

BETA DECAY

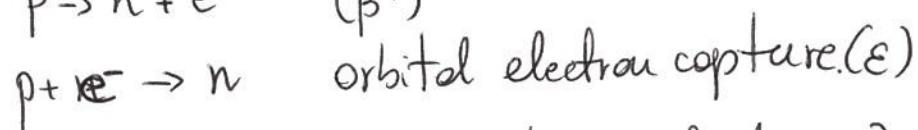
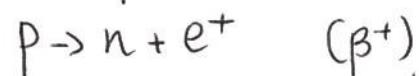
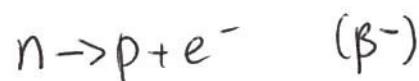
The emission of an ordinary electron from nuclei was among the earliest observation of radioactive phenomena.

The most basic β decay process is the conversion of one p to a n or a n into a p. In a nucleus β decay changes both Z and N by 1 unit: $Z \rightarrow Z \pm 1$; $N \rightarrow N \mp 1$ so that A remains constant.

β decay is a "convenient" way for nuclei to slide down the main probele of isobar nuclei approaching the stability valley.

The understanding of β decay has been achieved extremely slow, because often the experimental results were creating new puzzles difficult to be explained theoretically.

The basic processes of β decay are:



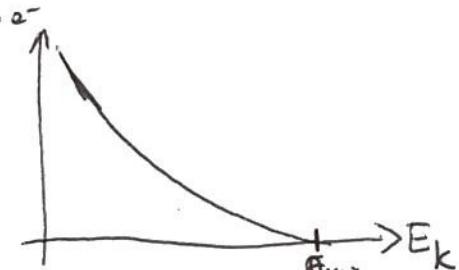
As we know these processes are not complete, in fact we do have to be involved. The β^+ and ϵ processes can occur only inside nuclei otherwise they are energetically forbidden for free protons.

ENERGY RELEASE IN β DECAY

The energy of the electron/positron emitted in β -decay was a confusing experimental results in 1920s.

Contrary to α particles, which are emitted with sharp well-defined energy equal to the difference in mass between initial & final states, less the small recoil energy, β energy is continuous.

It means that this cannot be a 2-body process.

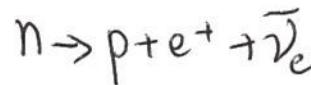


To account for the release energy, Pauli in 1931 proposed that there was emitted in the decay process a second particle (which Pauli wanted to call neutron, but Fermi re-named neutrino - little neutron one). The ν carries the "missing" energy and because it is a highly penetrat radiation it cannot be stopped in calorimeters. ~~HOW THE NEUTRINO WAS DETECTED.~~

Charge conservation requires the ν to be electrically neutral, and angular momentum conservation and statistical considerations in the decay give the neutrino a spin $1/2$.

Experiments shows that there are in fact 2 different kind of ν : $\bar{\nu}$ is emitted in β^- decay, while ν are emitted in β^+ and electron capture processes.

Let's evaluate the Q-values of the reactions, starting from β^- decay of free neutron (which have a $t_{1/2} \approx 10\text{ min}$)



$$Q = (m_n - m_p - m_e - m_{\bar{\nu}}) \cdot c^2$$

For neutron at rest

$$Q = T_p + T_e + T_{\bar{\nu}}$$

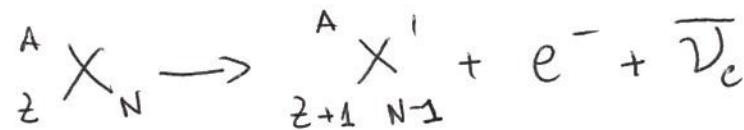
The recoil energy T_p can be ignored, since it amounts to only 0.3 keV. The $\bar{\nu}$ and e^- share the decay energy. The maximum e^- energy corresponds to the minimum ν energy. The measured maximum energy of electrons is $0.782 \pm 0.013\text{ MeV}$. Using the masses of n, p, e^- Q can be computed:

$$\begin{aligned} Q &= (m_n - m_p - m_e - m_{\bar{\nu}}) c^2 = (938,573 - 938,280 - 0,511) \text{ MeV} - m_{\bar{\nu}} c^2 \\ &= 0,782 \text{ MeV} - m_{\bar{\nu}} c^2 \end{aligned}$$

So within the uncertainty of 13 keV the ν mass is consistent with 0 ($m_{\bar{\nu}} \leq 13\text{ keV}$). Ideally we will consider $m_{\bar{\nu}} \approx 0$ here.

