What happens for large Energy? I.e. why the σ decreases?

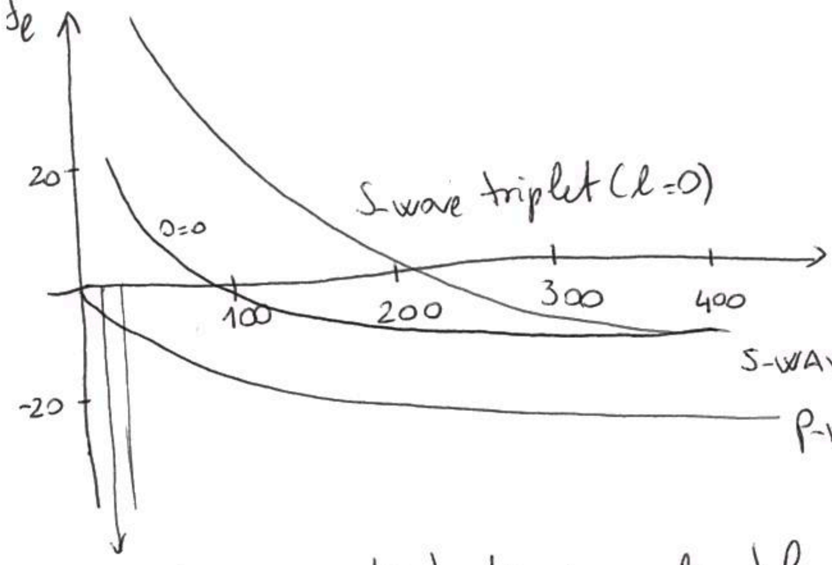


FIG 4.12. KRANE

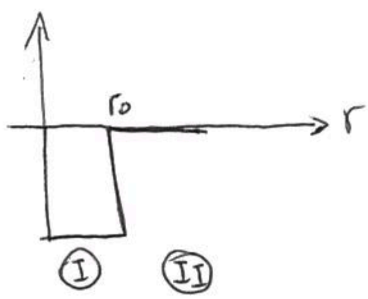
To separate ≠ components one have to measure $\frac{dN}{d\Omega}$ in ≠ θ ranges: $\ell=0$ is isotropic while ≠ ℓ have preferred directions

$E \lesssim 1$ MeV p contribute is negligible and only s-wave matters.

Up to ~300 MeV S is positive then it changes ~~and~~ sign. The effect of S-wave gets negative! ⇒ The nuclear charge become repulsive at short distance

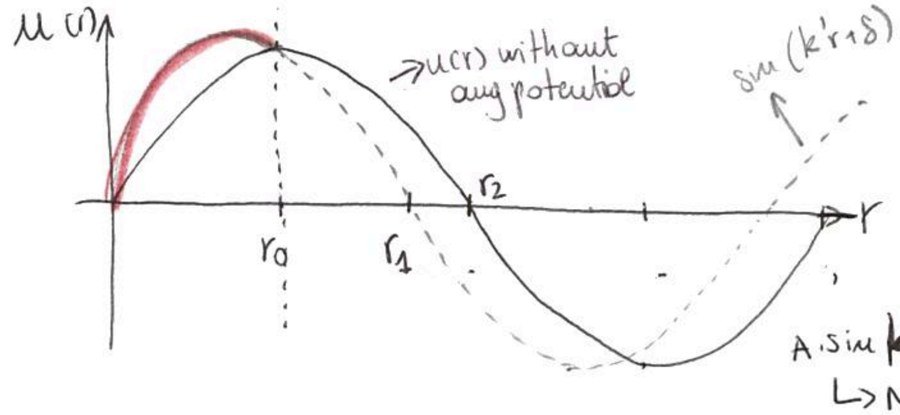
What is the meaning of the fact that ~~why~~ δ in ~~[illegible]~~ changes sign? → I.e. why it becomes repulsive at short range.

Let's consider deuteron again. The solution for square well potential gives us: analogy



$$u_I = A \sin kr \qquad k = \sqrt{\frac{2m(V_0+E)}{\hbar^2}} \leftarrow \text{SHRINKING}$$

$$u_{II} = C' \sin(k'r+\delta) \qquad k' = \sqrt{\frac{2mE}{\hbar^2}} \leftarrow \text{SHIFTING}$$



A sin k'r
↳ NO POTENTIAL

$k > k'$ ⇒ periodicity become smaller and changes of $\frac{2\pi}{k}$ → k large ⇒ smaller period

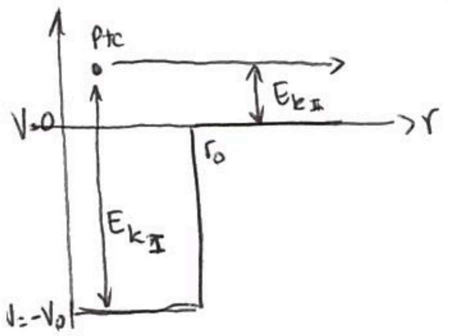
outside the periodicity ~~[illegible]~~ have to be the same, $\boxed{\text{BUT}}$ the w.f. has to move! To maintain continuity.

