

THE "CLASSICAL" EXPERIMENTAL TESTS OF THE FERMI THEORY

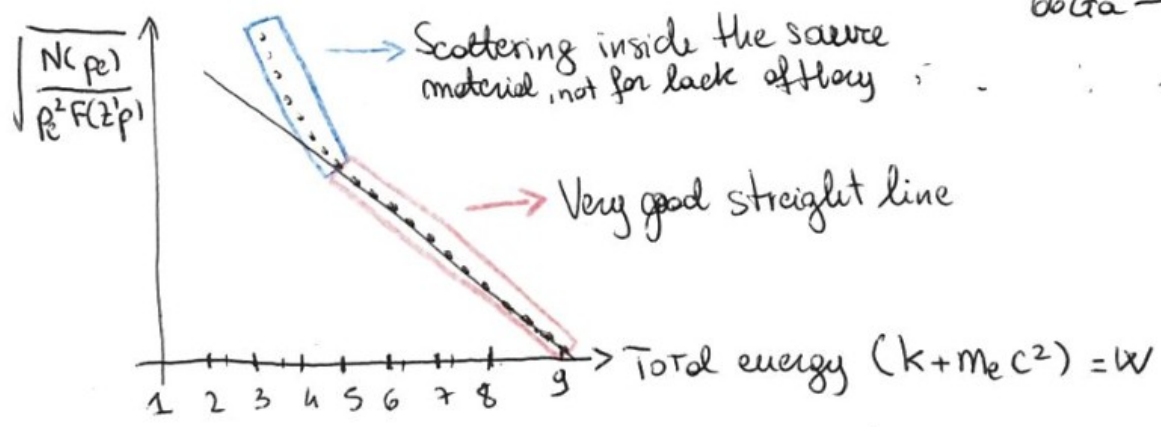
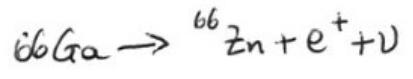
SHAPE OF β SPECTRUM

The equation of the $N(p)$ can be rewritten as (in the allowed approximation)

$$(Q-K) \propto \sqrt{\frac{N(p_e)}{p_e^2 F(Z', p)}}$$

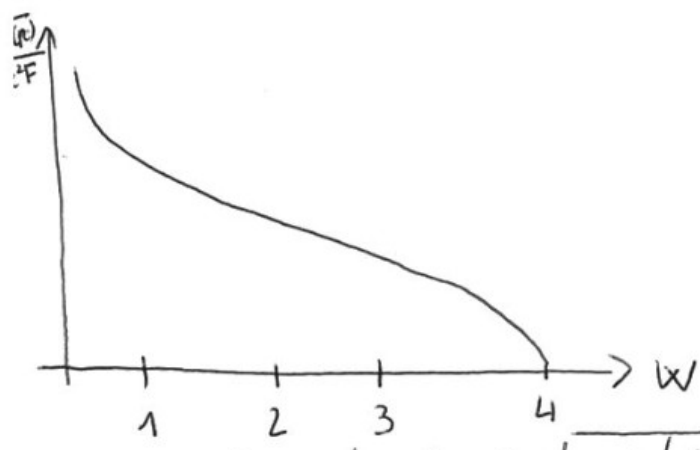
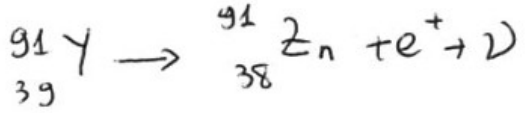
\Rightarrow plotting $\sqrt{\frac{N(p_e)}{p_e^2 F(Z', p)}}$ vs K should be a straight line, which intercepts

the x-axis at the decay energy Q . Such plot is called Kurie plot (or Fermi-Kurie plot or Fermi plot). As an example

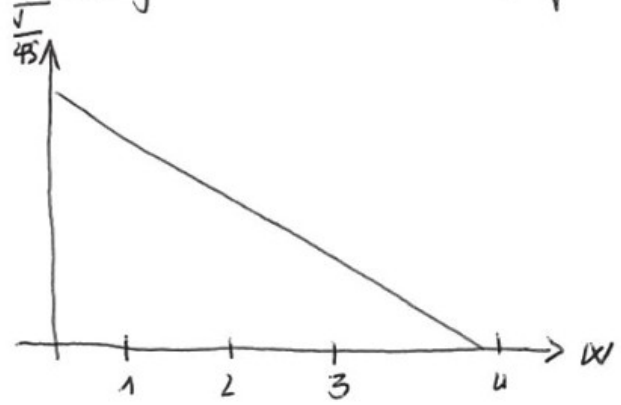


The Fermi-Kurie plot is ok for many β decays.

