

ALPHA DECAY

(65)

α were first identified as the least penetrating radiation emitted by naturally occurring materials.

In 1903 Rutherford measured their charge/mass ratio deflecting them in electric and magnetic fields. In 1909 Rutherford showed that α particle are He-4 nuclei.

Many nuclei, especially those of naturally occurring radioactive series, decay through α emission.

The emission of other nuclei than ${}^4\text{He}$ is extremely rare, so there must be a reason why nuclei choose α emission over other decay modes.

α emission is a Coulomb repulsion effect. It becomes increasingly important for heavy nuclei because Coulomb increase with size (dZ^2) ^{after} the binding energy $\propto A$.

α is emitted "spontaneously" because it's very stable and has a relatively small mass w.r.t. the original nucleus. (\Rightarrow it's kinematics!)

E.g. if we consider an alpha-emitter like ${}^{232}\text{U}(72\gamma)$ we can compute

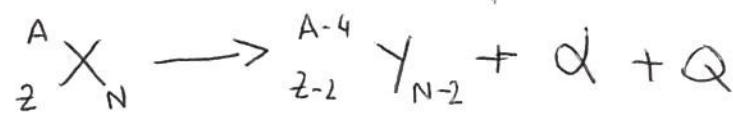
EMITTED PTC	Energy released (MeV)
n	- 7,26
$\text{p}({}^1\text{H})$	- 6,12
$\text{d}({}^2\text{H})$	- 10,70
$\text{t}({}^3\text{H})$	- 10,24
${}^3\text{He}$	- 9,92
${}^4\text{He}$	+ 5,41
${}^5\text{He}$	- 2,59

so the emission with α is the only energetically favoured. For lighter nuclei (like ${}^{12}\text{C}$ or ${}^8\text{Be}$) the energy is > 0 , but the weight of these decays are way smaller than the decay with α .

This suggest that to be an α -emitter a nucleus does not only obey to the rule "energetically favoured", but also ~~less~~ other conditions have to be found. I.e. the disintegration constant must not be too small, otherwise α emission will occur rarely. The $t_{1/2}$ of an α -emitter must then be $\leq 10^{16}$ y. Most nuclei with $A > 190$ (and many with $150 < A < 190$) are energetically unstable under α emission, but only $\frac{1}{2}$ meet the other requirements. (disintegration constant not too small / $t_{1/2} \leq 10^{16}$ y).

BASIC α DECAY PROCESSES

The spontaneous emission of an α ptc can be represented as:



From the conservation of motion law it is possible to evaluate the momentum of X and Y

$$P_Y = P_\alpha = P$$



The Kinetic Energy

$$\frac{P_Y^2}{2m_Y} + \frac{P_\alpha^2}{2m_\alpha} = Q$$

$$\frac{P^2}{2} \left(\frac{1}{m_Y} + \frac{1}{m_\alpha} \right) = Q$$

$$\frac{P^2}{2} = \left(\frac{m_\alpha * m_Y}{m_Y + m_\alpha} \right) Q$$

$$E_{k\text{ of}} \rightarrow k_\alpha = \frac{P^2}{2m_\alpha} = \frac{Q m_Y}{m_Y + m_\alpha} = Q \quad \frac{A-4}{A} = Q \left(1 - \frac{4}{A} \right)$$

$$\rightarrow k_Y = \frac{P^2}{2m_Y} = \frac{Q m_\alpha}{m_Y + m_\alpha}$$

α energy is a more easily accessible quantity

E_α is what it is measured in the detector.

For $A \approx 200$ $Q \left(1 - \frac{1}{200}\right) = 0.98Q \Rightarrow$ The α takes most of the momentum. The

remaining nucleus takes only 2% of the total energy released in the reaction as recoil energy.

This recoil energy is not entirely negligible. For a Q-value of 5 MeV the recoil nucleus has $E_y \approx 100$ keV. This $E_y \gg$ B.E. of atoms in solids \Rightarrow if close to the surface the nucleus can escape from the source.

If the decay is part of an decay chain \Rightarrow the daughter is radioactive and the recoil might end with a spread of radioactive material.

