NEUTRON INDUCED REACTIONS

Neutron are unaffected by Coulomb barrier, so they can easly penetrate the nucleus and initiate nuclear reactions even at very low energy (eV or less).

On the other hand the lack of Coulomb interaction present some experimen problems when they are used as nuclear probes.

Free neutrons are instable under weak decay, with a lifetime of ≃ 11 mi

-Neutron SOURCES

Neutrons camot be accelerated, but it is possible to start from high-energy neutrons and reduce their energy through collisions ~~of~~ with atoms of various materials. The slowing of n is called "moderating" of n.

The resulting neutrons can have very low energy which by convention are called:

THERMAL	$E \simeq 0{,}025$ eV
EPITHERMAL	$E \simeq 1$ eV
SLOW	$E \simeq 1$ keV
FAST	$E = 100$ keV ÷ 10 MeV

-Neutrons Sources

α-Berillium Source: The reaction responsible for the discovery of the neutron can be used to produce a source of n suitable for use in laboratory. The stable isotope of ^9Be has a relatively loosely bound neutron (1,7 MeV of B.E). Typical α ptc from radioactive decay (5-6 MeV) striking a ^9Be can produce a neutron:

Energy of n up to 15 MeV

$$^4He + {}^9Be \rightarrow {}^{12}C + n$$

PHOTONEUTRON SOURCE: Similarly to (α, n) source, it is possible to use (γ, n) reaction to produce n.

Energy of n ≃ 20 keV

$$\gamma + {}^9Be \rightarrow {}^8Be + n$$

- SPONTANEUS FISSION : E.g. [illegible] Cf (2,65y) spontaneus fission produces 4 n/fission. n produced with such process have $E \simeq 1\text{-}3$ MeV

- NUCLEAR REACTIONS

$$^3H + d \rightarrow {}^4He + n$$
$$^9Be + {}^4He \rightarrow {}^{12}C + n$$
$$^7Li + p \rightarrow {}^7Be + n$$
$$^2H + d \rightarrow {}^3He + n$$

≃ Monoenergetic n source at the desired energy

NEUTRON REACTIONS AND CROSS-SECTION

- The low-energy neutron cross-section shows a dependence $1/v$: Why?

Two different approaches can be used to reach the same conclusion.

Ⓐ $\sigma_{\text{Reaction}} = \pi (R + \lambda\!\!\!^{-})^2$

A primary modification to this estimate would include the reflection of incident nuclear w.f. at nuclear surface.

The transmission probability can be estimated as:

$$\sigma = \pi (R + \lambda\!\!\!^{-})^2 \frac{4 k_1 k}{(k_1 + k)^2}$$

V_0 = depth of the potential barrier

Where $k = \sqrt{2m(E + V_0)/\hbar^2}$ and $k_1 = \sqrt{2mE/\hbar^2}$

For low-energy neutrons $E \ll V_0$ and $k_1 \ll k$

$\lambda\!\!\!^{-} = k_1^{-1} \gg R$ So

$$\sigma = \pi (k_1^{-1})^2 * \frac{4 k_1 k}{k^2}$$

$$\sigma = \frac{4\pi}{k_1 k}$$

$$\sigma \simeq \frac{4\pi}{k k_1}$$

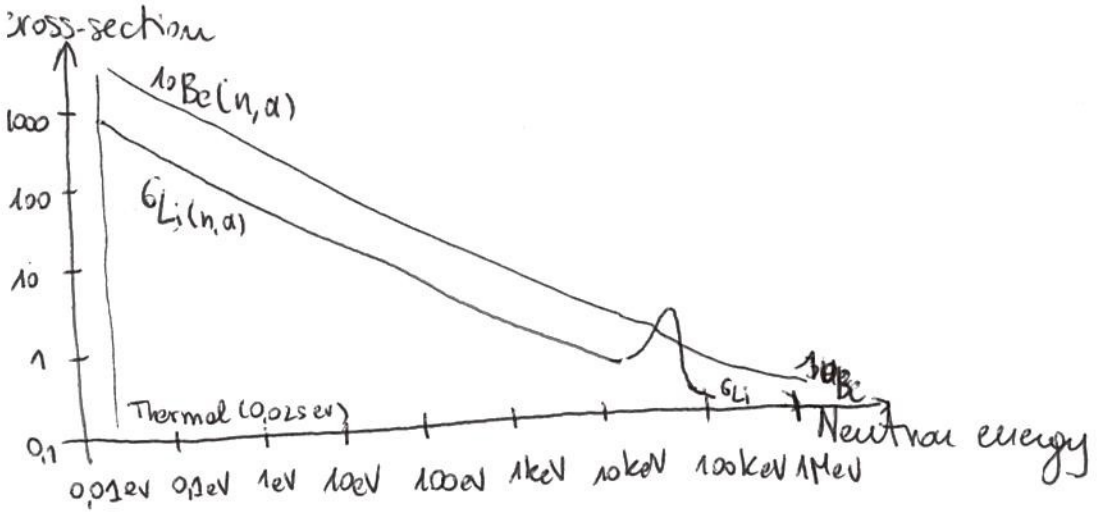
$$k_1 = \frac{p}{\hbar} = \frac{mv}{\hbar} \Rightarrow \sigma \propto \frac{1}{v}$$

A similar result can be obtained using the single-level resonance formula. Following neutron capture, the primary decay mechanism is γ emission, the probability of which is independent of any small variation in the resonance or indicident energy $\Rightarrow \Gamma$ is independent from neutron energy.

Γ_n (i.e. the neutron width which refers to the entrance channel) depends on dn/dE which is $\propto$ to $1/v$.

