromagnetic ratios $g(i)$. For orbital motion

$$\mu_i^{(orbital)} = g_\ell(i) \, \vec{l_i}$$

$$g_\ell(i) = \begin{cases} 1\mu_N & \text{for } p \\ 0 & \text{'' } n \end{cases}$$

Wher μ_N is the nuclear magneton $\frac{e\hbar}{2m_p} \vec{l_i}$

⇒ One can think to generalize the result writing:

$$\mu_J = \frac{e}{2m_p c}\int d^3x \ \psi^*(r') J \psi(r')$$

But this is not possible due to the gyromagnetic anomaly of the spin. In fact, an electron ~~has~~ with a spin $S = \frac{1}{2}$ (in unity of ") has a magnetic moment equal to the Bohr magneton.

It is the same as:

$$\mu_S = \frac{e}{m_0 c}\vec{S} = 2\mu_0 \vec{S}$$

where $\vec{S}$ in $\hbar$ units. The relation can be rewritten as:

$$\mu_S = \mu_0 g_e \vec{S}$$

, where g_e is the gyromagnetic ratio of the electron, $g_e = 2$

In the case of nucleons this is more complicated since their gyromagnetic ratios are:

$$\begin{cases} g_{sp} = 2\times(2,79\ldots) \\ g_{sn} = 2\times(-1,91\ldots) \end{cases}$$

So

$$\mu_{S(p)} = \frac{e}{2m_p c} g_p \vec{S} \qquad \mu_{S(n)} = \frac{e}{2m_n c}\cdot g_n \vec{S}$$

Since p and n have spin $= \frac{1}{2}$

$$\mu_{S(p)} = \underbrace{\frac{e\hbar}{2m_p c}}_{=\mu_N} \cdot 2,79 = \qquad \mu_{S(n)} = \underbrace{\frac{e\hbar}{2m_n}}_{=\mu_N} \times(-1,91)$$

Assuming that the structure of the bound nucleon inside a nucleus is the same one in its free state, g_p e g_n can be used to describe also

p and n inside the nucleus.
In terms of gyromagnetic ratios, the magnetic dipole operator can be written as a function of the orbital angular momentum operator $\vec{l}_i$ and the intrinsic operator $\vec{S}_i$ of each nucleon.
For a deuteron, only 2 nucleons are involved and

$$\mu_d = g_p \vec{S}_p + g_n \vec{S}_n + \vec{l}_p$$

Intrinsic spin operator

angular momentum of protons

Since the mass of p and d are $\simeq$ the same we can assume that each of the two carries half of the orbital angular momentum associated with their relative motion i.e. $\vec{l}_p = \frac{1}{2}\vec{L}$, we have

$$\mu_d = g_p \vec{S}_p + g_n \vec{S}_n + \frac{1}{2}\vec{L}$$

where $\vec{L}$ is the deuteron orbital angular momentum operator.

Expected value of the magnetic dipole operator.

The expected value of μ_d depends on M i.e. the projection of $\vec{J}$ on the z-axis. By convention, the magnetic moment is defined as the expectation value of z component. The expected value can be evaluted like

$$\langle \psi | \mu_z | \psi \rangle$$

in a ~~stereo~~ classical approac
μ_z is maximum in the direction of the magnetic field.
In deuteron $J = 1$ (total angular momentum) and z component can have 3 values $\mu_{J_z} = 1$ $\mu_{J_z} = 0$ $\mu_{J_z} = -1$ $\mu_{J_z} = J = 1$ gives the maximum
So

$$\langle \psi | \mu_z | \psi \rangle \Rightarrow \underbrace{\langle J, M_J = 1}_{\vec{S}=1} | \mu_z | J, M_J = 1 \rangle$$

$$\vec{J} = \vec{L} + \vec{S} \Rightarrow S_z = 1 \quad S_1 = \frac{1}{2} \quad S_2 = \frac{1}{2} \Rightarrow \psi = |\uparrow\uparrow\rangle$$

$$\vec{S} = \vec{S}_1 + \vec{S}_2$$

$$S_z = S_{1z} + S_{2z} \Rightarrow S_z = 1$$

$\langle \uparrow\uparrow | g_e$ $\quad \boxed{\mu_z = [g_e \cdot l_z + g_{sp} s_{zp} + g_{sn} \cdot s_{zn}]\mu_N}$

$$\langle \mu_z \rangle = \langle \uparrow\uparrow | g_{sp} S_{zp} + g_{sn} S_{zn} | \uparrow\uparrow \rangle \mu_N =$$