In case of forbidden decays, the standard Kurie plot does not give a straight line.



linearity can be restored if $\sqrt{N(p_e) / p_e^2 F(Z', p) S(p_e, p_\nu)}$ is plotted



$S(p_e, p_\nu)$ is called SHAPE FACTOR

TOTAL DECAY RATE

To find total decay rate the $d\lambda$ expression $d\lambda = \frac{2\pi}{h} g^2 |M_{fi}|^2 (4\pi)^2 \frac{p_e^2 dp_e p_\nu^2}{h^6} \frac{dq}{dE_f}$ have to be integrated

$$\lambda = \frac{g^2 |M_{fi}|^2}{2\pi^3 h^7 c^3} \int_0^{p_{max}} F(Z', p_e) p_e^2 (Q - T_e)^2 dp_e$$

The integral will depend only on Z' and on E_0 ($p_{max} = \sqrt{E_0^2 - m_e^2 c^4}$) so

$$f(Z', E_0) = \frac{1}{(mc)^3 (mc)^2} \int_0^{p_{max}} F(Z', p_e) p_e^2 (E_0 - k)^2 dp_e$$

Where all the constants are included to make f without dimensions. $F(Z', E_0)$ is called Fermi integral and is tabulated for values of Z' and E_0 . With $\lambda = 0,69/t_{1/2} \Rightarrow$

$$ft_{1/2} = 0,69 \frac{2\pi^3 h^7}{g^2 m_e^5 c^4 |M_{fi}|^2} \frac{A}{|M_{fi}|}$$

Comparative half-life or ft value

\Rightarrow it gives a way to compare β decay probabilities in nuclei i.e. in different $M_{fi} \Rightarrow$ In nucleus w.f.

Also in β -decay there is an enormous range of half-lives in β decay: ft ranges from 10^3 to 10^{20} s. FROM MIN TO GY

The decays with the shortest comparative half-lives are known as SUPERALLOWED decays. In case of superallowed decays with ψ_i in 0^+ and ψ_f in 0^+ f_i is easy to be calculated and is $\sqrt{2}$. In case of $0^+ \rightarrow 0^+$ decays the ft value should be a constant. \Rightarrow Within uncertainties that is the case. In case of $0^+ \rightarrow 0^+$ the decay constant g can be evaluated:

$$g = 0,88 \times 10^{-4} \text{ MeV} \cdot \text{fm}^3$$

$$J \cdot m^3 = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} \cdot \text{m}^3$$

$\hookrightarrow N \cdot m$

Let's compare it with other variables \rightarrow We have to make it dimensionless

$$[G] = M L^5 T^{-2}$$

$$h = J \cdot s = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} \cdot \text{s} = M L^2 S^{-1} \quad c = m/s$$

And there is no combinations of $h [M L^2 T^{-1}]$ or $c [L T^{-1}]$ which can be used to convert g into a dimensionless variable.

E.g. $\hbar c^3$ has a dimension $ML^2T^{-1} \cdot L^3T^{-3} = ML^5T^{-4}$ so $\frac{g}{\hbar c^3} = T^2$ (80)

We can introduce an arbitrary mass and choose the exponentials i, j, k such that $g/m^i \hbar^j c^k$ is dimensionless.

If $i=-2, j=3$ and $k=-1 \Rightarrow G = \frac{g}{m^2 \hbar^3 c^{-1}} = g \frac{m^2 c}{\hbar^3} \checkmark$

The mass that should be used is not clear. For nucleon-nucleon interaction the nucleon mass should be used. In this way $G = 1,0 \cdot 10^{-5}$.

The strong constant for π is ≈ 1 , so ordering the interactions:

pion - nucleon	1
e.m.	10^{-2}
β -decay (weak)	10^{-5}
gravitational	10^{-39}

Fermi theory is able to reproduce many aspects of the weak interaction, while it fails for others: a theory which describe the weak interaction in terms of exchange ptc (i.e. bosons) is more successful in describing such interaction.

THE MASS OF THE NEUTRINO

The Fermi theory is based on the assumption that the rest mass of ν is 0. This hypothesis can be tested by measuring the energy of β^+ and β^- decays at the end point and see how it behaves.