For from resonance $E \ll E_R$

$$\sigma \simeq \frac{\pi}{k^2} \frac{\Gamma_n \Gamma}{E_R^2 + \frac{\Gamma^2}{4}} \propto \frac{1}{v} \qquad \text{since } \Gamma_n \propto \frac{1}{v}$$



In the resonance region there is no exact theory for predicting the location of the resonance. The structure may be dominated by a single isolated resonance (e.g. $_{48}$Cd), or complex structure as in U

$\Rightarrow$ On line resource (oecd-nea.org/Janis) to look for updated cross-sections

NUCLEAR FISSION

The development of nuclear physics occurred very rapidly in the 1930s. Following the discovery of n by Chadwick in 1932 it was a natural next step to study the effect of exposing various nuclei to neutrons.

Pioneering work was done by Fermi & collaborators that exposed different materials to neutrons and studied the induced radioactivity following neutron capture.

They discovered that many nuclei decay by β^- emimission following neutron capture as they compensate for the extra neutron.

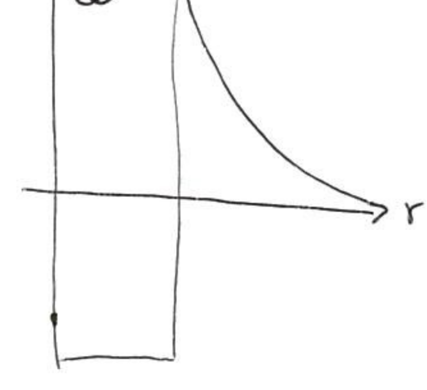
Next step was to use the neutrons to produce "transuranic" elements i.e. elements heavier then the heaviest naturally occurring element (i.e. the U)

Then it was discovered that if U is bombarded by neutron, after the n capture a large energy ($\simeq$100 MeV) is released (recall that α are emitted with 4÷5 MeV). Following this observation, Maitner and Frisch in 1939 propose that Uranium nuclei after the absorption of neutron are highly unstable and split nearly in half or "fission".

Fission results primarly from the competition between the nuclear and Coulomb forces in heavy nuclei.

The B.E. increases with A, while Coulomb repulsion of protons is faster, $\propto Z^2$.

We can regard the emission of heavy fragments as a decay process similar to α decay, then we can regard heavy nuclei as residing very close to the Top of the Coulomb barrier, where it is thin and easly penetrable.



Fission can occur spontaneously as a natural decay process or it can be induced through the absorption of a relatively low-energy particle.

WHY NUCLEI FISSION

The energetic preference for nuclei to fission can be understood from the B.E. per nucleon.

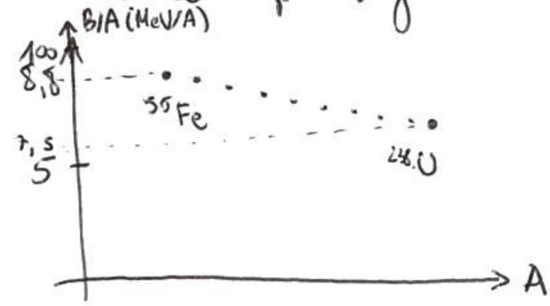
A heavy nucleus in the U region has a B.E. of $\simeq 7{,}6$ MeV/nucleon.

If ^{238}U divide into 2 equal fragments with $A \simeq 119$ their $BE/A \simeq 8{,}5$ MeV. Going to a more tighly bound system means that some energy have to be released; $^{238}_{92}U$ has $-7{,}6 \text{ MeV} \times 238 = -1809$ MeV of B.E, whil 2 nuclei of $^{119}_{46}Pd$ have a B.E. of $-2 \times 119 \times 8{,}5 \text{ MeV} = -2033$ MeV.

Palladio / Palladium

To conserve energy, the final state must include 214 MeV extra, which can appear in a variety of forms (n, β, γ emissions from fragments) but primarly ($\sim 80\%$) as kinetic energy of the fragments as Coulomb repulsion drive them apart.

In calculating decay probabilities, there is a term that depends on the energy release: the more the energy is released, the more ways there are for the decay products to share energy, the greater the number of final states to decay into and the higher the decay probability (i.e. $\frac{dn}{dE_f}$ increases). With such a large energy released, fission probability would be super big while the B.E. curve is "climbed"



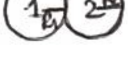
While the fission decay mode does exist it is not nearly probable as our discussion might indicate.

It cannot compete successfully with the Spontaneus α-decay of ^{238}U ($t^{\alpha}_{1/2} = 4{,}5 \cdot 10^{9}$ y, $t^{FISSION}_{1/2} = 10^{6}$ y).

Moreover fission is an important decay process only for $A > 250$.

What inhibits the fission process is the Coulomb barrier.

If we divide ^{238}U into 2 identical fragments at are just touching at their surface



with $R_1 = R_2 = 1{,}25\,(119)^{1/3} = 6{,}1$ fm

The Coulomb barrier is: $V = \frac{1}{4\pi\varepsilon_0}\frac{Z_1Z_2e^2}{R} = 1,44\ \text{MeV}\cdot\cancel{\text{fm}}\ \frac{(46)^2}{12,2\ \cancel{\text{fm}}} = 250\ \text{MeV}$

To be compared with E = 214 MeV.

The Coulomb barrier prevents the fragments from separating and the decay probability is small because the barrier cannot be penetrated.