$$= \Big[g_{sp} \underbrace{\langle \overset{p}{\uparrow}\overset{n}{\uparrow} | S_{zp} | \overset{p}{\uparrow}\overset{n}{\uparrow} \rangle}_{\text{operates only on } p} + g_{sn} \underbrace{\langle \overset{p}{\uparrow}\overset{n}{\uparrow} | S_{zn} | \overset{p}{\uparrow}\overset{n}{\uparrow} \rangle}_{\text{operates only on } n} \Big] \mu_N$$

$$\langle \mu_z \rangle = \left[g_{sp} \cdot \langle \uparrow\uparrow | \uparrow\uparrow \rangle + g_{sn} \langle \uparrow\uparrow | \uparrow\uparrow \rangle \right] \mu_N = [2{,}7928 - 1{,}943]\mu_N$$

$$= 0{,}8798 \ \mu_N$$

Expectation

Value obtain under these assumption $\quad \mu_z = 0{,}874$ = Measured

- Central potential
- Ground state $l = 0$
- g_{sp}, g_{sn} are the same for free and bound systems.

Expectation and measured value are not equal (tiny difference but relevant). Why?

We can make some hp:

1) The internal structure of protons and neutron are modified by the fact that we have a bound state. ⇒ g_{sp} and g_{sn} might be different

2) Contributions from meson exange could be play a role

3) There is a small $l=2$ state in the ground state

1) Is extremly unlikely (d is a loosely bound state) (33)

2) This might play a role

3) This is the mostly likely hp. Plus distinguish between 2 and 3 is a fundametal physics question that we will not address here.

How much of $l=0$ and $l=2$ goes in the ground state?

If $l=2$, $S=1$ $J=1$

$\vec{J} = \vec{L} + \vec{S}$

$J_z = L_z + S_z$ $\qquad M_L + M_S = 1 \qquad Y_l^{m_l}$

$M_J = 1 \Rightarrow M_l + M_S = 1$

$l=2$ $m_l = 2, 1, 0, -1, -2$

$S=1$ $m_S = 1, 0, -1$

$$\begin{cases} m_l = 2 & m_S = -1 \Rightarrow Y_2^2 \,|\downarrow\downarrow\rangle \\ m_l = 1 & m_S = 0 \Rightarrow Y_2^1 \,\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \\ m_l = 0 & m_S = 1 \Rightarrow Y_2^0 \,|\uparrow\uparrow\rangle \end{cases}$$

$$|J=1, M_J=1, L=2, S=1\rangle = \sqrt{\frac{3}{5}}\, Y_2^2 \,|\downarrow\downarrow\rangle - \sqrt{\frac{3}{10}}\, Y_2^1 \left[\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)\right] + \sqrt{\frac{1}{10}}\, Y_2^0 \,|\uparrow\uparrow\rangle$$

Clebsh-Gordon coefficient

Each componets can be used to evaluate μ_z and averaged

$\langle \mu_z \rangle_{L=2} = 0{,}3101\, \mu_N$

$\langle \mu_z \rangle_{L=0} = 0{,}8798\, \mu_N$

$\langle \mu_z \rangle_{OBS} = 0{,}8574\, \mu_N$

$\psi = a\,(L=0) + b\,(L=2)$

$$\begin{cases} \langle \mu_z \rangle = |a|^2\, 0{,}8798\, \mu_N + |b|^2\, 0{,}3101\, \mu_N = 0{,}8574\, \mu_N \\ |a|^2 + |b|^2 = 1 \end{cases}$$

$\Rightarrow |a|^2 = 0{,}96 \quad |b|^2 = 0{,}04 \Rightarrow$ The ground state is $l=0$ in 96% of cases and

(33b)

Table A-1: Some useful Clebsch-Gordan coefficients.