When we'll do the calculations, the ^{TOT} energy of ν is its kinetic energy. For the electron both kinetic (T_e) and rest mass energy $m_e c^2$ have to be taken into consideration: $E_\nu = T_e + m_e c^2$. Since $E_\nu \approx \text{MeV}$, $m_e \approx 0.5 \text{ MeV}$ $T \gg m_e c^2 \Rightarrow$ we have to use relativistic kinematics.

β^- decay in
For a generic nucleus:



$$Q_{\beta^-} = \left[\underbrace{\left(m({}_{Z}^A X_N) - Z m_e \right)}_{\text{Nuclear mass}} - \left(m({}_{Z+1}^A X'_{N-1}) - (Z+1) m_e \right) - m_e \right] c^2$$

\ominus Atomic B.E.

All m_e are cancelled in β^- case.

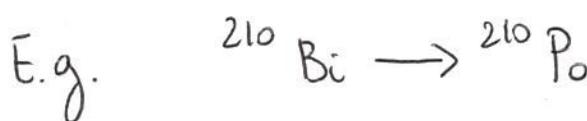
$$Q_{\beta^-} = \left[m({}_{Z}^A X_N) - m({}_{Z+1}^A X'_{N-1}) \right] c^2$$

The masses are the ATOMIC masses of parent and daughter etc.

$$Q_{\beta^-} = T_e + E_{\bar{\nu}}$$

And each has its maximum when the other approaches zero:

$$(T_e)_{\max} (E_{\bar{\nu}})_{\max} = Q_{\beta^-}$$



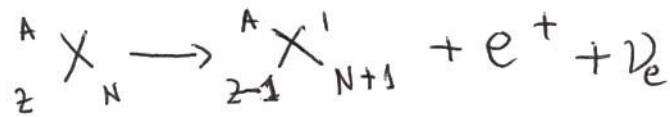
$$Q_{\beta^-} = \left[m({}^{210}\text{Bi}) - m({}^{210}\text{Po}) \right] c^2 =$$

$$= (209,984095 \mu - 209,982868 \mu) (931,0502 \text{ MeV}/\mu)$$

$$= 1,161 \text{ MeV.}$$

As shown in the example

In case of β^+ decay:



$$\begin{aligned} Q_{\beta^+} &= \left[m\left({}_{Z}^{A}X_N\right) - m\left({}_{Z-1}^{A}X'_{N+1}\right) - m_e \right] c^2 = \\ &= \left[m\left({}_{Z}^{A}X\right) - Z m_e \right] - \left[m\left({}_{Z-1}^{A}X'_{N+1}\right) - (Z-1) m_e \right] - m_e \right] c^2 \\ &= \left[m\left({}_{Z}^{A}X\right) - m\left({}_{Z-1}^{A}X'_{N+1}\right) - 2 m_e \right] c^2 \end{aligned}$$

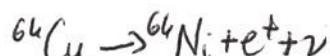
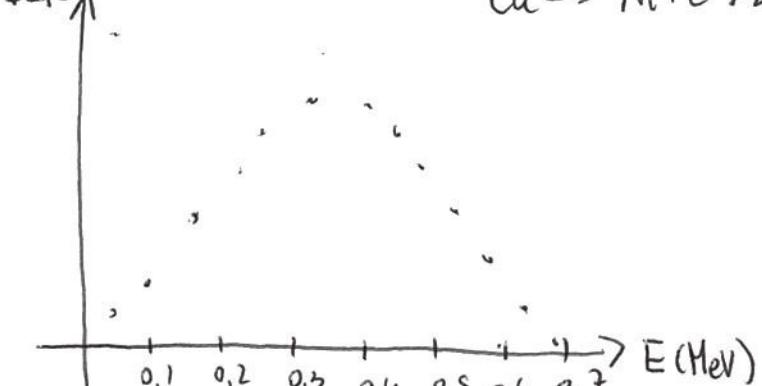
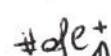
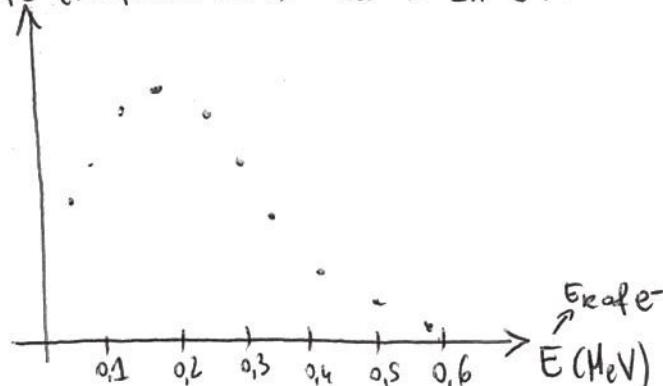
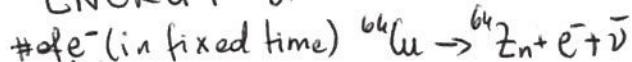
In this case $2m_e$ is not negligible because the Q -values are \approx MeV and $2m_e \approx 1$ MeV.