When we evaluated the NCK in Fermi's theory we said that if $m_\nu = 0 \Rightarrow$

$$E_\nu = p_\nu c \Rightarrow \frac{dN}{dE_f} = c p_\nu^2 p_e^2 dp_e \frac{dp_\nu}{dE_f}$$

$$\frac{dp_\nu}{dE_f} = \frac{1}{c} \Rightarrow \text{This won't be valid if } m_\nu \neq 0. \text{ In this case}$$

$$E_\nu = m_\nu c^2 + \frac{p_\nu^2}{2m_\nu}$$

$$E_f = m_e c^2 + m_\nu c^2 + k + \frac{p_\nu^2}{2m_\nu} \quad dE_f = \frac{p_\nu dp_\nu}{m_\nu} \quad (\text{AT A FIXED } k)$$

$\Rightarrow \frac{dp_\nu}{dE_f} = \frac{m_\nu}{p_\nu} \neq \frac{1}{c} \Rightarrow$ In this case the shape of the spectrum would change considerably

$$p_\nu = \sqrt{2m_\nu(Q-k)} \quad \frac{p_\nu^2}{2m_\nu} = Q-k$$

$$\frac{dn}{dE_f} = C p_e^2 dp_e p_\nu$$

↑
Before it was p_ν^2 !

$$\Rightarrow N(k) = C_1 \underbrace{\sqrt{k(k+2m_e c^2)}}_A \underbrace{(k+m_e c^2)}_B \cdot p_\nu$$

$$p_\nu = \begin{cases} (Q-k) & \text{if } m_\nu = 0 \quad \textcircled{A} \\ \sqrt{2m_\nu(Q-k)} & \text{if } m_\nu \neq 0 \quad \textcircled{B} \end{cases}$$

The effect of the ν mass will be visible in the tail part.

If we evaluate $\frac{dN}{dk}$ we would get $N(k)$ vs Q

Ⓐ $m_\nu = 0$

$$\frac{dN}{dk} = C_1 \left[\sqrt{k(k+2m_e c^2)}(k+2m_e c^2) \cdot 2(Q-k) + (Q-k)^2 \cdot \frac{d}{dk} [A] \right]$$

When $Q=k$ both terms are 0 \Rightarrow the $N(k)$ will approach 0 horizontally

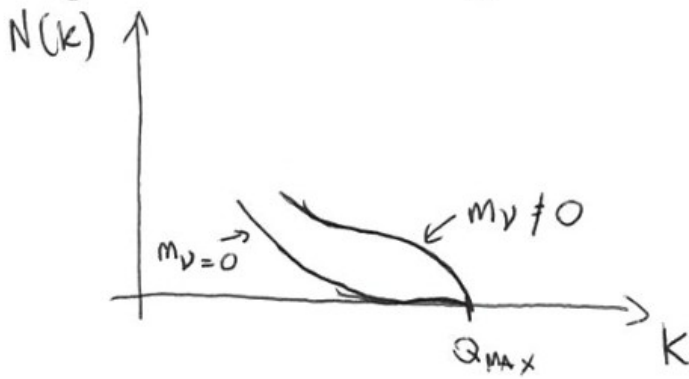
Ⓑ $m_\nu \neq 0$

$$\frac{dN}{dk} = C_1 \left\{ \left[A \cdot \frac{d}{dk} B \right] + \sqrt{2m_\nu(Q-k)} \cdot \frac{d}{dk} A \right\}$$

$\frac{1}{2m_\nu \sqrt{Q-k}} \Rightarrow$ When $Q=k$ the ^{2^o} term goes to ∞
 \Rightarrow Drastic change in slope!

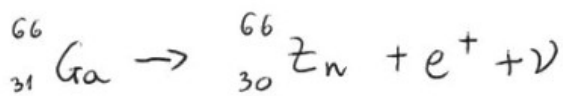
Studying the end-point region the expected behaviour is

(81)



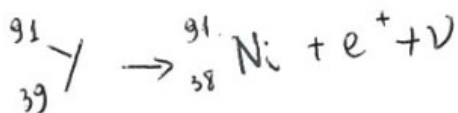
ANGULAR MOMENTUM AND PARITY SELECTION RULES

So, we have observed that:



OK FERMI/KURIE PLOT

$$0^+ \rightarrow 0^+$$



NOT OK FERMI-KURIE PLOT

$$\frac{1}{2}^- \rightarrow \frac{5}{2}^+$$

\Rightarrow The essential physics difference is from J^π !

$$I_P^\pi \rightarrow I_D^\pi + \underbrace{J(e + \nu)}_{\text{This makes the difference.}}$$

ALLOWED DECAYS

In the allowed approximation ψ_e and ψ_ν are expanded such that they start from the origin: $\psi_i = \frac{1}{\sqrt{V}} \left[\underset{\uparrow}{1} + \frac{\vec{p}_i \cdot \vec{r}}{\hbar} + \dots \right]$

In this case e and ν CANNOT carry any orbital momentum, and the only change in the angular momentum of nucleus must result from spins of e^- and ν , each of which has $s = \frac{1}{2}$.

The spins can be parallel $S=1$ or antiparallel $S=0$.