$\langle \frac{1}{2} m_s \ell m_\ell | jm \rangle$

j	$m_s = +\frac{1}{2}$	$m_s = -\frac{1}{2}$
$\ell + \frac{1}{2}$	$\sqrt{\frac{\ell + \frac{1}{2} + m}{2\ell + 1}}$	$\sqrt{\frac{\ell + \frac{1}{2} - m}{2\ell + 1}}$
$\ell - \frac{1}{2}$	$\sqrt{\frac{\ell + \frac{1}{2} - m}{2\ell + 1}}$	$-\sqrt{\frac{\ell + \frac{1}{2} + m}{2\ell + 1}}$

$\langle 1 m_s \ell m_\ell | jm \rangle$

j	$m_s = +1$	$m_s = 0$	$m_s = -1$
$\ell + 1$	$\sqrt{\frac{(\ell+m)(\ell+m+1)}{2(\ell+1)(2\ell+1)}}$	$\sqrt{\frac{(\ell-m+1)(\ell+m+1)}{(\ell+1)(2\ell+1)}}$	$\sqrt{\frac{(\ell-m)(\ell-m+1)}{2(\ell+1)(2\ell+1)}}$
ℓ	$\sqrt{\frac{(\ell+m)(\ell-m+1)}{2\ell(\ell+1)}}$	$\frac{-m}{\sqrt{\ell(\ell+1)}}$	$-\sqrt{\frac{(\ell-m)(\ell+m+1)}{2\ell(\ell+1)}}$
$\ell - 1$	$\sqrt{\frac{(\ell-m)(\ell-m+1)}{2\ell(2\ell+1)}}$	$-\sqrt{\frac{(\ell-m)(\ell+m)}{\ell(2\ell+1)}}$	$\sqrt{\frac{(\ell+m+1)(\ell+m)}{2\ell(2\ell+1)}}$

$$\begin{pmatrix} j & 0 & j' \\ -m & 0 & m' \end{pmatrix} = \langle jmj' \, -m' | 00 \rangle = \frac{(-1)^{j-m}}{\sqrt{2j+1}} \delta_{jj'} \delta mm' \qquad \langle jm00|j'm'\rangle = \delta_{jj'}\,\delta_{mm'}$$

$$\begin{pmatrix} j & 1 & j \\ -m & 0 & m \end{pmatrix} = (-1)^{j-m} \frac{m}{\sqrt{j(j+1)(2j+1)}}$$

$$\begin{pmatrix} j & 2 & j \\ -m & 0 & m \end{pmatrix} = (-1)^{j-m} \frac{3m^2 - j(j+1)}{\sqrt{(2j-1)j(j+1)(2j+1)(2j+3)}}$$

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ 0 & 0 & 0 \end{pmatrix} = \begin{cases} (-1)^g \sqrt{\frac{(2g-2j_1)!(2g-2j_2)!(2g-2j_3)!}{(2g+1)!}} \dfrac{g!}{(g-j_1)!(g-j_2)!(g-j_3)!} & \text{if } 2g = \text{ even} \\ 0 & \text{if } 2g = \text{ odd} \end{cases}$$

where $2g = j_1 + j_2 + j_3$

$m_s = -1$

$m = 1$

$\ell = 2$

$\ell+1 \quad \sqrt{\frac{2}{2(3)(5)}} = \sqrt{\frac{1}{15}}$

$\ell \quad \sqrt{\frac{1(\cancel{4})}{\cancel{4}(3)}} = -\sqrt{\frac{1}{3}}$

$\boxed{\ell-1 \quad \sqrt{\frac{4\cdot 3}{4(4+1)}} = \sqrt{\frac{3}{5}}}$

$m_s = 0$

$m = 1$

$\ell = 2$

$\sqrt{\frac{2\cdot 4}{3\cdot 5}} = \sqrt{\frac{8}{15}}$

$\frac{-1}{\sqrt{2(3)}} = -\sqrt{\frac{1}{6}}$

$\boxed{-\sqrt{\frac{1\cdot 3}{2(5)}} = -\sqrt{\frac{3}{10}}}$

$m_s = 1$

$m = 1$

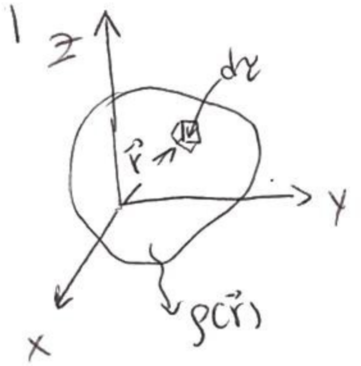
$\ell = 2$

$\boxed{\sqrt{\frac{1\cdot 2}{\cancel{4}_{2}(5)}} = \sqrt{\frac{1}{10}}}$

DEUTERON ELECTRIC QUADRUPOLE MOMENT

A similar conclusion (i.e. deuteron in ground state is a linear combination of S and D states) can be drawn when the electric quadrupole moment is considered.