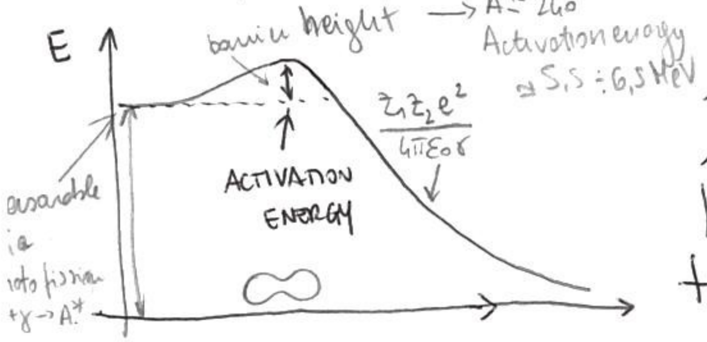
This is anyhow a very crude calculation, because the energy might vary by 10-20%.

For example if ^{238}U split into 2 ≠ particle such as $^{79}_{30}Zn$ and $^{159}_{62}Sm$ (samario), the Coulomb barrier reduces from 250 to 221 MeV. Also the Coulomb barrier calculation based on a sharp edge is quite unlikely to be correct.

What is certainly true is that the height of the Coulomb barrier is roughly equal to the energy released in fission, and that there are some nuclei that have a reasonably good chance to penetrate.

Other nuclei may be close to the barrier such that a little amount of energy is enought to induce fission.

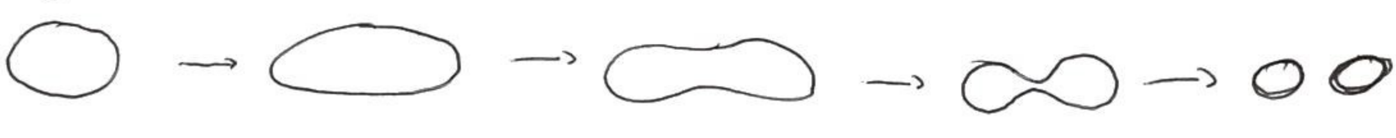
The ability of a nucleus to undergo induced fission will depend on the energy of the intermediate system. For some nuclei, absorption of thermal neutron (E ≃ 0,025 eV) may be sufficient to push them over the barrier, while for others fast (MeV) neutrons may be required.



A more detailed calculation of the energy needed to induce fission is shown here: this is based of ~~the~~ calculations based on "liquid-drop" model, which treats only average nuclear properties.

An istructive approach to understand fission can be obtained from semi-empirical mass formula.

Let's consider the effect on B.E. of an initially spherical nucleus that gradually stretch.



The stretched nucleus can be seen as an ellipsoid. It's volume is $\frac{4}{3}\pi a b^2$, where a is semimajor and b is semi minor axes.

The deviation of the ellipsoid from a sphere of radius R in terms of a distorsion parameter ε (eccentricity)

$$a = R(1+\varepsilon) \qquad R^3 = R(1+\varepsilon) b^2$$

$$b = R(1+\varepsilon)^{-1/2}$$

$$\nearrow \frac{4}{3}\pi R^3 = \frac{4}{3}\pi a b^2$$

$R^3 = ab^2 \Rightarrow$ the volume keeps constant as the distorsion increases.

As a sphere is stretched and distorted into an ellipsoid, its surface increases as $S = 4\pi R^2\left(1 + \frac{2}{3}\varepsilon^2 + \ldots\right)$ and then the difference in energy (i.e. the decrease in B.E) is:

$$\Delta E = B(\varepsilon) - B(\varepsilon = 0) =$$

$$= -a_s A^{2/3}\left(1 + \frac{2}{5}\varepsilon^2 + \ldots\right) - a_c Z^2 A^{-1/3}\left(1 - \frac{1}{5}\varepsilon^2 + \ldots\right) +$$

$$+ a_s A^{2/3} + a_c Z^2 A^{-1/3}$$

$$\simeq \left(-\frac{2}{5} a_s A^{2/3} + \frac{1}{5} a_c Z^2 A^{-1/3}\right)\varepsilon^2$$

If the second term is > than the first one ΔE is positive $\Rightarrow$ We gain Energy with the stretching process. Such nucleus will fission at the end $\Rightarrow$ Spontaneous fission occur if:

$$\frac{1}{5} a_c Z^2 A^{-1/3} > \frac{2}{5} a_s A^{2/3}$$

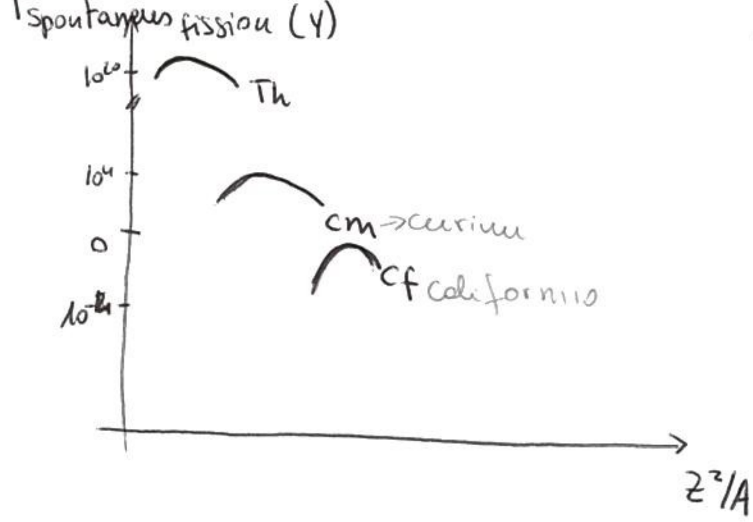
Using $a_s = 16{,}8$ MeV and $a_c = 0{,}72$ MeV we obtain

$$\frac{Z^2}{A} > 47$$

This estimate must be modified somewhat to account for Q.M. barrier penetration, which permits spontaneous fission even if the energy is negative!