Q-value vs half-life time

Already in the '20 it was observed that α emitters with large disintegration energies have short half-lives and conversely,

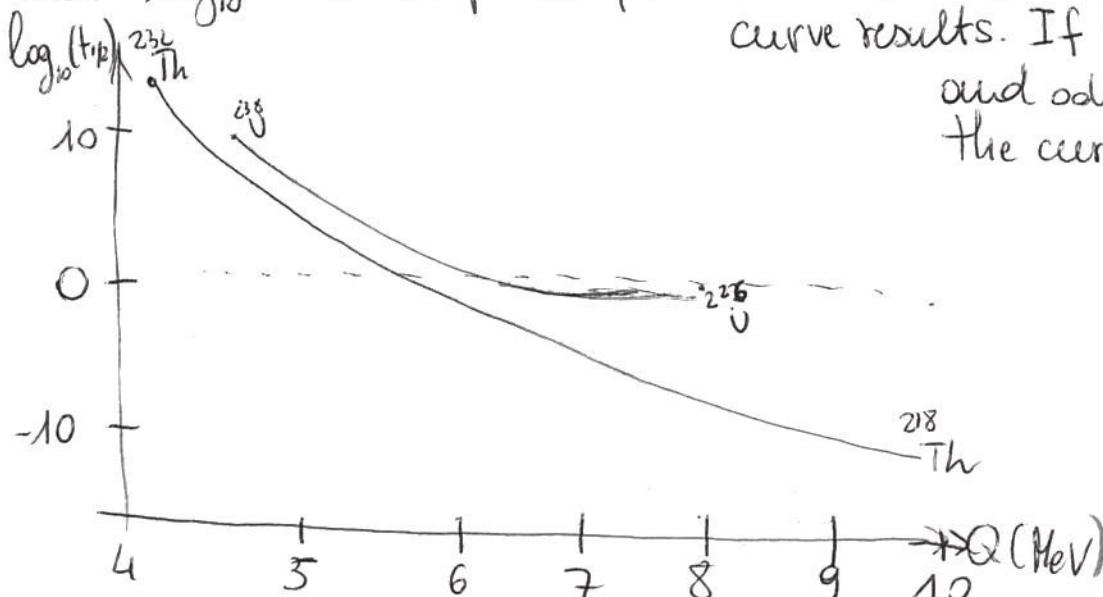
But, Q-values range between 4-10 MeV for α emitters, while $t_{1/2}$ varies from μs to Gy (giga-years)!

E.g. ^{232}Th ($t_{1/2} = 1.4 \times 10^10 \text{ y}$; $Q = 6.08 \text{ MeV}$)

^{218}Th ($t_{1/2} = 1 \times 10^{-7} \text{ s}$; $Q = 9.85 \text{ MeV}$)

The theoretical explanation of this effect (1928 Geiger-Nuttal) was one of the first triumphs of QM. (see later)

When $\log_{10} t_{1/2}$ vs Q is plotted for even-even nuclei a very smooth curve results. If Even-odd, odd-even and odd-odd are shown the curve is not that smooth