If $S = 0 \uparrow \downarrow \Rightarrow$ FERMION DECAY \Rightarrow In the allowed approximation $l=0$ $l=0$

$$\Delta I = |I_i - I_f| = 0 \Rightarrow I_p^\pi = I_d^\pi \quad \text{ALLOWED } \beta \text{ DECAYS}$$

If $S = 1 \uparrow \uparrow \Rightarrow$ GAMOW-TELLER DECAY

$$\Rightarrow I = 1 \quad (\text{TOTAL ANGULAR MOMENTUM } \vec{I} = \vec{L} + \vec{S})$$

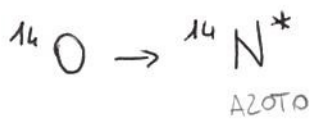
$$\vec{I}_R = \vec{I}_F + \vec{1} \Rightarrow \begin{cases} \vec{I}_d = +1 \\ \vec{I}_d = 0 \\ \vec{I}_d = -1 \end{cases}$$

Selection rules:

$$\Delta I = 0, 1$$

$$\Delta \pi \text{ (parity change)} = \text{no}$$

Examples of allowed β decay:



this is a $0^+ \rightarrow 0^+$ decay to an excited state of ${}^{14}\text{N} \Rightarrow$ This is a pure Fermi type allowed decay



this is a $0^+ \rightarrow 1^+$ decay: pure Gamow-Teller allowed decay (Other example: ${}^{13}_5\text{B} \rightarrow {}^{13}_6\text{C} (\frac{3}{2}^- \rightarrow \frac{1}{2}^-)$ or ${}^{230}_{89}\text{Pa} \rightarrow {}^{230}_{90}\text{Th}^* (2^- \rightarrow 3^-)$ protactinium)



$$\Delta I = 0 \left(\frac{1}{2}^+ \rightarrow \frac{1}{2}^+ \right) \text{ Both Fermi and Gamow Teller}$$

This is an example of mixed F+GT transitions where the exact proportion of F and GT is determined by the initial and final w.f.

It might be convenient to define the ratio γ of amplitudes (\Leftrightarrow matrix elements)

$$\gamma = \frac{G_F (M_F)}{G_{GT} (M_{GT})} \text{ Nuclear matrix elements}$$

If $g_F = g$ (defined for super allowed $0^+ \rightarrow 0^+$ decays) and for $n \rightarrow p$ decay

$$|M_F| = 1 \Rightarrow \gamma \propto G_F^2 M_F^2 (1 + \gamma^2) \text{ . From neutron decay rate}$$

$$\text{one can evaluate } \gamma = 0,467 \pm 0,003 \Rightarrow 82\% \text{ GT ; } 18\% \text{ F.}$$

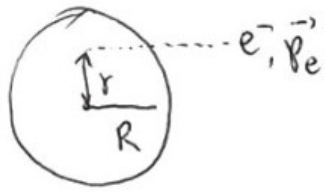
FORBIDDEN DECAYS

These decays are not actually forbidden, but as we saw they are less probable, but allowed if ^{cases where} the matrix element ~~does not~~ vanishes.

The most frequent occurrence of forbidden decays is when the initial and final states have opposite parities \Rightarrow the selection rules for allowed decay is violated.

To accomplish the change in parity e^- and ν must be emitted with an odd angular momentum relative to the nucleus.

What does it mean? $l = 1, 3, 5, \dots$



The angular momentum is defined as $\vec{p}_e \cdot \vec{r}$

The maximum angular momentum is obtained when the e^- is closest to the surface and \perp to the direction of \vec{r} ; in this case $l = p_e \cdot R \Rightarrow$ MAX l is for max angular momentum

Let's consider a 1 MeV electron ($\equiv k = 1 \text{ MeV}$)

$$\underbrace{\sqrt{p^2 c^2 + m_e^2 c^4}}_{\text{TOTAL ENERGY}} = \underbrace{m_e c^2 + k}_{\text{mon + kinetic energy}}$$

$$p^2 c^2 + (0,5)^2 \text{ MeV}^2 = (0,5+1)^2 \text{ MeV}^2$$

$$p^2 c^2 = (2,25 - 0,25) \text{ MeV}^2 \quad p_{\text{MAX}} \approx 1,4 \text{ MeV}/c$$

$$l_{\text{max}} = p_e \cdot R = 1,4 \text{ MeV}/c \cdot 5 \text{ fm} \Rightarrow \frac{\sqrt{l(l+1)} \hbar^2}{\sqrt{l_{\text{max}}^2}} = 7 \frac{\text{MeV} \cdot \text{fm}}{c \hbar}$$

$$\sqrt{l(l+1)} = \frac{7 \text{ MeV} \cdot \text{fm}}{\hbar c} = \frac{7 \text{ MeV} \cdot \text{fm}}{200 \text{ MeV} \cdot \text{fm}} \approx \frac{1}{3} \approx 0,3$$

$l(l+1) = 0,3^2 \quad l \ll 1 \quad \rightarrow$ It is unlikely to have $l=1$ w.r.t. $l=0$ but this

is more likely to occur than $l=3, 5, \dots$

Note that in this core $l=0$ is ACTUALLY FORBIDDEN!