When the function is 0? $k' r_2 = \pi$ (sin k'r)
$k' r_1 + \delta = \pi$ (sin(k'r+δ)) } We subtract and we get

$k'r_1 + \delta - k'r_2 = 0 \qquad \delta = k'(r_2 - r_1) > 0 \qquad \delta$ is positive.

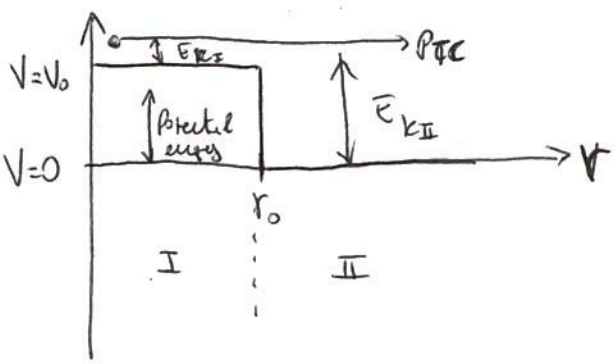
Let's suppose that the potential is repulsive instead of attractive.

ATTRACTIVE POTENTIAL



E_k (kinetic energy) decreases
⇒ the potential is attractive

REPULSIVE POTENTIAL

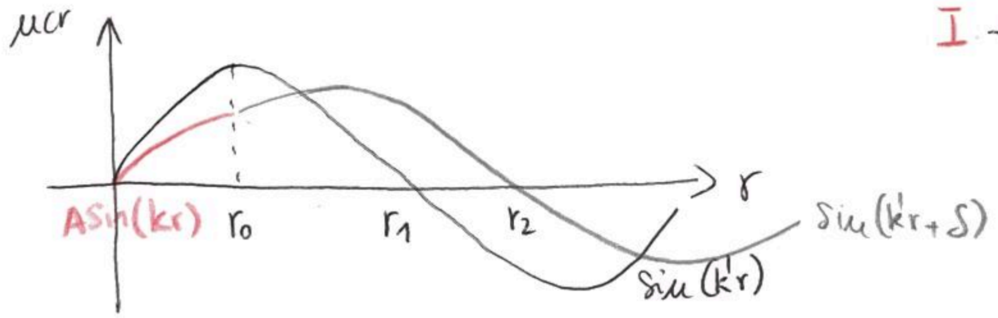


The separation is increasing ⇒ the potential is repulsive.

$u_I = A \sin kr \quad , \quad k = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$

$u_{II} = C' \sin(k'r + \delta) \quad k' = \sqrt{\frac{2mE}{\hbar^2}}$

In case of REPULSIVE potential $k < k'$ ⇒ period I > period II



I the period expands

$$\left.\begin{aligned} k'r_1 &= \pi \\ k'r_2 + \delta &= \pi \end{aligned}\right\} \text{SUBTRACTION}$$

$k'(r_1 - r_2) - \delta = \pi$

$\delta = k'(r_1 - r_2) < 0$

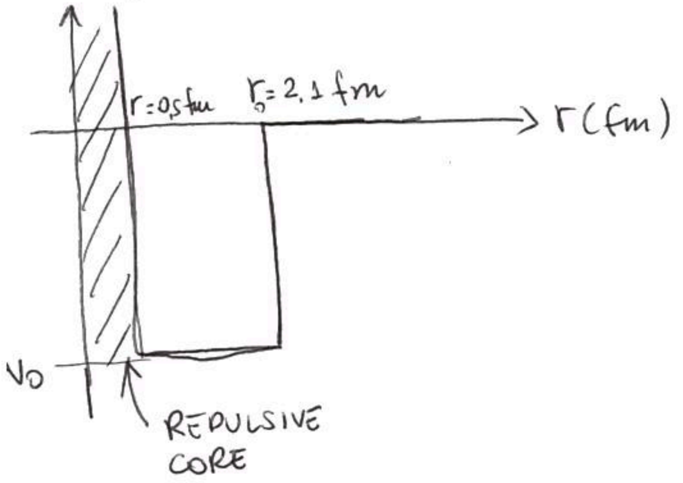
⇒ The sign of δ tells us if the actraction was attractive or repulsive.