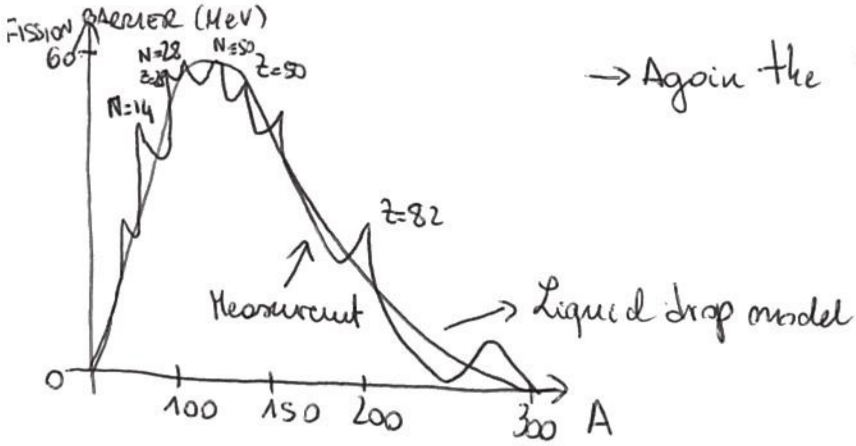
Moreover nuclei in the Uranium region have a permanent equilibrium deformation.

Nevertheless the parameter $\frac{Z^2}{A}$ does serve as raught indicator of the ability of a nucleus to fission spontaneosly.



The larger the value of Z^2/A, the shorter is the $t_{1/2}$ for spontaneous fission.
For $\frac{Z^2}{A} = 47$ the $t_{1/2} \simeq 10^{-20}$ s
So the fission would be istantaneusly. No such nuclei ~~exists~~ are known, but for A=300 and $Z/A \simeq 0,4 \Rightarrow Z^2/A = 48$ consistent with 0 activation energy for A=300

as shown here:



→ Again the "shell" structure appears!

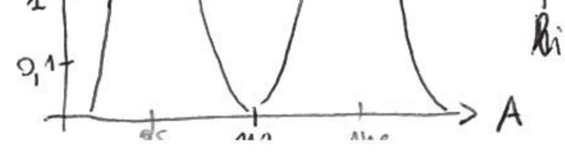
CHARACTERISTICS OF FISSION

- MASS DISTRIBUTION OF FRAGMENTS

A typical neutron induced fission reaction is:

$$^{235}U + n \longrightarrow {}^{93}Rb + {}^{141}Cs + 2n$$

which is possible for incident thermal energies (meV). The fission products are not determined [illegible] uniquely. The distribution must be simmetric about the center i.e. for heavy fragments there must be a light fragment BUT fission into equal (or nearly equal) fragments



is 600 times more probable than a decay with very different fragments
Most. probable decay of $^{235}U + n$ is $A_1 \simeq 95$ and $A_2 \simeq 140$

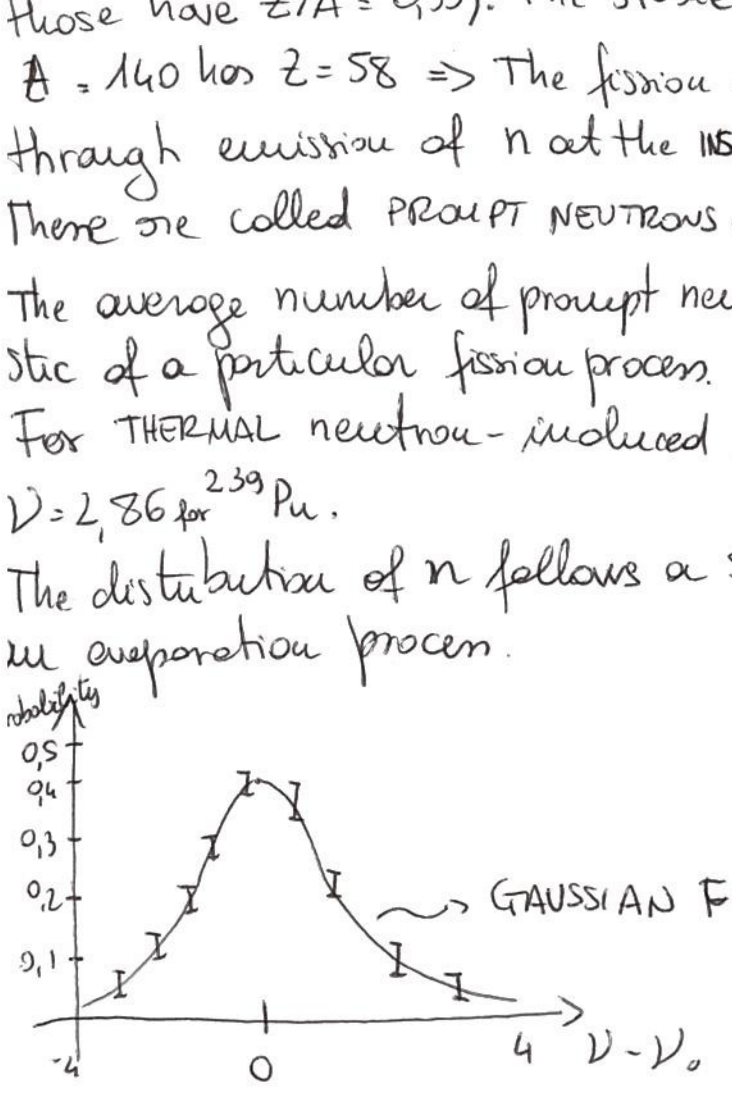
A convincing explanation of the mass distribution for low-energy processes has not been found.