$l=1$	First - forbidden		Also in this core decays can be	
$l=2$	Second - forbidden		FERMI	GAMOW-TELLER
$l=3$	Third - forbidden		\downarrow $S=0$	\downarrow $S=1$

For not-allowed β decay the selection rules are

$$l \neq 0 \Rightarrow \Delta I = 0, 1, 2 \quad \Delta \pi = \text{Yes}$$

E.g. $l=1 \quad \Delta \pi = \text{YES}$
 $S=0; J=1 \Rightarrow \Delta I = 0, \pm 1 \quad \equiv 1^{\text{st}}$ forbidden Fermi transitions

$l=1 \quad \Delta \pi = \text{YES}$
 $S=1; J=2, 1, 0 \Rightarrow \Delta I = 0, \pm 1, \pm 2 \quad \equiv 1^{\text{st}}$ forbidden Gamow-Teller transitions

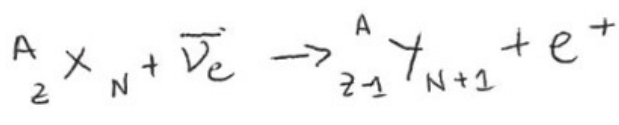
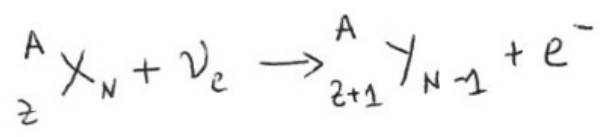
In cases of not-allowed transitions we already saw that the $\psi_{e,\nu}$ have to be expanded to higher order:

$$\psi_e = \frac{1}{\sqrt{V}} \left(\underset{\substack{\uparrow \\ l=0}}{1} + i \frac{\vec{p}_e \cdot \vec{e}}{\hbar} \underset{\substack{\uparrow \\ l=0,1}}{\quad} + \left(i * \frac{\vec{p}_e \cdot \vec{e}}{\hbar} \right)^2 + \dots \right)$$

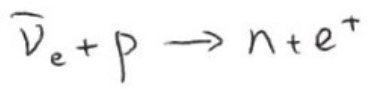
The inclusion of higher order l is included in the shape factor of the Fermi-Kurie plot.

THE DISCOVERY OF NEUTRINO

Shortly after the publication of the Fermi theory of β decay, Bethe & Peierls pointed out the ^{possibilities of} inverse β -decay. A nucleus capture a ν or $\bar{\nu}$ and eject e^- or e^+ .



The cross-sections of such inverse processes are expected to be extremely small due to the weakness of β decay. Let's consider ^{inverse} neutron decay.



The transition probability is given by:

$$\lambda = \frac{2\pi}{\hbar} |M_{fi}|^2 \frac{dn}{dE_f}$$

And the cross-section σ is defined as transition probability divided by the flux of incoming $\nu \Rightarrow$

$$\sigma = \frac{\lambda}{c} V = \frac{2\pi V}{\hbar c} |M_{fi}|^2 \frac{dn}{dE_f}$$

\uparrow Speed of ν \uparrow Volume

The difference w.r.t. what we have in neutron decay is in $\frac{dn}{dE_f} \Rightarrow$ Different probability for initial and final states.

Neglecting the recoil, but treating e^+ as a relativistic particle we obtain:

$$\frac{dn_e}{dE_f} = \frac{p_e^2}{2\pi^2 \hbar^3} \sqrt{\frac{dp_e}{dE_f}} = \frac{p_e^2}{2\pi^2 \hbar^3 c^2} \sqrt{E_e} = \frac{m_0^2 c V}{2\pi^2 \hbar^3} \omega \sqrt{\omega^2 - 1}$$

where ω is the reduced energy $\omega = E/m_0 c^2$. The M_{fi} correspond to that

of n in β decay in which both GT and F ~~are~~ contribution are present;

$$|M'_{fi}| = |M'_{fi}(F)|^2 + \frac{g_{GT}^2}{g_F^2} |M'_{fi}(GT)|^2$$

So the cross section is:

$$\sigma_c = \frac{g_F^2 m_0^2}{\pi \hbar^4} \left(|M'_{fi}(F)|^2 + \frac{g_{GT}^2}{g_F^2} |M'_{fi}(GT)|^2 \right) \cdot \omega \sqrt{\omega^2 - 1}$$

$$= \left(\frac{\hbar}{m_0 c} \right)^2 G^2 \left\{ |M'_{fi}(F)|^2 + \left(\frac{g_{GT}}{g_F} \right)^2 |M'_{fi}(GT)|^2 \right\} \omega \sqrt{\omega^2 - 1}$$