$l=1$ phase shift the δ sign in negative.

Higher energy ⇒ closer ptc. ⇒ The separation is smaller for higher energy

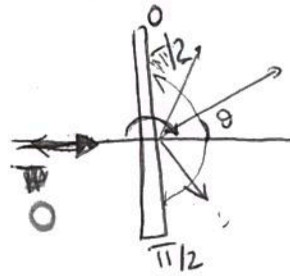
⇒ The change of sign means that the force become REPULSIVE "very close". Meaning that the separation among nucleons is ≃ 0,5 fm. ⇒ It is not possible to push to nucleons closer than 0,5 fm. This observation also lead ~~to~~ to what we already observed: There is a saturation density!

Nuclei are ⇒ not very dense object ⇒ The square well potential should be modified a bit.



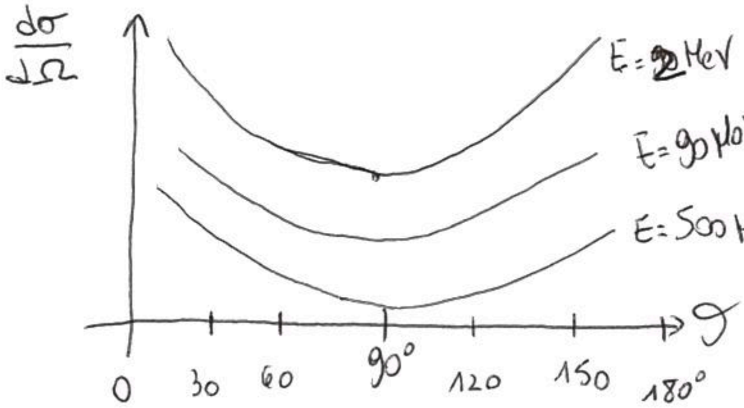
EXCHANGE FORCE MODEL

Let's look at the n-p differential cross-section.



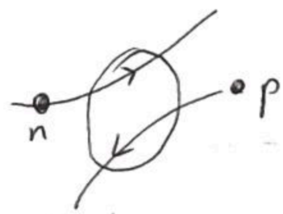
In general one should think that the $\frac{dN}{d\theta}$ should decrease starting from $\vartheta = 90°$ ($\theta = 90°$ Max → decrease + 0 x $\vartheta = 180°$)

But if the measurement is done what can be observed is



Why there is a peak at $\vartheta = 0°$?

This can be interpred using the exchange force model:



In the interaction, when p e n are close enought p and n can "exchange" charge explaing the peak at π!

The "exchange" is done at the level of "nuclear field"

In fact, in parallel to what happens in e.m. force the same processes happen at the level of nuclear forces.

In E.M interaction, we starts from 2 charges

$\overset{\bullet}{q_1}$ $\overset{\bullet}{q_2}$ → classically, q_1 ~~apply~~ a force on q_2 $F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$

When we move to "fields" → q_1 creates a field which apply a force on q_2

Moving to Q.E.D., the field is not continuos, but is quantified.
The quantization is given by the photon γ ⇒ the $q_1 \cdot q_2$ interaction can be seen in terms of γ exchange: the γ are the mediator of E.M. force and are <u>VIRTUAL</u> → we cannot observe them.

Moving to nuclear force, if we have two nucleons

$\overset{\bullet}{N_1}$ $\overset{\bullet}{N_2}$ The interaction is mediated from an exchange of particle $N_1 \to ptc \to N_2$

Which particle mediates the interaction?
Particle can have mass and charge and can ~~have an energy which can be as large as the~~ exist only as long as the indetermination principle allows.

$\Delta E = M_\pi c^2$ $\Delta E \Delta t \simeq \hbar$ ⇒ $\Delta t = \frac{\hbar}{\Delta E} = \frac{\hbar}{M_\pi c^2}$

The range of the interaction have to be $\lesssim \Delta t \cdot c = \frac{\hbar c}{m_\pi c^2}$

The range of the interaction is $\simeq 2fm$ ⇒ $M_\pi c^2 = \frac{\hbar c}{2fm} = \frac{200\ MeV\ fm}{2fm} \simeq 100$ M

The mass of the exchange particle is $\simeq 100\ MeV/c^2$ ⇒ The exchange mass ptc is massive (range of E.M. is ∞ ⇒ Massless mediator) and virtual, but not IMMAGINARY!