In electrostatics, the potential due to an arbitrary charge distribution of point far away from the source is characterized by the distance and by the moments of a multipole expansion of the source. In nuclear charge distribution the first moment which do not vanish is the quadrupole mome



Monopole moment = charge

Dipole = $\int \rho(\vec{r})$

Tripole

Quadrupole moment:

$$e\,Q_{ij} = \int \underbrace{\rho(\vec{r})}_{\substack{\text{Charge}\\ \text{density}\\ \text{at } \vec{r}}} \big(\underbrace{3\,x_i x_j}_{\substack{\text{Coordinates}\\ \text{of the volume}\\ \text{elements}}} - \vec{r}\,\underbrace{\delta_{ij}}_{\text{Delta di Dirac}}\big)\,\underbrace{d\tau}_{\text{Volume}}$$

$x_1 = x$
$x_2 = y$
$x_3 = z$

$$e Q_{xx} = \int \rho(\vec{r}) \left[3x^2 - \vec{r}\right] d\tau$$

$$e Q_{xy} = \int \rho(\vec{r}) \left[3xy\right] d\tau$$

....

In experiments one can measure Q_{zz} $\Rightarrow$ $e Q_{zz} = \int \rho(\vec{r}) \left[3z^2 - \vec{r}\right] d$

If the charge distribution is spherically symmetric $\rho(\vec{r}) = \rho(r)$ and $e Q_{ij} = 0$.

$$e Q_{zz} = \int \rho(r) \left[3z^2 - r^2\right] d\tau = \int \rho(r) \left[3y^2 - r^2\right] d\tau = \int \rho(r) \left[3x^2 - r^2\right] d\tau$$

$$\boxed{e Q_{zz} = (3z^2 - r^2)}$$

The electric quadupole momet is defined as the expectation velue of Q_0 in the substrate of maximum M

$$Q_A = \langle J\ M=J \,|\, Q_0 \,|\, J M=J \rangle$$

Based on angular momentum considerations, any nuclear state with $J<1$ cannot have quadupole momet different from zero. The expectation value $\langle J, M | Q_0 | J, M \rangle$ vanishes if the 3-angular momenta involved J, 2, and J, cannot be coupled together to form a closed triangle.

At the same time, since Q_0 operates only in the coordinate space, it is independent of the total intrinsic spin S.

This means that the orbital angular momentum L of the state must also be greater or equal to 1.

For this reason the expectation value of Q_0 vanishes in the S_1-state.

The existence of non vanishing quadupole moment is therefore a direct evidence of the presence of ^1D-component in the deuteron ground state.

Experimentally, Q_{zz} (deutone) $= 2{,}88 \cdot 10^{-27}$ cm^2

EXERCISE

A deuteron can be seen as a bound state of a p and a n which interact through a squre well potential of with $b = 1{,}9 \cdot 10^{-15}$ m and a depth $V_0 = 40$ MeV in an $l=0$ state.

a) Calculate the probability that the proton moves within the range of the neutron.

Use the approximation that $m_p = m_p = M$ $kb = \frac{\pi}{2}$, where $k = \sqrt{\frac{M(V_0 - \varepsilon)}{\hbar^2}}$

and ε is the binding energy of the duteron

b) Find the mean-square radius of the deuteron.

The interaction may be considered as beetween two ptc of mass M, so the reduded mass of the system is $\mu = \frac{1}{2} M$. The potential energy is

$$V(r) = \begin{cases} -V_0 & r < b \\ 0 & r > b \end{cases}$$

where r is the radius between p and n. The system energy is $E = -\varepsilon$.

For $l=0$ states, let the wave function would be $\psi = \frac{u(r)}{r}$ and the Schrödinger equation will be

$$\frac{d^2u}{dr^2} + \frac{2\mu}{\hbar^2}(E - V)u = 0$$

For $r \le b$ $\quad \frac{d^2u}{dr^2} + k^2 u = 0 \quad k = \sqrt{\frac{M(V_0 - \varepsilon)}{\hbar^2}}$

$r > b$ $\quad \frac{d^2u}{dr^2} - k_1^2 u = 0 \quad k_1 = \sqrt{\frac{M\varepsilon}{\hbar^2}}$

With boundary conditions $\psi = 0$ at $r=0$; ψ finite at $r = \infty$ and the continuity of the derivative:

$$u(r) = A\sin(kr) \quad r \le b$$
$$u(r) = Be^{-k_1(r-b)} \quad r > b$$

$$A \sin(kb) = B$$
$$kA \cos(kb) = -k_1 B$$
$$\Rightarrow \cot(kb) = -\frac{k_1}{k} = -\sqrt{\frac{\varepsilon}{V_0 - \varepsilon}}$$