For higher energy reaction the most favorable mass distribution is that with 2 nuclei with the same mass.

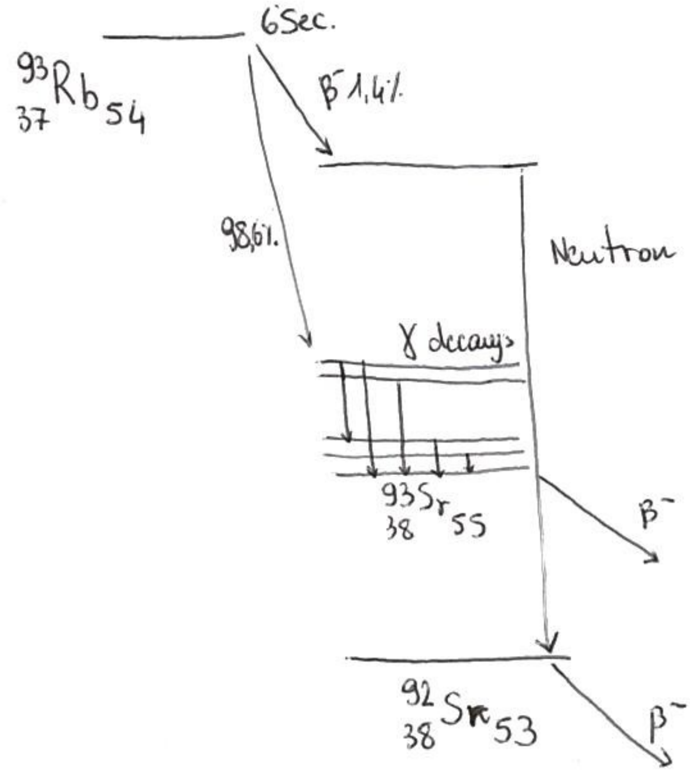
-NUMBER OF EMITED NEUTRONS.

Let's consider the same reaction we considered up to now.
^{235}U has 92 protons that have to be shared among the fission fragments that have $A = 95$ and $A = 140$ according to what we show before.
The resulting nuclei could share the 92 p following mass proportion. If it is the case, the formed nuclei would be $^{95}_{37}Rb_{58}$ and $^{140}_{55}Cs_{85}$. These nuclei are extremly neutron rich (stable nuclei in this A region have $Z/A \simeq 0{,}41$ those have $Z/A = 0{,}39$). The stable $A = 95$ isobar has $Z = 42$ and stable $A = 140$ has $Z = 58$ ⇒ The fission fragments shed this neutron excess through emission of n at the INSTANT of the fission ($\Delta t \leq 10^{-16}$ s)
There are called PROMPT NEUTRONS.

The average number of prompt neutron is called ν and is characteristic of a particular fission process.
For THERMAL neutron-induced fission $\nu = 2{,}48$ for ^{233}U, $\nu = 2{,}42$ ^{235}
$\nu = 2{,}86$ for ^{239}Pu.
The distribution of n follows a statistical behavior as expected from an evaporation process.



In addition to prount neutrous, delayed neutrons ($\Delta t \gtrsim$ seconds) (Th
sre emitted following the β decay of fission fragments.
A typical core can be depicted as:



Following the 6 seconds β decay of $^{93}_{37}Rb$, $^{93}_{38}Sr$ is left in a high excited state: the energy of this state exceed the neutron sepsration energy (S_n). It can decay by neutron emission in competition with γ. Neutron branching ratio is 1,4%.

=> Delayed n occur $\simeq$ 1/100 fission but these n sre enential to contron a nuclar reactor, since no mechanical system could react to an anomalus amount of prount neutrons, but it is possible to control reaction through delayed neutrons.