The force mediator are PIONS : $\underbrace{\pi^+, \pi^-}_{m = 139,6 MeV}$, π^0 $m = 135\ MeV$

Force between nucleons is mediated by π (strong force is the residual o color charge in quark).

n-p interaction can be mediated by π^+, π^- & π^0

$n \to n + \pi^0$ | $p \to p + \pi^0$
$p \to p + \pi^0$ | $n \to n + \pi^0$

$\Delta t = \frac{r}{c} = \frac{2fm}{3 \cdot 10^8 m/s} \simeq 10^{-23}\ s$
The reaction can be not energetically balanced for $t < \Delta t$

$n \to p + \pi^-$ | $p \to n + \pi^+$
$p + \pi^- \to n$ | $n + \pi^+ \to p$

The same is valid

p-p interaction can be mediated only by π^0

$p \to p + \pi^0$ | $p \to n + \pi^+$

$p + \pi^0 \to p$ | $p + \pi^+ \to$??

This can be ~~related~~ to $m_{\pi_{ch}} \neq m_{\pi_0}$ (Mass difference)

Square well potential must be changed into a phenomenological potential e.g. one π exchange potential (also known as Yukawa potential)

$$V(r)_{OPEM} \propto \frac{e^{-\mu r}}{r} \left[\text{spin dependence, non central} \right]$$

π exchange is responsible for the nuclear attraction.

For shorter range interaction, the exchange of ω may ~~explain~~ contribute to the repulsive core, while the ϕ exchange can provide the spin-orbit part of the interaction.

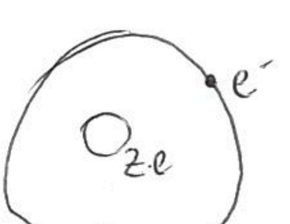
NUCLEAR MODELS

~~It~~ At this point we would like to extend the ideas developed up to now (i.e. understand the nuclear force) to heavier nuclei.
This is not an easy task, since going from 2 to "many" add a lot of technical difficulties.
The equations cannot be solved analytically but only numerically.
Moreover, the nuclear force do not acts only between 2-nucleus, but also the 3-body, 4-body, ecc force have to be taken into account.
An for such problem there is no classical analog.
In principle it is possible to do additional scattering experiment in ~~but~~ the 3-body force, but however the limit where microscopical approach obscures rather than illuminates the "essential physics" is soon reached.
In order to study the nuclei, we decide to use an oversimplified theory, but mathematically tractable and full of physics insight. If the theory is fairly succesfull we can improve it by adding additional terms creating a "NUCLEAR MODEL".
A nuclear model is a simplified view of nuclear structure, but with all the essential nuclear physics inside. To be consider a "model", a theory must describe reasonably describe previous measurements and must be able to predict additional properties which can be measured by new experiments.

|SHELL MODEL|

In atomic physics, the theory based on shell for electrons has provided a remarkable success in describing the complicated details of atomic structure.

H-like atom



$$H = -\frac{\hbar^2}{2m}\nabla^2 - \frac{Ze^2}{4\pi\varepsilon_0 r}$$

→ MULTI - e⁻

$$H = \sum_i \left[-\frac{\hbar^2}{2m}\nabla_i^2 + \left[-\frac{Ze^2}{4\pi\varepsilon_0 r_i} + \sum_{i,j} \frac{e^2}{4\pi\varepsilon_0 |\vec{r}_i} \right.\right.$$

i = electrons
j = electrons - i-electron

Interaction term

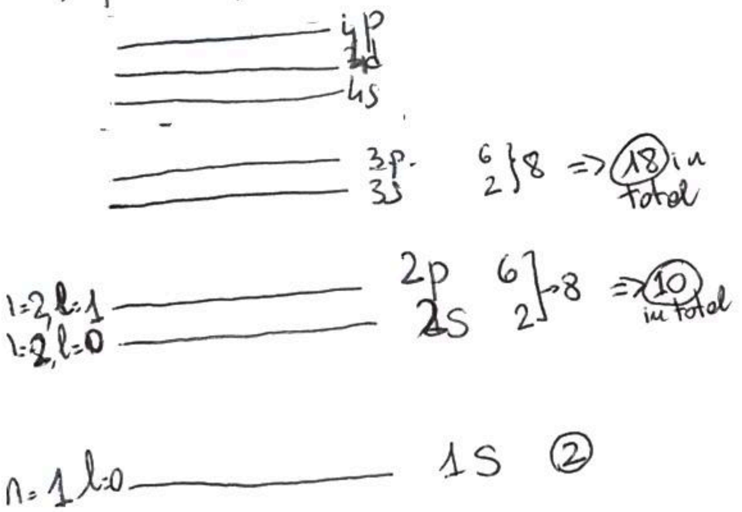
To take into accout the interaction, an effective potential $V(r)$ is employed and this potential is a "non-Coulomb" potantiel