Q-VALUE vs A

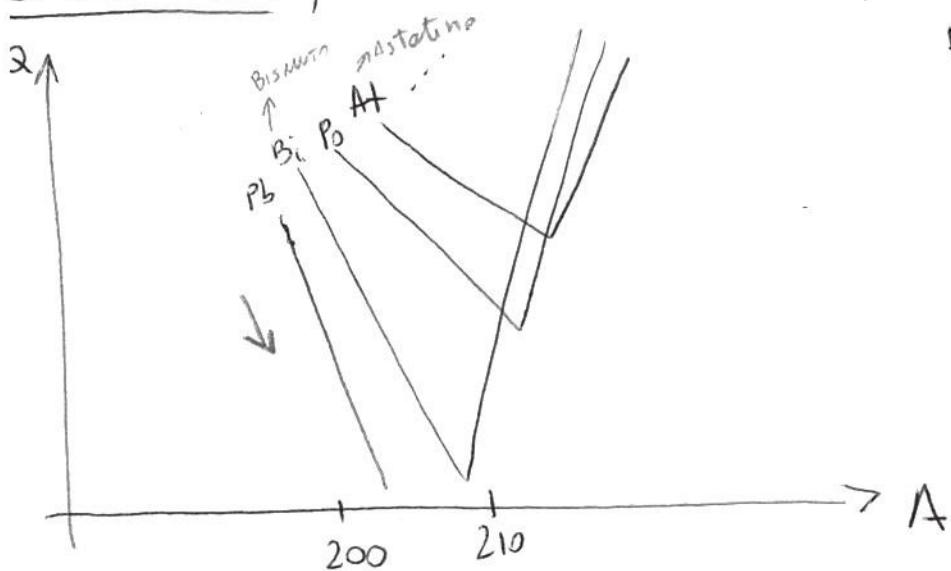


Fig 8.2 KRANE

For $A > 212$, adding a neutron to the nucleus reduces the disintegration energy \Rightarrow increase the lifetime and so the nucleus become more stable

A discontinuity is seen for $A = 212$, where $Z = 126$ (i.e. magic number!) If the data are compared to the systematic dependence of Q and A

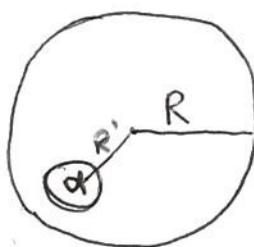
$$Q = B(^4\text{He}) + B(Z-2, A-L) - B(Z-A)$$

$$\approx 28.5 - 4\alpha_v + \frac{8}{3}\alpha_s A^{1/3} + 4\alpha_c Z A^{-1} (1 - 2/3A) - 4 \text{asym} \left(1 - 2 \frac{Z}{A}\right)^2 + 3 \alpha_p A^{-7/4}$$

A fair comparison is obtained (e.g. ^{232}Th) $Q_{\text{FARHATA}} = 5.71 \text{ MeV}$ $Q_{\text{HASS}} = 6.081$ indicating that all the assumptions we have done up to now are correct!

THEORY OF α EMISSION

The basic assumption of the theory is that the α particle is already formed inside the parent nucleus and that it can move outside it. There is no specific reason to believe this is what actually happens, but since it works well especially for even-even nuclei, there is no reason why to believe this is wrong.



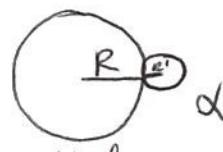
R is the parent nuclear radius

r is the distance between the center of the nucleus and the center of the α ptc.

Under the assumption that α is already inside the nucleus, inside Coulomb potential is \ll than nuclear force. Outside, it's true the opposite.

$V(r)$ = Nuclear attractive potential $r < R$

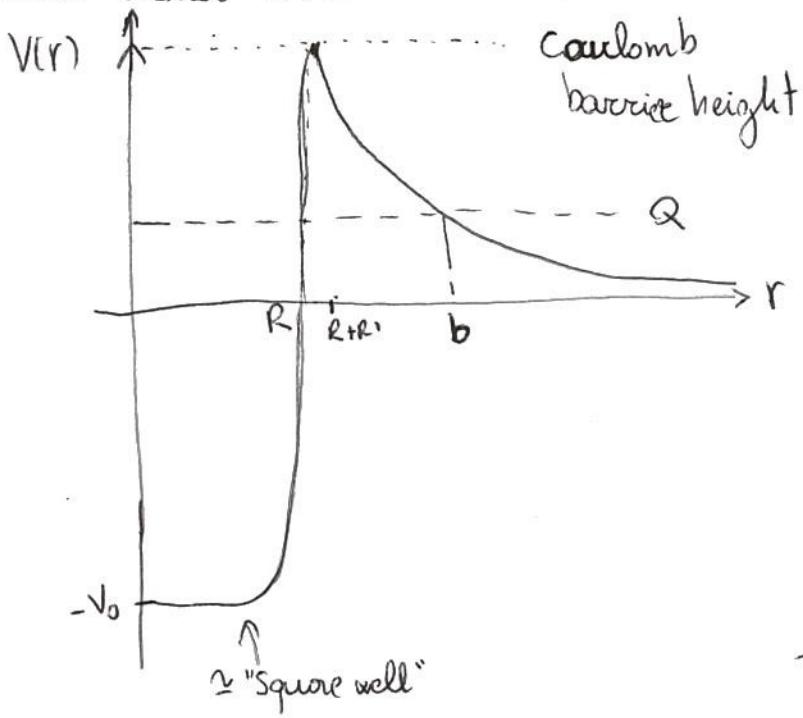
$\frac{1}{r}$ = Coulomb repulsive potential $r > R + R'$