In electron capture:

$$\begin{aligned} Q_E &= \left[m_N\left({}_{Z}^{A}X_N\right) + m_e - m_N\left({}_{Z-1}^{A}X'_{N+1}\right) \right] c^2 \\ &= \left[m\left({}_{Z}^{A}X_N\right) - Z m_e + m_e - \left[m\left({}_{Z-1}^{A}X'_{N+1}\right) - (Z-1) m_e \right] \right] c^2 \\ &= \left[m\left({}_{Z}^{A}X_N\right) - m\left({}_{Z-1}^{A}X'_{N+1}\right) \right] c^2 \end{aligned}$$

Q -value of E.C. is larger than that of β^+ \Rightarrow In some cases only the E.C. is energetically possible.

ENERGY DISTRIBUTION OF β -decay FERMI THEORY OF THE DECAY



For α decay we observed that the Coulomb barrier gives the probability of decay \Rightarrow the lifetime of α nucleus. For β^+ there is too sufficient barrier to penetrate, and in case of β^- there is no barrier at all.

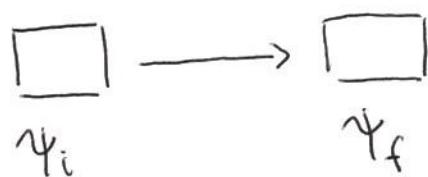
Moreover there are 3 other differences between $e\beta$ decay:

- 1) e^- and ν do not exist before the decay \Rightarrow the formation of the 2 ptc have to be taken into account.
- 2) e^- and ν must be treated relativistically
- 3) The continuous distribution of energy must be the result of a calculation.

The theory of β -decay was developed by Fermi in 1934, based on Pauli's neutrino hypothesis (from 1930).

The characteristics of the decay can be derived from the basic expression for transition probability caused by an interaction, that is weak w.r.t the interaction that forms the quasi-stationary states.

This is true for β decay since the characteristics $\tau_{1/2} \approx 50$ minutes are \gg than the nuclear time ($\approx 10^{-20}$ s)



The probability of transition would be:

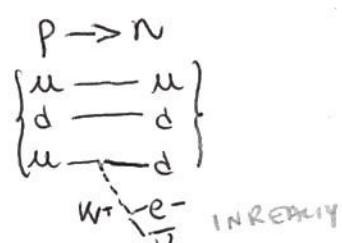
$$\lambda = \frac{2\pi}{\chi} \left| H_{if} \right|^2 \frac{dN}{dE_f}$$

H_{if} = Interaction hamiltonian

$$H_{if} = \langle \psi_f | H | \psi_i \rangle = \int \psi_f^* (H) \psi_i d\tau$$

Takes care of actual decay

FERMI GOLDEN RULE
- Time dependent perturbation theory



Volume of interest

$\frac{dn}{dE_F}$ = density of states which can be found at each E_F

↑
Shape of the distribution

Now let's have a look at Ψ

$$\Psi_i = \Psi_p \quad \Psi_f = \Psi_d \quad \Psi_e \quad \Psi_\nu$$

NUCLEAR W.F.

e^- , $\bar{\nu}$ can be treated as free particles
+ some corrections due to Coulomb interaction.

$$H_{if} \rightarrow \int \Psi_d^* \Psi_e^* \Psi_\nu^* H \Psi_p dV$$

$\left. \begin{array}{l} \Psi_e = \frac{1}{\sqrt{V}} e^{i \frac{\vec{p}_e \cdot \vec{r}}{\hbar}} \\ \Psi_\nu = \frac{1}{\sqrt{V}} e^{i \frac{\vec{p}_\nu \cdot \vec{r}}{\hbar}} \end{array} \right\}$ FREE WAVE-FUNCTION normalized
WITHIN the volume V