$$\Rightarrow H = -\frac{\hbar^2}{2m}\nabla^2 + V(r)$$

It ends up with a "shell model" where the different electrons are disposed. In the model spin and angular momentum degeneracy are taken into account and different "energy levels" (=> i.e. shell) are "created".

The Energy of each level is characterize by quantum numbers (n, ℓ) and the gaps betweent the different energy levels are different.

So, for example, we have.

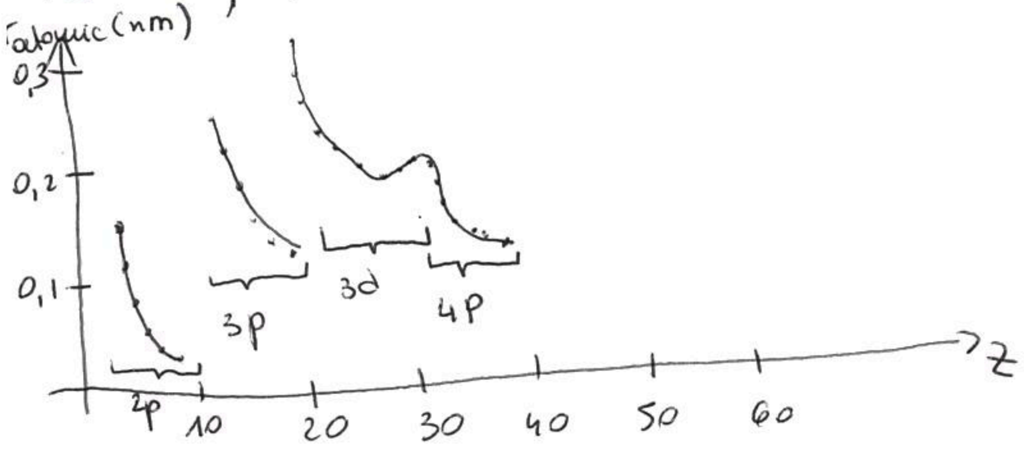


In s shell 2 electrons can be present, p-shell 6 and in d-shell 10.

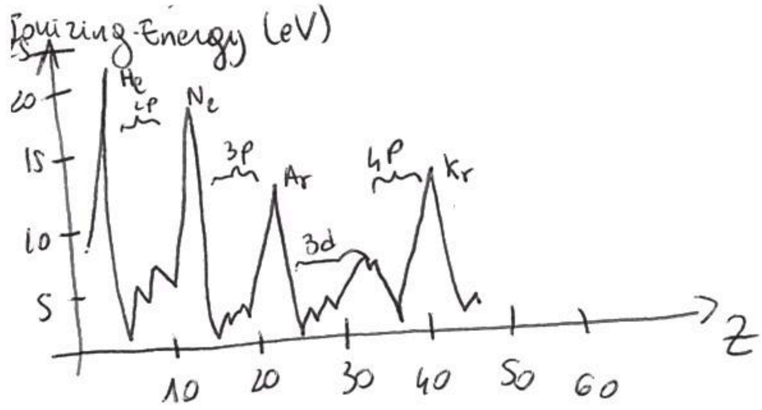
The levels closes "cumulatively" so $n=1$ closes with $2e^-$, $n=2$ with 10, $n=3$ with 18 and so forth.

The model is succesfull in describe "periodicity" of atom characteristics which are seen in the periodic table.