Nucleus

Where R' is the α -ptc radius

Between R and $R+R'$ there should be "something"



$V_0 \approx 30-60$ MeV

Coulomb potential:

$$V(r) = \frac{(Z-2)e \cdot 2e}{4\pi\epsilon_0 r}$$

Nucleus

→ hyperbolic behavior

The potential is (as usual) in 3-d, but we will use only the central potential assumption ($\ell=0$)

The energy of the α -ptc is $\approx 4-5$ MeV.

Let's estimate other numbers which have to be considered. Let's see how high is the Coulomb barrier when $R = R + R'$

$$V_c = \frac{2(Z-2)e^2}{4\pi\epsilon_0 r}$$

≈ 1.64 MeV fm

$$\text{Let's use } A = 200 \rightarrow R = (1.25 \text{ fm}) A^{1/3} \\ = 1.25 \times 6 \text{ fm} \approx 7.5 \text{ fm}$$

$Z = 72$

$$V_c = \frac{140 \cdot 1.64 \text{ MeV fm}}{9 \text{ fm}} \approx 25 \text{ MeV}$$

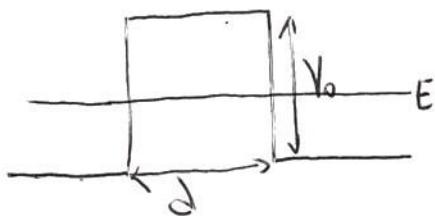
$R' = 1.5 \text{ fm}$

$r = R + R' \approx 9 \text{ fm}$

It ranges between 20 and 30 MeV

$Q \approx 4-8 \text{ MeV} \ll V_0 \Rightarrow$ The region $R=b$ is classically forbidden
 And the barrier can be surmounted or
 better penetrated only using the
 tunnelling \Rightarrow There is a probability for the
 α to go outside the nucleus

Let's evaluate it: The text book example is like this:



The probability to penetrate the barrier
 is $e^{-2\gamma d}$

$$P_{\text{TUNNEL}} = e^{-2\gamma d}$$

$$\gamma = \sqrt{\frac{2m}{\hbar^2} (V_0 - E)}$$

Let's assume now that the region $[R-b]$ can be divided into infinitesimal rectangles so that γd becomes $\int \gamma(r) dr$ and

$$\gamma = \sqrt{\frac{2m}{\hbar^2} (V_0 - E)} \quad \text{with} \quad V = \frac{(Z-2)2e^2}{4\pi\epsilon_0 r}$$

The probability becomes:

$$P_{\text{TUNNEL}} = e^{-2 \int \gamma(r) dr} = e^{-2G}$$

$$\gamma(r) = \sqrt{\frac{2m}{\hbar^2} (V(r) - E)} \quad \parallel \quad E = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 b}$$

$$G = \sqrt{\frac{2m}{\hbar^2} \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0} \int_R^b \frac{1}{r} - \frac{1}{b} dr}$$

I

To solve I we have to use substitution method using

$$r = b \sin^2 \theta$$

$$I = \int \sqrt{\frac{1}{b} (\cot^2 \theta)} 2b \sin \theta \cos \theta d\theta =$$

$$= \int 2\sqrt{b} \cos^2 \theta d\theta = \sqrt{b} \int (1 + \cos 2\theta) d\theta = \sqrt{b} \left(\theta + \frac{\sin 2\theta}{2} \right)$$

limits of integration: $r=R$ and $r=b$

$$r=R \rightarrow \sin \theta = \sqrt{\frac{R}{b}}$$

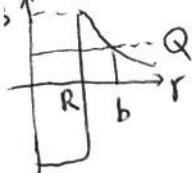
$$r=b \rightarrow \sin \theta = \frac{\pi}{2}$$

$$G = \sqrt{\frac{2m}{h^2} \frac{z_1 z_2 e^2}{4\pi\epsilon_0}} \sqrt{b} \left[\frac{\pi}{2} - \left[\sin^{-1} \left(\sqrt{\frac{R}{b}} \right) \right] + \sqrt{\frac{R}{b}} \sqrt{1 - \frac{R}{b}} \right] =$$