For an electron of $E_k \perp 1 \text{ MeV}$: $E_k = \sqrt{p_e^2 c^2 + m_e^2 c^4} - m_e c^2$
we want to evaluate p_e

$$\Rightarrow [(1 + 0,5) \text{ MeV}]^2 = p_e^2 c^2 + (0,5)^2 \text{ MeV}^2$$

$$2,25 \text{ MeV}^2 = p_e^2 c^2 + 0,25 \text{ MeV}^2 \quad p_e^2 c^2 = 2 \text{ MeV}^2 \quad p_e c = \sqrt{2} \text{ MeV}$$

$$p_e \cdot c = 1,44 \text{ MeV}$$

$$\text{So } \frac{\vec{p}_e \cdot \vec{r}}{\hbar} = \frac{1,44 \frac{\text{MeV} \times \text{fm}}{c}}{\hbar c} r = \frac{1,44 \cdot \text{MeV} \cdot \text{fm}}{200 \text{ MeV} \cdot \text{fm}} r \approx 7 \cdot 10^{-3} r \ll 1$$

It means that the exponential can be expanded

$$\Psi_e = \frac{1}{\sqrt{V}} e^{i \frac{\vec{p}_e \cdot \vec{r}}{\hbar}} \approx \frac{1}{\sqrt{V}} \left[1 + i \cdot \frac{\vec{p}_e \cdot \vec{r}}{\hbar} \right] \approx \frac{1}{\sqrt{V}}$$

$$\Psi_\nu \approx \frac{1}{\sqrt{V}} \left[1 + i \cdot \frac{\vec{p}_\nu \cdot \vec{r}}{\hbar} \right] \approx \frac{1}{\sqrt{V}}$$

This approximation
is called
"ALLOWED" APPROXIMATION

$$H_{if} = \frac{1}{V} \int \underbrace{\Psi_s^* H \Psi_p d\gamma}_{\text{Short range interaction}} = \frac{1}{V} M_{if}$$

(77)

Nuclear matrix element

$\lambda \propto \frac{dN}{dE_f} \Rightarrow$ Density of states of final energy E_f
 \Rightarrow Slope of the distribution of p_{te} .

How many are them?

If we assume a 3dimensional space with coordinates p_x, p_y, p_z the locus of a point which represent a specific momentum p can be seen as

$$|p| = \sqrt{p_x^2 + p_y^2 + p_z^2} \Rightarrow \text{a sphere of radius } p.$$

The locus of points which represent a sphere of radius $p+dp$ is a spherical shell of radius p & thickness dp , with a volume = $4\pi p^2 dp$.
 If the electron is confined within a box a volume V (\Rightarrow Only to cancel out normalization) \Rightarrow The number of electrons in final state, dN_e , with a momenta from p to $p+dp$ is:

$$dN_e = \frac{4\pi p^2 dp V}{h^3}$$

where h^3 is introduced to have a pure number.

The same is valid for ν :

$$dN_\nu = \frac{4\pi p^2 dp V}{h^3}$$

So dN (both for e^- and ν) is

$$dN = \frac{4\pi p_e^2 dp_e V}{h^3} \cdot \frac{4\pi p_\nu^2 dp_\nu V}{h^3} = \frac{(4\pi)^2 p_e^2 p_\nu^2 dp_e dp_\nu V^2}{h^6}$$

So, the partial decay rate is:

$$d\lambda = \frac{2\pi}{h} g^2 |M_{fi}|^2 (4\pi)^2 \frac{p_e^2 dp_e p_\nu^2}{h^6} \cdot \frac{dp_\nu}{dE_f}$$

$$M_{fi} = \int \Psi_f^* H \Psi_i \Rightarrow \text{nuclear matrix element}$$

$$E_f = m_e c^2 + E_K^e + E_\nu \equiv m_e c^2 + k + p_\nu c \quad dE_f = dp_\nu c$$

$$\text{So } \frac{dp_\nu}{dE_f} = \frac{1}{c} \quad \text{At a fixed } E_p$$

The number of particles at a determined k

$$N(k) dk = C_0 p_e^2 p_\nu^2 dp_e$$

We have to use relativistic expression

$$p_e^2: \quad E_e^2 = p_e^2 c^2 + m_e^2 c^4 = (k + m_e c^2)^2 = k^2 + 2k m_e c^2 + m_e^2 c^4$$