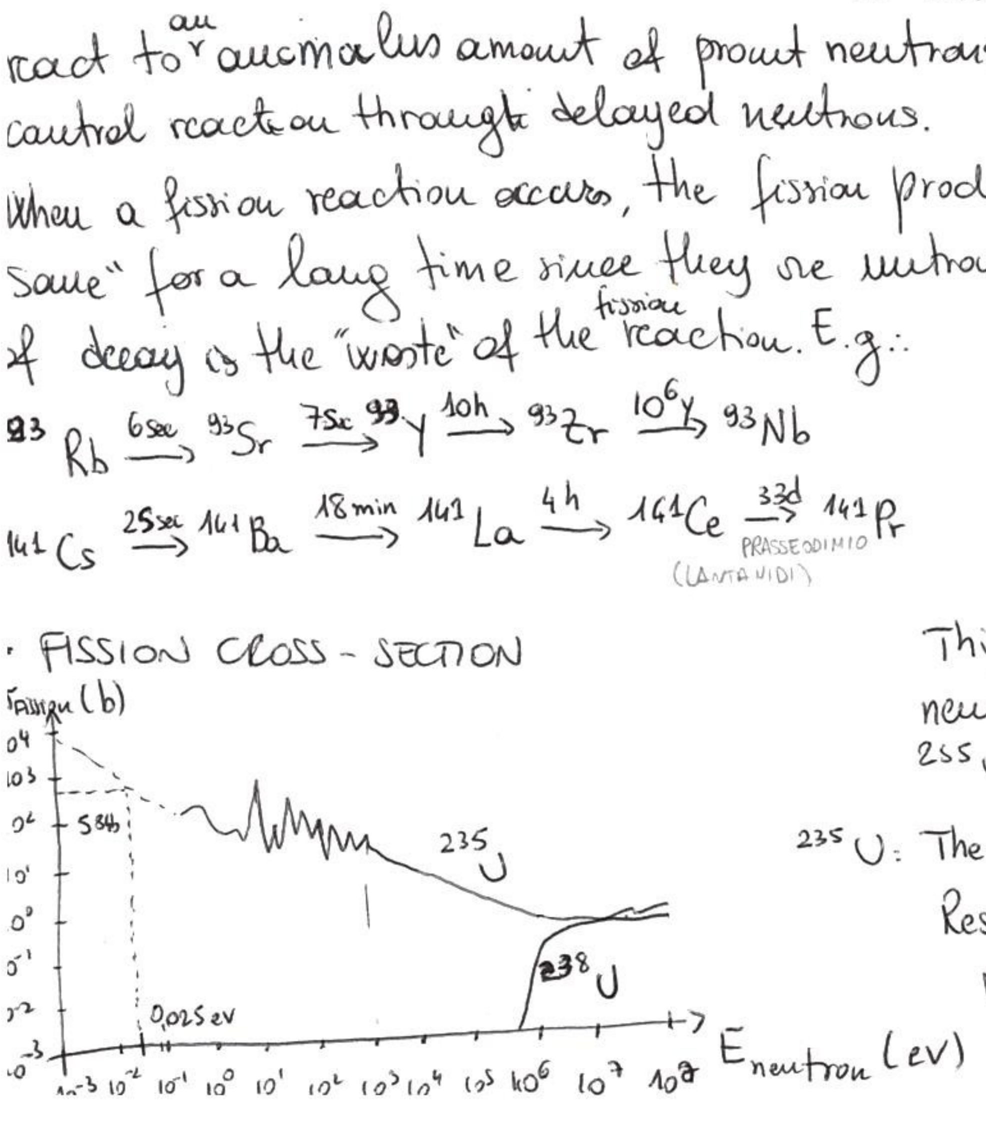
When a fission reaction occurs, the fission products do not remains "the same" for a long time since they sre neutron rich. The final product of decay is the "waste" of the fission reaction. E.g.:

$$^{93}Rb \xrightarrow{6sec} {}^{93}Sr \xrightarrow{7sec} {}^{93}Y \xrightarrow{10h} {}^{93}Zr \xrightarrow{10^6 y} {}^{93}Nb$$

$$^{141}Cs \xrightarrow{25sec} {}^{141}Ba \xrightarrow{18min} {}^{141}La \xrightarrow{4h} {}^{141}Ce \xrightarrow{33d} {}^{141}Pr$$

PRASSEODIMIO
(LANTANIDI)

· FISSION CROSS - SECTION



This graph represents neutron induced fission of ^{235}U and ^{238}U

^{235}U: Thermal region: $\sigma \propto \frac{1}{v}$

Resonances region for $1 < E < 100 eV$

$\sigma_{Fission}$ (584b) $> \sigma_{SCATTERING}$ (9 b) $>$ $\sigma_{radiative\ capture}$ (97 b)

$\sigma_{\text{Thermal } n} \cong 10^{3}\,\sigma_{\text{FAST } n}$

${}^{238}U$: No fission in thermal region: the reason for that is related to pairing term in liquid-drop model.

ENERGY IN FISSION

When ${}^{235}U$ capture a neutron to form the compound state ${}^{236}U^{*}$ the exitation energy is:

$$E_{EX} = \left[m({}^{236}U^{*}) - m({}^{236}U)\right]c^{2}$$

The energy of compound state can be found directly from mass energies of ${}^{235}U$ and n assuming k_n is small (as it is in thermal region) and negligible.

$$m({}^{236}U^{*}) = m({}^{235}U) + m(n) = 235{,}043924\,u + 1{,}008665\,u =$$
$$= 236{,}052589\,u$$

$$m({}^{236}U) = 236{,}045563\,u$$

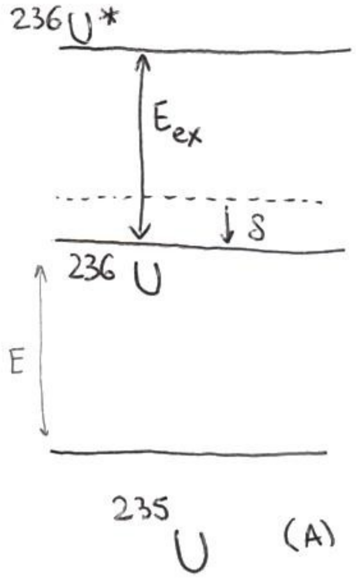
$E_{EX} = 6{,}5\ \text{MeV}$ → Energy of the "extied" ${}^{236}U^{*}$

The "ativation energy" (i.e. energy needed to overcome fission barrier) for ${}^{236}U$ is calculated to be 6,2 MeV ⇒ The energy needed to excite ${}^{236}U$ into fissionable state is exceeded by the energy one gets adding a neutron to ${}^{235}U$ ⇒ ${}^{235}U$ can be fissioned with 0-energy neutron as shown by the σ_{Fission}.