$\cos^2 \theta = 1 - \sin^2 \theta$

$$= \sqrt{\frac{2m}{h^2} \frac{z_1 z_2 e^2}{4\pi\epsilon_0}} \sqrt{b} \left[\frac{\pi}{2} - \sin^{-1} \sqrt{\frac{R}{b}} + \sqrt{\frac{R}{b}} \left(1 - \frac{R}{b} \right) \right]$$

If we define $B = \frac{z_1 z_2 e^2}{4\pi\epsilon_0 R} \Rightarrow R = \frac{z_1 z_2 e^2}{4\pi\epsilon_0 B} \quad \frac{R}{b} = \frac{Q}{B}$



$$Q = \frac{z_1 z_2 e^2}{4\pi\epsilon_0 b} \Rightarrow b = \frac{z_1 z_2 e^2}{4\pi\epsilon_0 Q}$$

$$G = \sqrt{\frac{2m}{h^2} \frac{z_1 z_2 e^2}{4\pi\epsilon_0} \frac{z_1 z_2 e^2}{4\pi\epsilon_0 Q}} \left[\cos^{-1} \sqrt{\frac{Q}{B}} - \sqrt{\frac{Q}{B} \left[1 - \frac{Q}{B} \right]} \right]$$

$Q \approx 5 \text{ MeV}$

$B \approx 30 \text{ MeV}$

$\frac{Q}{B}$ small

$\frac{Q^2}{B^2} \rightarrow$ can be neglected

$$G \approx \frac{z_1 z_2 e^2}{4\pi \epsilon_0} \sqrt{\frac{2m}{h^2 Q}} \left[\underbrace{\frac{\pi}{2} - \sqrt{\frac{Q}{B}}}_{\downarrow} - \sqrt{\frac{Q}{B}} \right]$$

Using the relation $\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x \approx \frac{\pi}{2} - x$

$$G \approx \frac{z_1 z_2 e^2}{4\pi \epsilon_0} \sqrt{\frac{2m_\alpha}{h^2 Q}} \left[\frac{\pi}{2} - 2\sqrt{\frac{Q}{B}} \right] \approx \frac{z_1 z_2 e^2}{4\pi \epsilon_0} \sqrt{\frac{2m_\alpha \cdot 2}{h^2 m_\alpha v_\alpha^2}} \left[\frac{\pi}{2} \right]$$

$$\rightarrow Q = \frac{m_\alpha \cdot v_\alpha^2}{2}$$

$$G \approx \frac{z_1 z_2 e^2}{4\pi \epsilon_0 h v_\alpha}$$

\rightarrow After many stages of approximations

$$P = e^{-2G}$$

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

$\lambda = p$ of decay \times unit time

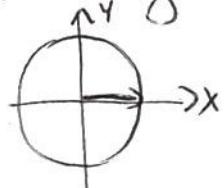
||

$$\lambda = f \cdot p$$

||
of attempt
 \times unit time

probability
for attempt

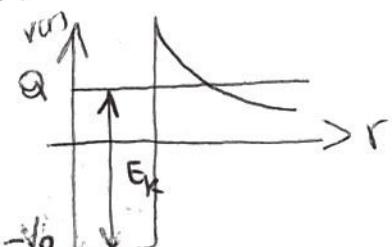
How large is f ? Let's use a semi-classical approach: If the α moves along x



$$f = \frac{v_0}{2R}$$

, where v_0 is the velocity of α inside the nucleus \neq outside the nucleus.

v_0 can be determined



$$E_k = E_{pot} \Rightarrow \frac{1}{2} m v_0^2 = V_0 + Q$$

$$mv_0 = \sqrt{\frac{2(V_0 + Q)}{m}} \Rightarrow$$

$$f = \sqrt{\frac{Q + V_0}{2mR^2}} \quad \lambda = P f$$

$$\Rightarrow \lambda = \sqrt{\frac{Q + V_0}{2mR^2}} \exp[-2G]$$

Putting all together one gets (for Th isotopes)

A	Q	$t_{1/2}$ calculated	$t_{1/2}$ measured
220	8,95	$3,3 \cdot 10^{-7}$	10^{-5}
...			
232	6,08	$2,6 \cdot 10^{16}$	$4,4 \times 10^{17}$

\Rightarrow The calculation is not exact, but is able to reproduce the trend of the $t_{1/2}$ within 1-2 orders of magnitude over a range of more than 20 orders of magnitude.