$$p_e^2 c^2 = k (k + 2 m_e c^2)$$

$$p_e^2 = \frac{1}{c^2} k (k + 2 m_e c^2) \quad [k = E_K \text{ of electron}]$$

$$p_\nu^2: \quad Q = k + p_\nu c$$

$$p_\nu = \frac{Q - k}{c} \quad p_\nu^2 = \frac{(Q - k)^2}{c^2}$$

$$dp_e = \frac{1}{c} \frac{1}{2\sqrt{k(k+2m_e c^2)}} (2k + 2m_e c^2) dk$$

$$N(k) dk = C_0 \frac{k(k+2m_e c^2)(Q-k)^2 (k+m_e c^2)}{\sqrt{k(k+2m_e c^2)}} dk$$

$$N(k) = C_1 \sqrt{k(k+2m_e c^2)} (Q-k)^2 (k+m_e c^2)$$

PREDICTION OF
FERMI THEORY OF
 β -decay. (18)

This expression is \approx ok for β^- , but not ok for β^+ : up to now we have considered e^- and e^+ as the same ptc. Where the \neq nature of the 2 ptc enters in our calculations? The Coulombs have not been considered up to now \Rightarrow The ψ_e have to be modified.

$$\psi_e = \frac{1}{\sqrt{V}} e^{i \frac{\vec{p} \cdot \vec{r}}{\hbar}}$$

\rightarrow Coulomb interaction between e^\pm and the nucleus must be included, and the ψ_e have to be multiplied by the "Fermi Function" $F(z_d, k)$

\uparrow
 z (Atomic number) of daughter nucleus

Here we will use NON-RELATIVISTIC expressions, why? Because Coulomb interaction have effect only in the low-energy part of the interaction, where the velocity are "small" ($\ll c$)

$$F(z_d, k) = \frac{2\pi M}{1 - e^{-2\pi M}}$$

$$M = \pm \frac{z_d e^2}{4\pi \epsilon_0 \hbar v} \quad \textcircled{*} \quad v = \text{Speed of } e^\pm$$

+ : β^- decay

- : β^+ decay

With the inclusion of such expression:

$$N(k) = C_1 \sqrt{k(k+2m_e c^2)} (Q-k)^2 (k+m_e c^2) \cdot F(z_d, k)$$

How large is M : $\frac{z_d e^2}{4\pi \epsilon_0 \hbar v} = \frac{z_d \cdot 1.44 \text{ MeV.fm}}{\hbar c} \cdot \sqrt{\frac{m_e c^2}{2k}} = \frac{z_d \cdot 1.44}{200} \sqrt{\frac{0.511}{2k}}$

$$\frac{e^2}{4\pi \epsilon_0} = 1.44 \text{ MeV.fm} \quad \text{thus: } \frac{1}{2} m v^2 = k \rightarrow v^2 = \frac{2k}{m} \quad \hbar v = \hbar c \sqrt{\frac{2k}{mc^2}}$$

What is still missing is the effect of the nuclear matrix element, M_{fi} , which up to now has been considered to not influence the shape of the spectrum. This approximation is called "ALLOWED APPROXIMATION" is often a very good approximation, but there are some cases M_{fi} vanishes in the allowed approximation giving no shape at all. In such cases the plane wave function have to be taken into account introducing an additional momentum dependence.

Such cases are called "FORBIDDEN DECAYS": these decays are not absolutely forbidden but they occur with less probability \Rightarrow larger lifetime.

The degree to which a transition is forbidden depends on how far the expansion of the W.F. ^{we} have to go. \Rightarrow 1st term is first-forbidden decays, next term is second forbidden decay and so on.

Angular momentum and parity selection rules restrict the kinds of decay can occur.