G_F is expressed in terms of G^2 . The σ_c is integrated in 4π (both of ν and e^+)

For $|M'_{fi}| \approx 5$ and the value of G , the $\sigma_c \approx$

$8 \cdot 10^{-44} \text{ cm}^2$	$E_\nu / m_0 c^2 = 4,5$
$20 \cdot 10^{-44} \text{ cm}^2$	$= 5,5$
$180 \cdot 10^{-44} \text{ cm}^2$	$= 10,8$

For $\bar{\nu}_e + p \rightarrow n + e^+$ reaction the threshold energy is $E_{\bar{\nu}_e} = 1,8 \text{ MeV}$.

For $E_{\bar{\nu}_e} = 2,8 \text{ MeV}$ the $E_{e^+} = 1 \text{ MeV}$

The mean free path $l = \frac{1}{n \cdot \sigma_c}$, where $n = \text{nuclei} \times \text{cm}^{-3}$.

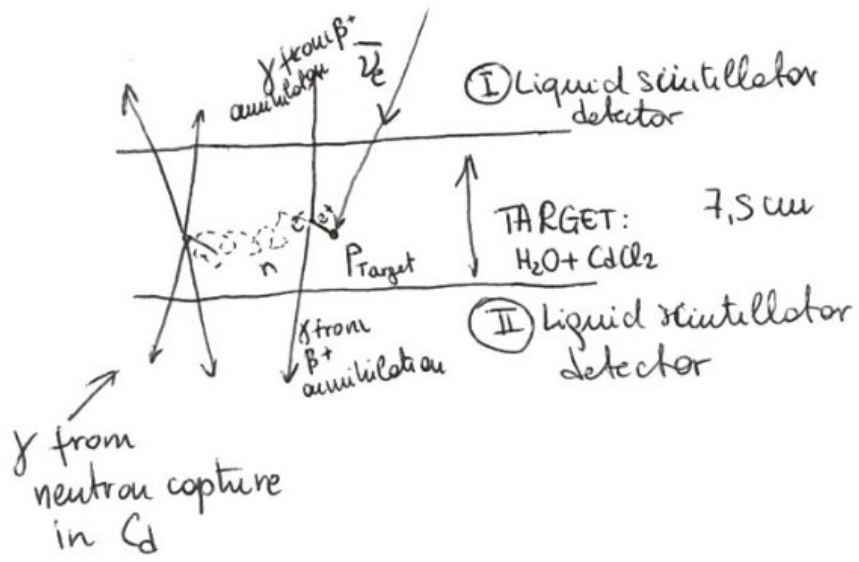
For protons in water $n \approx 3 \cdot 10^{22} \Rightarrow l \approx 3 \cdot 10^{20} \text{ cm} \approx 300 \text{ light years}$.

The reaction was studied by Cowan and Reines in the late 50's (58 and '59) using a 1000 MW reactor as source of $\bar{\nu}$. The flux of $\bar{\nu}$ was $\approx 10^{13} / \text{cm}^2 \cdot \text{s}$.

The target consisted on water containing $\approx 10^{28}$ p in which some CdCl_2 was dissolved: some reactions were observed. A positron produced in the capture reaction:

$\bar{\nu}_e + p \rightarrow n + e^+$ quickly annihilates with ^{an} electrons within 10^{-9} s producing $\approx 511 \text{ KeV}$ γ which travels in opposite directions.

The recoiling neutron slow down ~~reacts~~ through collisions with protons⁽⁸⁴⁾ and is then captured by Cd nuclei by the reaction $Cd(n, \gamma)Cd^*$ producing a γ with total energy = 8 MeV. The neutron capture process takes $\approx 10^{-5}$ s so a characteristic signature of 2 simultaneous 511 keV γ -ray + a neutron captured few μ s later indicates a capture reaction.



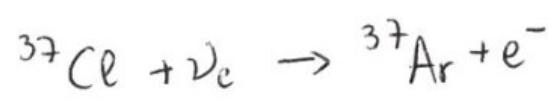
The expected rate of events is $F \cdot N \cdot \sigma_c \cdot \epsilon$ ← Efficiency of detection ($\approx 3 \cdot 10^{-2}$)

↑ Flux of $\bar{\nu}_e$ ↑ # of pins the target Capture cross-section

$R \approx 1$ / hours (\approx to what was observed).

ARE ν and $\bar{\nu}$ the SAME particle?