The approximations we did, do not consider:

- 1) The ψ_i e ψ_f (wave functions) needed in the correct evaluation of decay probability
- 2) The angular momentum carried by the α
- 3) We assumed spherical nuclei with $r = R_0 A^{1/3}$ even if for $A \geq 30$ they are quite deformed.

Even if the calculations are not fully corrected and oversimplified, a good estimate of the $t_{1/2}$ is reached.

It also enable us to understand why other decays into light pte are not commonly seen, even if allowed by the Q-value.

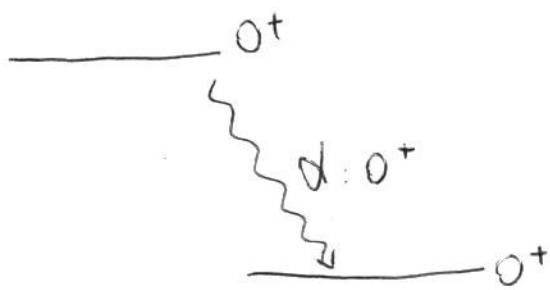
E.g. $^{220}\text{Th} \rightarrow ^{12}\text{C} + ^{208}\text{Po}$ would have a Q value of 32,1 MeV,

so a $t_{1/2} = 2,3 \times 10^6$ s for ^{220}Th decay into ^{12}C which is 13 orders of magnitude larger than the decay with $\alpha \Rightarrow$ Not easily observable

ANGULAR MOMENTUM AND PARITY IN α DECAYS

We have neglected up to now to discuss the angular momentum carried by the α ptc. In a transition from ~~I_i~~ ^{initial state} with angular momentum I_i to a final state F with angular momentum I_f , the angular momentum of α can range between $|I_i + I_f|$ to $|I_i - I_f|$.

α is a ptc composed by 2p and 2n. (even-even) $\Rightarrow I_\alpha = 0^+$ ptc. For even-even nuclei which emits α and go into a ground-state nuclear we can say

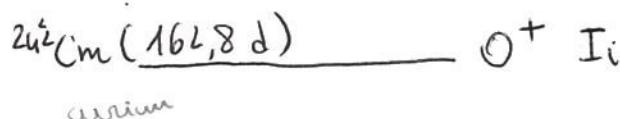


and there are no violations of the parity selection rule:

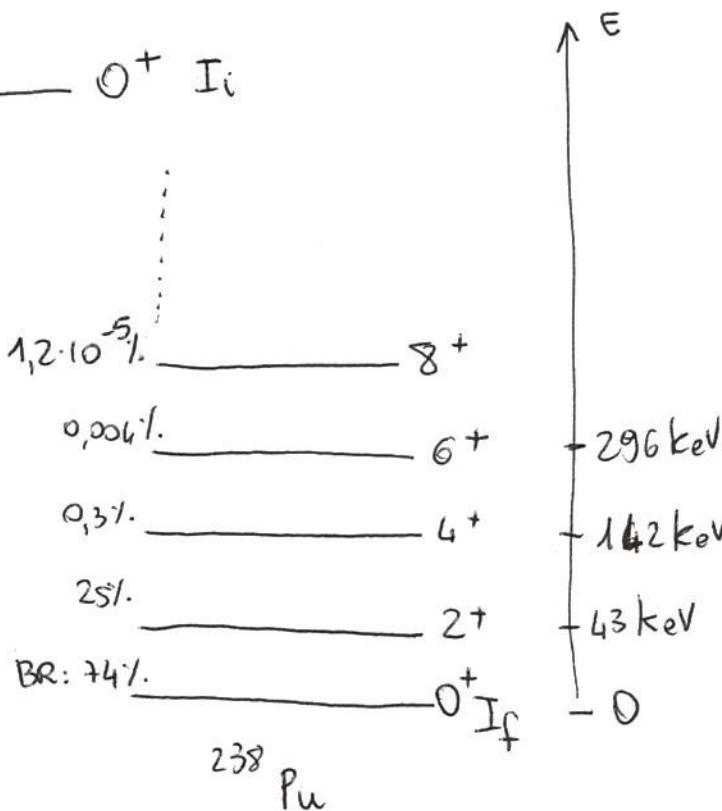
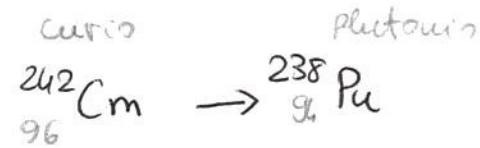
If the initial and final parities are the same I_α must be even otherwise it must be one. The last d must be $(-1)^{I_\alpha}$.