A similar calculation for ${}^{238}U + n \rightarrow {}^{239}U^{*}$ gives $E_x = 4{,}8$ MeV which is << than the activation energy of ${}^{239}U$ ($E_{\text{ACTIVATION}} = 6{,}6$ MeV) ⇒ Neutrons of at least MeV energy are required for fission of ${}^{238}U$. (As shown by the σ_{Fission} plot)

⇒ The primary explanation of fissionability of ${}^{235}U$ and ${}^{238}U$ lies in the difference between their exitation energies. But why the 2 energies are so different? It can be understood in terms of pairing energy term δ of the semi empirical mass formula.

If we illustrate the effect of δ we will have:

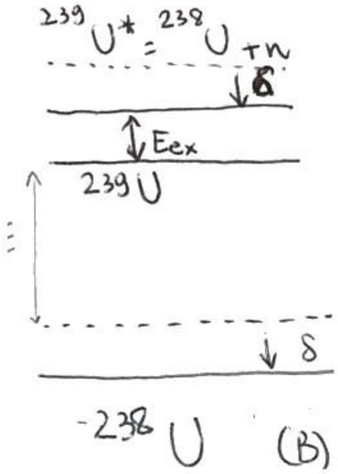


→ NO PAIRING (^{236}U)

→ ADD PAIRING ⇒ Increase B.E ⇒ Reduce mass
($\delta \cong 0{,}56$ MeV)

↓

E_{EX} for $^{236}U^*$ is increaded by a factor δ (w.r.t. the absence of δ ter



→ E_{EX} is correspondingly lower ⇒ The dirrerence betwen A and B is $\simeq 2\delta = 1.1$ MeV ⇒ Very close to the observed difference.

^{238}U the grand state is LOWERED Before capture

In general odd-N nuclei have a much larger termal neutron cross-section then even N, and this is all due to the pairing force!

ENERGY RELEASED IN THE REACTION: WHERE DOES IT GO?

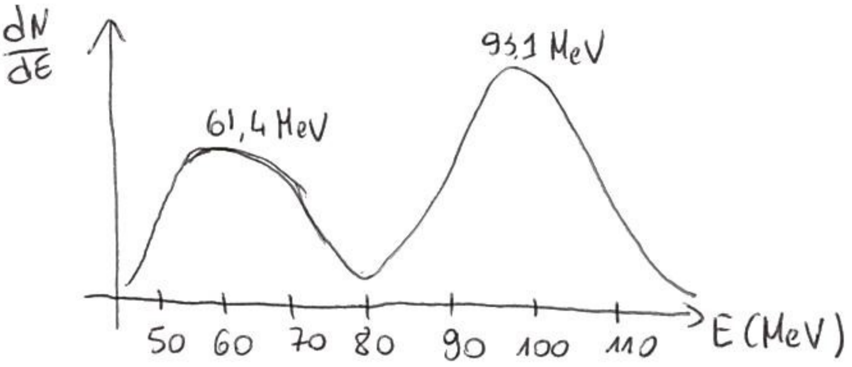
Let's consider a specific reaction:

$$^{235}U + n \rightarrow {}^{236}U^* \rightarrow {}^{93}Rb + {}^{141}Cs + 2n$$

Using the masses, the Q-value is 181 MeV.

Other final product will give energy releases of ≃ the same magnitude. It is reasonable to assume 200 MeV as $\langle E \rangle$ relased in the reaction* ⇒ Kinetic energy of the fragments. The energy is splitted as:

* This is also consistent with the estimate from Coulomb repulsion



Fission of ^{235}U

Two high probability are obtained at about 66 and 98 MeV. Because neutrons carry very little momentum, the kinetic energy of fragment 1 and 2 will be similar and opposite

$$p_1 = p_2 \qquad m_1 v_1 = m_2 v_2 \implies$$

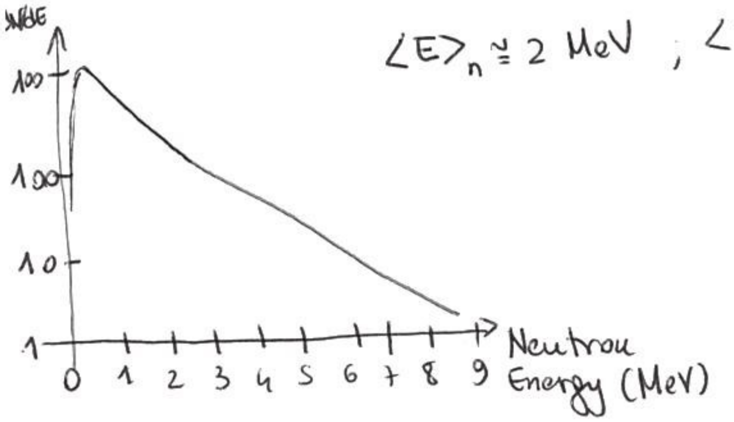
$$\frac{\frac{1}{2} m_1 v_1^2}{\frac{1}{2} m_2 v_2^2} = \frac{m_2}{m_1}$$

The ratio $66\,\text{MeV}/98\,\text{MeV} = 0,67$ consistent with $(140/95)^{-1} = 0,68$