The experiment coined by Davis (1955) were based on the observation of inverse reaction of electron capture which for ^{37}Ar



15 out using $\bar{\nu}_e$. The experiment of Davis did not observed such reaction

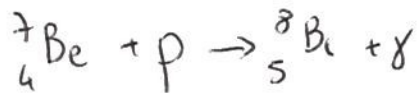
indicating that ν and $\bar{\nu}$ are not the same ptc.

Even though $\bar{\nu}$ capture by ^{37}Cl is a forbidden process, the reaction



could be used to detect a ν flux. ^{37}Ar which is formed is unstable and decays back to ^{37}Cl with a $t_{1/2} = 35$ days. The electron capture produces an excited ^{37}Cl with k-shell vacancy \Rightarrow Auger electron and x-ray emission.

This ν capture reaction was used to detect ν emitted in nuclear, stellar reactions, ... such as:



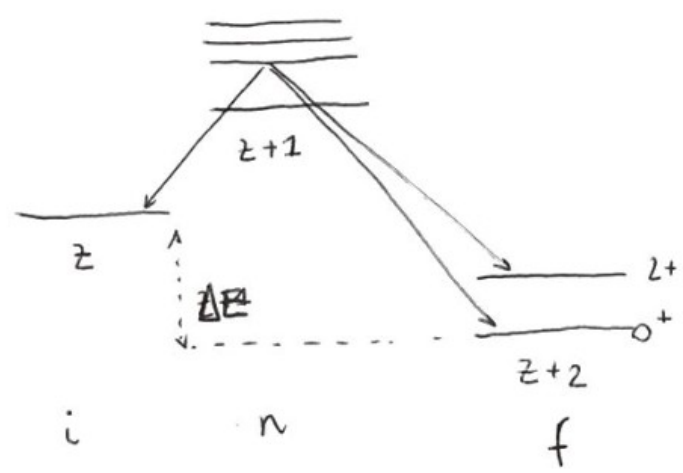
Which produce only a small number of ν in sun burning experiment but with enough energy to be initiate the ν capture in ^{37}Cl .

~~Solar~~ Experiment performed by Davis led to the surprising observation of the ν nuclear flux to 1/3 of the expected one. Opening the

"solar neutrino problem". [Solved by the possibility of ν to oscillate \Rightarrow they MUST have mass]

Double β decay

Near the bottom part of "mon valley" a double β decay can occur. It might happen if: ${}^A_Z X_N$ has an adjacent nucleus ${}^A_{Z+1} Y_{N+1}$ with higher mass while the nucleus ${}^A_{Z+2} Y'_{N-2}$ has a lower mass, with an energy difference ΔE .



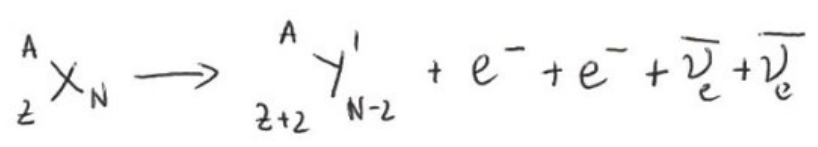
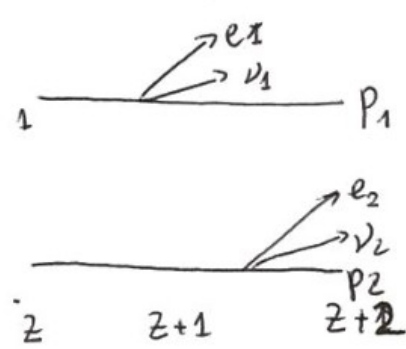
The ^{direct} transition from ${}^A_Z X$ to ${}^A_{Z+2} Y$ is forbidden, since β decay only changes a neutron into a proton.

Decay possibilities do exist as second-order decay processes, indicating that the intermediate state $|n\rangle$ are virtual states. It is possible to obtain:

$$\langle f | H_{int} | i \rangle = \sum_n \frac{\langle f | H_{int} | n \rangle \langle n | H_{int} | i \rangle}{E_i - E_n}$$

and a simple estimate for double-beta decay process is given by the square of the standard beta decay matrix element. In more complex calculations the individual matrix elements have to be evaluated.

Double-beta decay is a second order process:



with the emission of 2 electrons and 2 anti-neutrinos.

Nuclei which undergo $2\text{-}\beta$ decay: $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$ or ^{96}Zr , ^{100}Mo , ^{116}Cd , ^{124}Sn , ^{238}U .