The situation can get more complicated we consider the fact that after an α decay the daughter ptc can be ^{many} excited state. (Where the spin-parity can be $\neq 0^+$ even for even-even nuclei).

As an example let's consider the α decay of



curium



If the α decays in the excited state 2^+ it means that its angular momentum is 2^+ . This could happen but it has a "price" to be paid. The price is the terms that depends from angular momentum in the potentials.

$$V_{\text{eff}} = V(r) + \frac{l(l+1)\hbar^2}{2mr^2}$$

↑
central

\Rightarrow If the term that depends from angular momentum is included \Rightarrow The potential increases \Rightarrow The probability decreases (and \Rightarrow the B.R.).

\Rightarrow The same can be applied also for other states \Rightarrow The most probable excited state which will be populated will be 2^+ .

In case of even-even nuclei there are some decays which are absolutely forbidden by the parity selection rules. For example $0^- \rightarrow 3^-$ must have $l_a = 3$ which means that the π of initial and final state must be different \Rightarrow

$0^+ \rightarrow 3^-$ is allowed while $0^+ \rightarrow 3^+$ is forbidden.

For even-odd nuclei the situation is more complicated, and there are absolutely forbidden decays.

For example the decay $2^- \rightarrow 2^+$ must have odd π and angular momentum between $0 \leq l_a \leq 4$. \Rightarrow The only allowed values are $l_a = 1$ or 3 .

Which is the favored channel? From even-even nuclei it could be that the lowest l state is the most favored by one order of magnitude.

To determine the relative contributions of different l it is necessary to measure ^{angular} distributions. $l=1$ α -ptc will be $\propto Y_1(\theta, \varphi)$ while $l=3$ will be $\propto Y_3(\theta, \varphi)$: if we can measure angular distribution we could determine relative amounts of l values.

How? 1) Align spins in a magnetic field (\rightarrow but to keep them aligned a $T=0, 0.1$ K is needed!!)
 2) Measure angular distributions.