\Rightarrow The complete β^- spectrum includes 3 terms:

- 1) The statistical factor $p_e^2(Q-k)^2$ from the number of final states accessible to the emitted particles
- 2) The Fermi function $F(z_d, p)$ which accounts for nuclear Coulomb field
- 3) The nuclear matrix element $|M_{fi}|^2$ which accounts for the effects of initial and final nuclear states and that could include an additional terms which depends on $p_e \cdot p_\nu$

$$N(p) \propto p^2(Q-k)^2 F(z_d, p) |M_{fi}|^2 S(p_e, p_\nu)$$

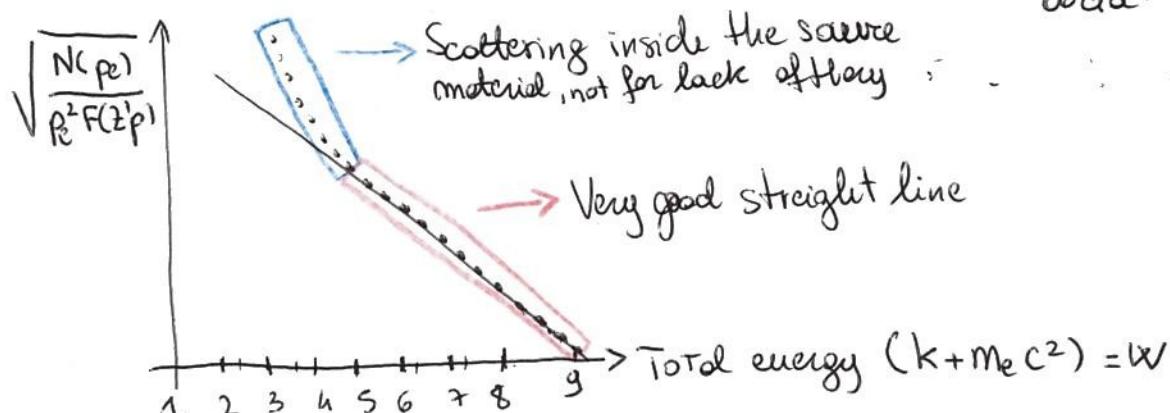
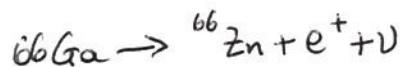
THE "CLASSICAL" EXPERIMENTAL TESTS OF THE FERMI THEORY

SHAPE OF β SPECTRUM

The equation of the $N(p)$ can be rewritten as (in the allowed approximation)

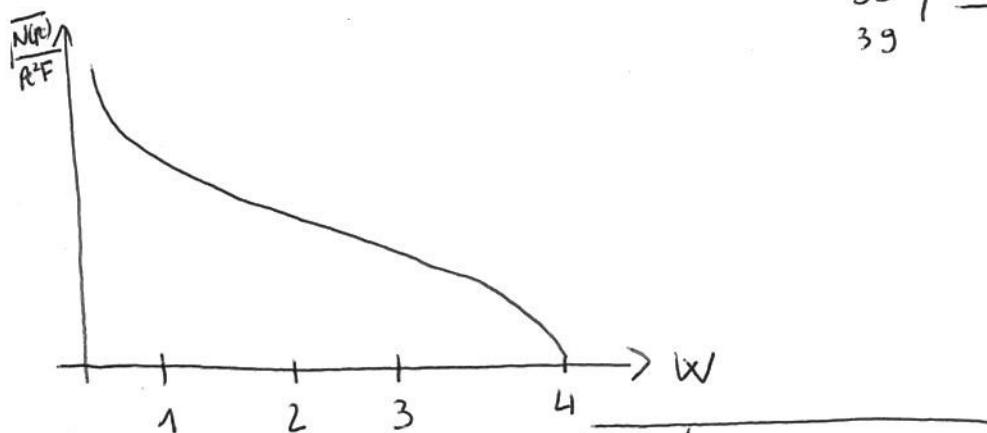
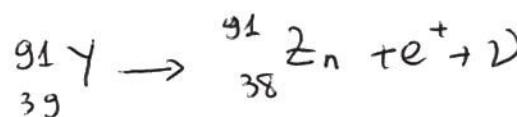
$$(Q - K) \propto \sqrt{\frac{N(p_e)}{p_e^2 F(z' p)}}$$

\Rightarrow plotting $\sqrt{\frac{N(p_e)}{p_e^2 F(z' p)}}$ vs K should be a straight line which intercepts the x-axis at the decay energy Q . Such plot is called Kurie plot (or Fermi-Kurie plot or Fermi plot). As an example



The Fermi-Kurie plot is ok for many β decays.

In case of forbidden decays, the standard Kurie plot does not give a straight line.



Linearity can be restored if $\sqrt{\frac{N(p_e)}{p_e^2 F(z' p)}} S(p_e, p_w)$ is plotted

$S(p_e, p_w)$ is called SHAPE FACTOR