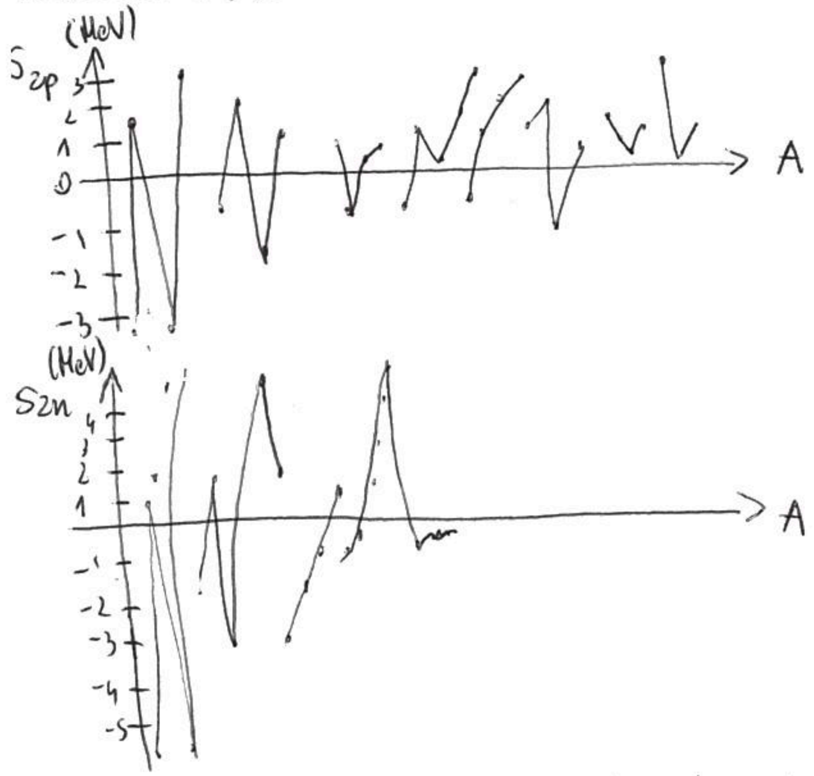
For example, if one looks at the atomic-radius and to the ionizing energy



A regular and smooth variation within the a sub-shell can be observed, but a rather drammatic change when 2 sub-shell are crossed can be seen. Moreover, it appears the the shells are "space-separated".



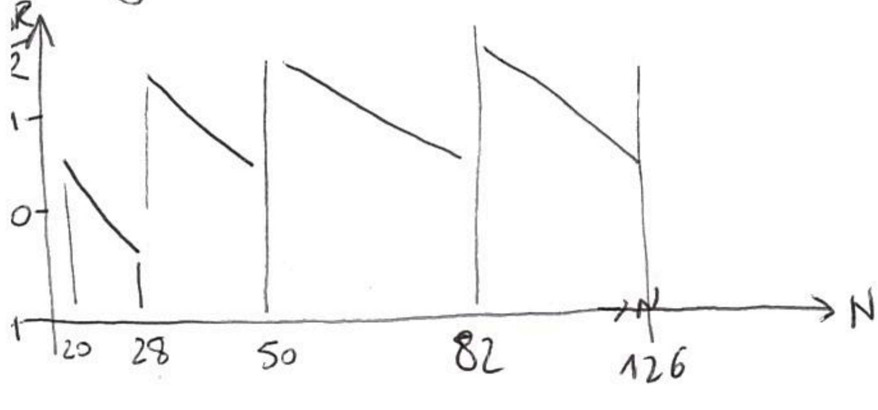
Let's now move to a "nuclar physics" measurement. If ~~man~~ one plot the "proton (and neutron) sepration energy" $S_{p,n}$ (i.e. the energy needed to extract one p(n) from a nucleus) the obtained results is:



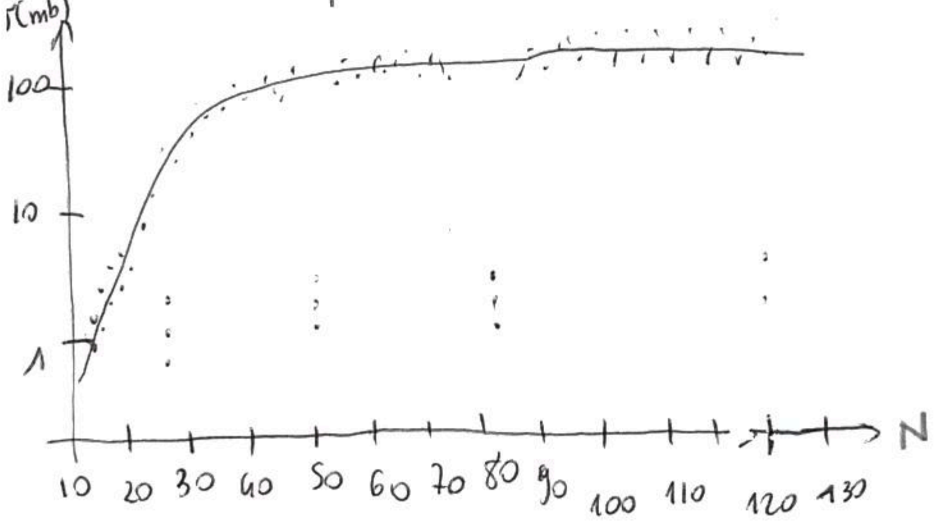
The sepration energy increases gradually with N or Z, except for sharp drops that occur at the same p and n numbers (i.e. ~~A~~N=Z= 8, 20, 28, 50, 82, 126)