OTHER ENERGY RELEASED IN THE REACTION

-NEUTRONS

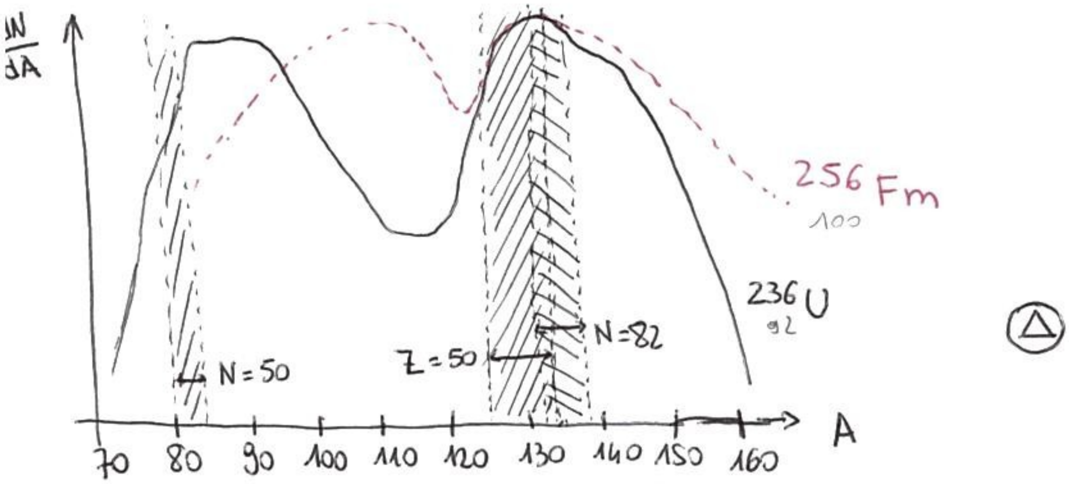
$\langle E\rangle_n \simeq 2$ MeV ; $\langle n\rangle_{neutron} \simeq 2,5$ $\implies \langle Energy\rangle_{neutron} \simeq 5$ MeV



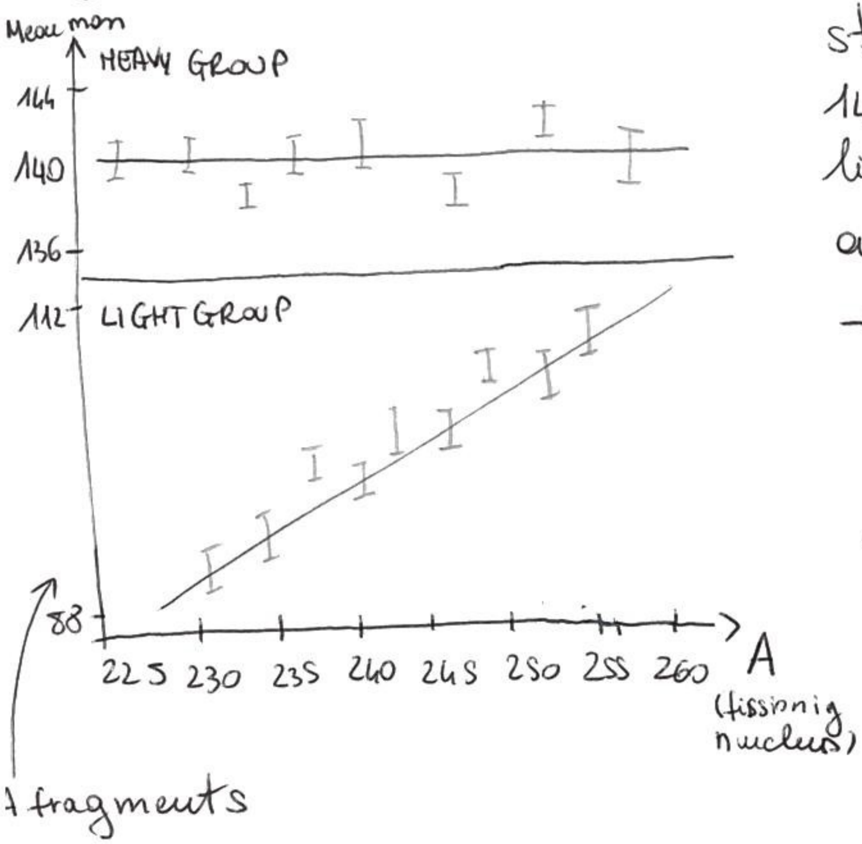
-other energies released
- prompt γ ($\simeq 8$ MeV) $\Delta t \simeq 10^{-14}$ s
- β decay of radioactive decay products ($\simeq 20$ MeV) | $\Delta t \simeq$ sec
- γ decay " ($\simeq 7$ MeV) | $\Delta t \simeq$ ps

FISSION AND NUCLEAR STRUCTURE

Fission is guenerally treated as a collective phenomenon according to the liquid-drop model -> It is not helpfull aneliticolly, but provides a mental image of the process.
But we will see that the structure of nucleus play a role. To do so let's consider the asymmetric mass distribution of fragments



Nomatter is the fissioning nucleus,
The heavy fragments overlaps quite well, while lighter fragments show a large deviation,
This can be better depict by looking at the average mass of heavy and light fragments



Average mass of heavy fragments stays nearly constant at about 140, while the average mass of lighter fragments increases linearly as A increases.

-> Throughout the entire range, the added nucleons all go to the lighter fragments, while in a liquid-drop fission one should expect that masses scales with the mass of ~~drop~~. fragment

The explanation of this unusual behaviour lies with the shell model.
The figure Ⓐ shows the region of magic numbers for the fragments.
For $Z = 50$ the stable nucleus has $Z/A = 0{,}40 \Rightarrow A = 125$ and neutron rich fission products range down to a minimum of $Z/A \simeq 0{,}38$ ($\Rightarrow A = 137 \Rightarrow 7$ neutrons from stability).
At the lower edge of the heavy fragments mon distubution there is a doubly magic nucleus $^{132}_{50}\mathrm{Sn}_{82}$. This excepionally stable configuration determines the lower edge of mon distubution of the heavier fragment.
No such effect occcurs for the lighter fragment, and indeed the mon distubution of light fragment has no overlap with single magic nuclei.