This was done e.g. for $^{253}\text{Es} \rightarrow ^{249}\text{Bk}$ 70b

Einsteinium Berkelium

$\frac{7/2^+}{\alpha} \xrightarrow{253}\text{Es}$

$\frac{17/2^+}{\alpha} \quad 6(0,0004)$

$\frac{15/2^+}{\alpha} \quad 4(0,0083)$

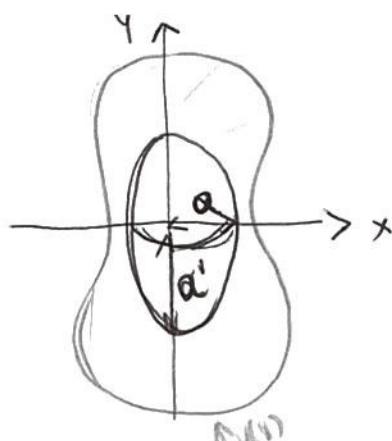
$\frac{13/2^+}{\alpha} \quad 4(0,083)$

$\frac{11/2^+}{\alpha} \quad 4(0,88)$

$\frac{9/2^+}{\alpha} \quad 2(59)$

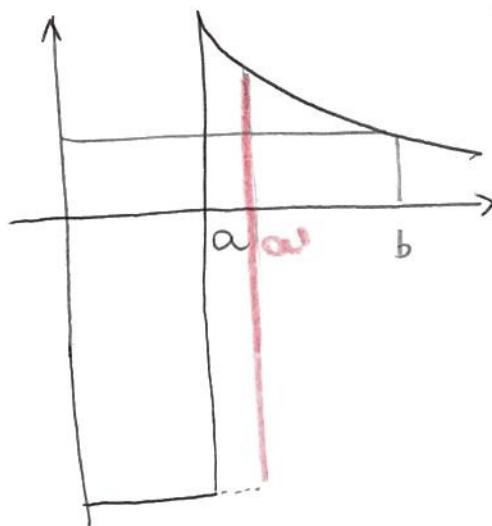
$\frac{7/2^+}{249}\text{Bk} \quad 0(78,6), 2(10.0), 4(0.3), 6(0.002)$
ld %.

Finally, we can see what happens to deformed nuclei. In fact, many α -emitter nuclei are deformed.



In which direction is it more probable that α ptc are emitted? (H1a)

The coulomb effect is larger at poles and lower at the equator =>



At the poles the barrier is lower! => The missions from poles is 3-4 more probabilities than the emission from equator!

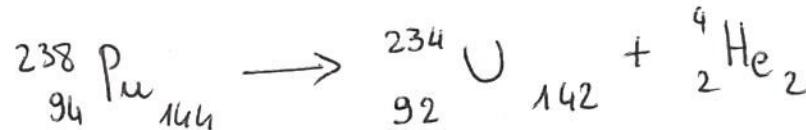
EXERCISE:

Plutonium (^{238}Pu , $Z=94$) has been used as power source in space flight.

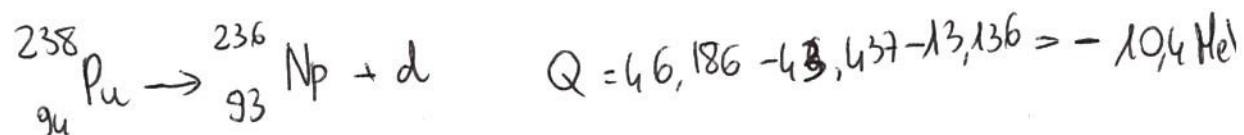
^{238}Pu has a decay half-life of 90γ ($2,7 \cdot 10^9$ sec).

- a) What are the Z and N of the nucleus which remains after the decay?
- b) Why is ^{238}Pu more likely to emit α 's instead of β 's?
- c) Each α is emitted with 5.5 MeV. What is the power released if there are 238 g of ^{238}Pu ($\Rightarrow 6 \cdot 10^{23}$ atoms)?
- d) If the power source of c) produces 8 times the minimum required to run a piece of the apparatus, how long the fuel will be enough to run that system?

- a) The daughter will have $N=142$ and $Z=92$



- b) This happens because the binding energy of α . For ^{238}Pu ,



Deuteron decay is forbidden!

- c) Because of the recoil of ^{234}U , the decay per ^{238}Pu is

$$E_\alpha = E_\alpha + E_U = \frac{P_\alpha^2}{2m_\alpha} + \frac{P_U^2}{2m_U} = E_\alpha \left(1 + \frac{M_\alpha}{M_U} \right) = 5,5 \text{ MeV} \left(\frac{238}{234} \right) = 5,6 \text{ MeV}$$

$$t_{1/2}(\text{f}^{238}\text{Pu}) = 90\gamma = 2,7 \cdot 10^9 \text{ s} \Rightarrow \lambda = \frac{\ln 2}{t_{1/2}} = 2,57 \cdot 10^{-10} \text{ s}^{-1}$$

For 238 g the energy released per second is:

$$\frac{dE}{dt} = E_d \quad \frac{dN}{dt} = E_d \times N_0 = 5,6 \text{ MeV} \cdot 2,57 \cdot 10^{40} \text{ s}^{-1} \cdot 6 \cdot 10^3 = 8,6 \cdot 10^{44} \text{ MeV/s}^{(72)}$$

5) As the amount of nuclei attenuates, so does the power output:

$$W(t) = W(0) e^{-\lambda t}$$

$$\text{When } W(t_0) = \frac{W(0)}{8}$$

$$t_0 = \ln 8 / \lambda = \frac{3 \ln 2}{\lambda} = 3 t_{1/2} = 270 \text{ y}$$

\Rightarrow The apparatus can run for 270 y.