Addional evidence of such structure exists also for other quantities such as:

Change of the nuclar radius



or neutron capture cross-section



The decrease of the $\sigma_{n.c.}$ decrease at $N = 28, 50, 82, 126$

All these measurements show a pattern and also ~~that~~ there are some Neutron numbers where sudden changes of behaviour occurs. These numbers are called "MAGIC NUMBERS" ($N = Z = 2, 8, 20, 28, 50, 82, 126$) and represents the effects of "filled shells". Any succesfull theory must be able to account for the existences of such shell closures.

The question of the existence of a nuclear potential is deal with by the fundamental assumption of the shell model: the motion of a single nucleons is governed by the potential caused by all the other nucleons.
What will matters here is a "Mean potential" which ~~she~~ must be used taking into account that in nuclear physics we have to deal with 2 ferminons (p and n) at a time (while in atomic physics only the e^- have to be taken into account).
Which potential should we use? For sure not the 2-body potential since it will introduce too many variables in the calculations.

$\boxed{\text{SHELL MODEL POTENTIAL}}$

Let's start from the easiest potential we can think to use here: the "infinite square well potential"

$V(r) = 0 \quad r < r_0$

$V(r) = \infty \quad r \geq r_0$

This is a central potential, so there is no dependence on θ and φ in the potential.
And the wave-function can be written as:

$$\phi(r,\theta,\varphi) = \frac{u(r)}{r} \underbrace{Y_l^m(\theta,\varphi)}_{\text{spherical harmonics}}$$

The potential does not depends on θ and φ, but the wave-function does.

The Shrödinger equation can be written as: $H\psi = E\psi \rightarrow Hu = Eu$

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \left[V(r) + \frac{l(l+1)\hbar^2}{2m_p}\right]u = Eu$$

degeneracy of each level

For $r < r_0$

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \frac{l(l+1)\hbar^2}{2m_p}u = Eu$$

Applying the boundary conditions that at $r=0$ and $r=r_0$ the w.f. must be 0, the solution of

Such equation is a Bessel function

$$u(r) = r \cdot J_l(kr)$$

Where J_l = Bessel function which depends on l

$k = \sqrt{\frac{2m_pE}{\hbar^2}}$ $\quad m_i = m_p$ because each n e p has the same Energy

and $J_l(kr_0) = 0$, $r_0 \cong 2\,\text{fm}$ $\qquad J_l = z^l\left(-\frac{1}{z}\frac{d}{dz}\right)^l\left(\frac{\sin z}{z}\right)$

$\Rightarrow$ So starting from $l=0$ a minimum (i.e. a solution) of Bessel function can be obtained $\rightarrow$ changing l one gets a "shell model"

Similar to what is seen in atomic physics for each n a different l can be associated

$l=0 \qquad n=1,2,3, \ldots$

$l=1 \qquad n=1,2,3 \ldots$

Here l and n do no represent the same thing as in atomic physics, but like there we can say that,

$l=0 \rightarrow s$

$l=1 \rightarrow p$

$l=2 \rightarrow d$

$l=3 \rightarrow f$

...

→ The energy levels are disposed like this:

1g
2p
1f
2s
1d
1p
1s

n counts the # of levels with l value

$\boxed{\text{Note that 1p does not exist in ATOMIC PHYSICS!}}$

If we count how many p(n) can be "accomodated" in such way

1g	18	58	
2p	6	40	
1f	14	~~34~~	→ NOT A MAGIC NUMBER!
2s	2	20	↳ The square-well potential is not ok
1d	10		
1p	6	8	Let's try an-other one
1s	2	2	