For $kb = \frac{\pi}{2} \Rightarrow A \simeq B$ and $\cot(kb) = 0$ ($V_0 \gg \varepsilon \equiv l=0$ state)

To normalize

$$1 = \int_0^\infty |\psi(r)| \, d\Omega = \int_0^\infty |\psi(r)| 4\pi r^2 dr = 4\pi A^2 \int_0^b \sin^2(kr)\, dr + 4\pi B^2 \int_b^\infty e^{-2k_1(r-b)} dr$$

$$\simeq 2\pi A^2 b \left(1 + \frac{1}{bk_1}\right)$$

$$\Rightarrow A \simeq B \simeq \left[2\pi b \left(1 + \frac{1}{bk_1}\right)\right]^{-1/2}$$

The probability of the proton moving within the range of the force of
of neutron is :

$$P = 4\pi A^2 \int_0^{b} \sin^2(kr)\, dr = \left(1 + \frac{1}{k_1 b}\right)^{-1}$$

As $\quad k = \frac{\sqrt{M(V_0 - \varepsilon)}}{\hbar} = \frac{\pi}{2b} \quad \Rightarrow \varepsilon \simeq V_0 - \frac{1}{Mc^2}\left(\frac{\pi \hbar c}{2b}\right)^2$

$$= 11{,}8 \text{ MeV}$$

And $\quad k_1 = \frac{\sqrt{Mc^2 \varepsilon}}{\hbar c} = 5{,}3 \cdot 10^{14}\ m^{-1}$

$$\Rightarrow P = \left(1 + \frac{1}{5{,}3 \cdot 10^{14} \times 1{,}9 \cdot 10^{-15}}\right)^{-1} = 0{,}50$$

The mean square radius is

$$\overline{r}^2 = \langle \psi | r^2 | \psi \rangle_{r<b} + \langle \psi | r^2 | \psi \rangle_{r>b}$$

$$= 4\pi A^2 \left[\int_0^b \sin^2(kr)\, r^2 dr + \int_b^\infty e^{-2k_1(r-b)}\, r^2 dr \right] \simeq 5{,}8 \cdot 10^{-30}$$

$$\Rightarrow \quad \left(\overline{r}^2\right)^{1/2} = 2{,}4 \cdot 10^{-15} \text{ m}$$

---

c) What happens if the depth of the potential increases?
d) What happens if the B.E. is decreased?

EXERCISE

)A proton and a neutron can undergo radioactive capture at rest:

$$p + n \rightarrow d + \gamma$$

Find the energy of the photon emitted in this capture.
Is the recoil of the deuteron important?

a) The energy released in the radioactive capture is:

$$Q = \left[m_p + m_n - m_d\right]c^2 = \left[m_p + m_n - m_d\right]c^2 = E_B = 2{,}225\,\mathrm{MeV}$$

This energy appears as kinetic energy of the photon and recoil of deuteron

$$Q = \underbrace{pc}_{E_K} + \underbrace{\frac{p^2}{2m_d}}_{E_{RECOIL}}$$

$$(pc)^2 + 2m_d c^2 (pc) - 2 m_d c^2 Q = 0$$

$$pc = m_d c^2 \left(-1 + \sqrt{1 + \frac{2Q}{m_d c^2}}\right)$$

As $\frac{Q}{2m_d c^2} \ll 1 \Rightarrow pc \simeq m_d c^2 \left(-1 + 1 + \frac{Q}{m_d c^2}\right) \simeq \frac{Q}{c}$

So the recoil energy of deuteron become:

$$E_{recoil} = \frac{p^2}{2m_d} = \frac{Q^2}{2m_d c^2} = \frac{(2{,}225)^2\,\mathrm{MeV}^{\cancel{2}}}{2 \times 1875\,\cancel{\mathrm{MeV}}} \simeq 1{,}3 \cdot 10^{-3}$$

$$\frac{E_{recoil}}{E_\gamma} = \frac{1{,}3 \cdot 10^{-3}\,\mathrm{MeV}}{2{,}225\,\mathrm{MeV}} \simeq 6 \cdot 10^{-4}$$

$\Rightarrow$ The recoil of deuteron do not affect the energy of the γ ($\Rightarrow$ effect at the 10^{-4} le