The lifetime estimates are of the order of $T \approx 10^{20}$ y for a $\Delta E \approx 5 \text{ m.e.v.}$

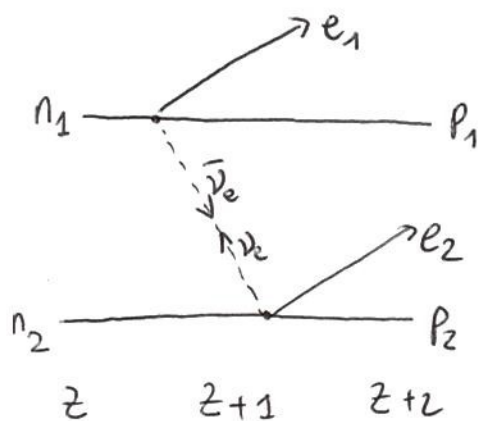
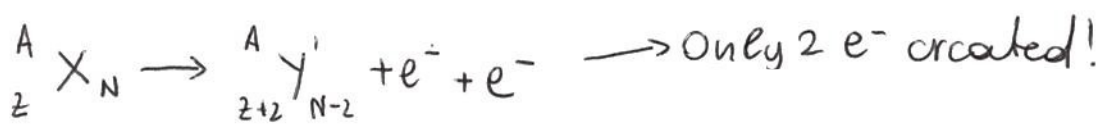
Double- β decays carry a number of very intriguing aspects related to neutrino physics. For example, it was suggested that in Relativistic Quantum mechanics a ν might exist in a way that the charge conjugate state of the original neutrino is identical to (Majorana) or different (Dirac) from the ν itself; i.e:

$$C|\nu_e\rangle \equiv |\bar{\nu}_e\rangle \equiv |\nu_e\rangle \quad \text{Majorana}$$

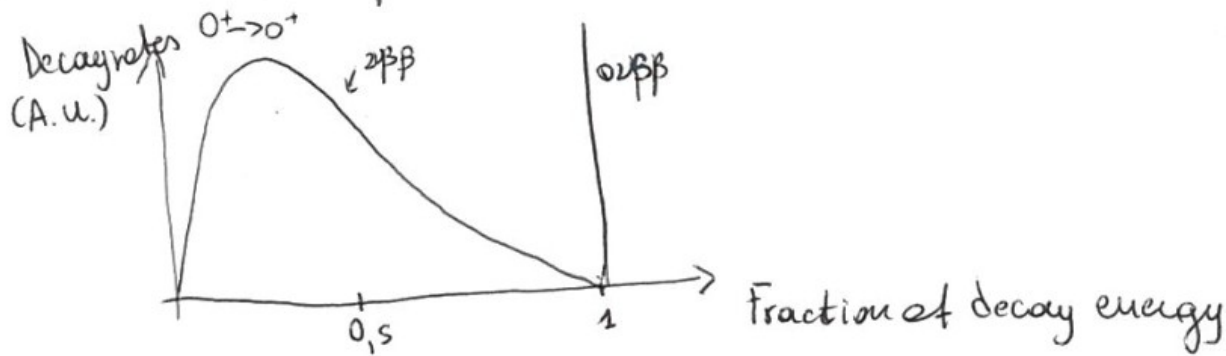
$$C|\nu_e\rangle \equiv |\bar{\nu}_e\rangle \neq |\nu_e\rangle \quad \text{Dirac}$$

Majorana particles appear in a natural way in GUT theories which unify strong and electro-weak interactions, with the possibility that the lepton number is no longer conserved.

A double-beta decay process can be imagined in a way where the emitted $\bar{\nu}$ is absorbed as ν giving rise to $2e^-$ process:



The energy sum of the 2 electrons is constant, in contrast with the usual classic process. The transition probability turns out to be $\approx 10^5$ shorter than the $(2e^-, 2\bar{\nu})$ process.



The distinction between Majorana and Dirac ptc was tested by Davis experiment. The reaction



were not observed. So Davis concluded that $\bar{\nu}_e$ was different from ν_e .

This distinction had to be changed in the light of Wu experiment, which indicated that the weak interaction violates the parity conservation, implying that ν has a specific helicity (LEFT-handed for ν and RIGHT-handed for $\bar{\nu}$)

In light of that, Davis experiment and the forbidden character of neutrinoless double beta decay can more easily be explained.

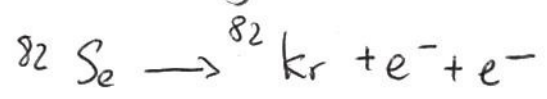
ν and $\bar{\nu}$ never match because of the difference in helicity needed in the reverse reaction:



likewise the neutrinoless double beta decay is forbidden even if ν has Majorana character. Note that all these statements are valid only if neutrino is massless, since for massive ptc helicity is a not fixed quantum number.

In fact in 1987 it was presented the first experimental evidence for

neutrino less beta decay:



such process has a lifetime $T \approx 1,1 \times 10^{20} \text{ y}$