The 3-d Harmonic potential → again this is a central potential and we can solve it for u

$$V(\vec{r}) = \frac{1}{2} m \omega^2 \vec{r}^2 \qquad \phi(r) = \frac{u(r)}{r} Y_l^m(\theta, \varphi)$$

$$-\frac{\hbar^2}{2m}\frac{d^2 u}{dr^2} + \left(\frac{1}{2} m\omega^2 r^2 + \frac{l(l+1)\hbar^2}{2mr^2}\right) u = E u$$

$$E = \left(2n + l + \frac{3}{2}\right)\hbar\omega$$

$$= \left(N + \frac{3}{2}\right)\hbar\omega$$

$n = 0, 1, 2$

$l = \overset{s}{0}, \overset{p}{1}, \overset{d}{2}, \overset{f}{3}$

$\Downarrow$

$N = 0, 1, 2, 3, \dots$

~~With~~ this potential, all the energy level ~~are~~ equally separated

If $N = 0 \Rightarrow E = \frac{3}{2}\hbar\omega$ $\quad N = 0 \Rightarrow n = 0, l = 0 \rightarrow 1s$

$N = 2n + l$

22 + 14 + 6 —— $\boxed{1h, 2f, 3p}$ → this can be "extrapolated")

18 + 10 + 2 —— 1g, 2d, 3s

14 + 6 —— $1\overset{12}{f}$, $\cancel{2}\overset{6}{p}$ 40

BREAK!

2 + 10 —— $2\overset{2}{s}$, $1\overset{10}{d}$ 20

SAME ENERGY

6 —— 1p 8

$\hbar\omega$

2 —— 1s 2

$3/2\,\hbar\omega$

$N = 1 \quad n = 0, l = 1 \rightarrow 1p$

$N = 2 \quad n = 1, l = 0 \rightarrow 2s$
$\quad\quad\quad n = 0, l = 2 \rightarrow 1d$

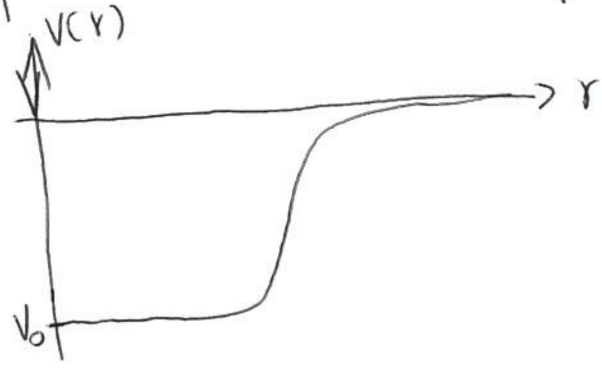
$N = 3 \quad n = 0, l = 3 \rightarrow 1f$
$\quad\quad\quad n = 1, l = 1 \rightarrow 2p$

$N = 4 \quad n = 0, l = 4 \rightarrow 1g$
$\quad\quad\quad n = 1, l = 2 \rightarrow 2d$
$\quad\quad\quad n = 2, l = 0 \rightarrow 3s$

s	p	d	g	h
2	6	10	18	22

Also in this case the magic numbers stops at 20... Also this potential is not ok. This should not be a "big surprise", infact both the potentials we have considered are ∞, which is inrealistic in a nucleus ⇒ A finite potential must be used.
The "easy" options are a) A finite square well potential or b) a finite square well potential with exponential tails (to round the tails of the potential).
The calculations were done in the 40, and it turns out that the levels remain the same as for the ∞ potentials.

The "most realistic" potential that can be used is a "Wood-Saxon" potential. This $V(r)$ is ≃ flat until R_0 and then goes to 0



$$V(r) = -\frac{V_0}{1 + e^{\frac{r-R}{a}}}$$

The sign ⊖ tells us that the potential is ATTRACTIVE.

$R = R_0 A^{1/3}$ $a = .52$ fm $V_0 \simeq 50$ MeV

Such potential was tried, but the change of potential shape do not change the shell closers, whic are insensitive to the actual shape of the potential.

How could we modify the potential to obtain the proper magic numbers? For sure we don't have to modify the potential too much, hence we have to add a term to improve the situation.

In '49 it was demonstrated that the term which can cure the problem is the "spin-orbit" potential.

The idea is the same as in atomic physics, but the differences introduced by such a contribution in atomic physics are of the order of % of eV. Such interaction won't play any role in nuclear physics, but the idea can be applied to the spin-orbit contribution of the STRONG force.

The contribution has to be added to ANY potential in the hamiltonian

$$H = H_0 + f(r)\,\vec{l}\cdot\vec{s}$$

The "central" H_0 commutes with L^2, L_z, S^2, S_z ⇒ You can have

$H_0 = -\frac{\hbar^2}{2m}\nabla^2 + V(r)$ a definite angular momentum and spin in the energy eigen function ⇒ For each energy a definite set of L^2, L_z, S^2 and S_z.

The spin-orbit terms make the Hamiltonian as "non-commutative", in the sense that H commutes with L^2 and S^2 but not with L_z, S_z