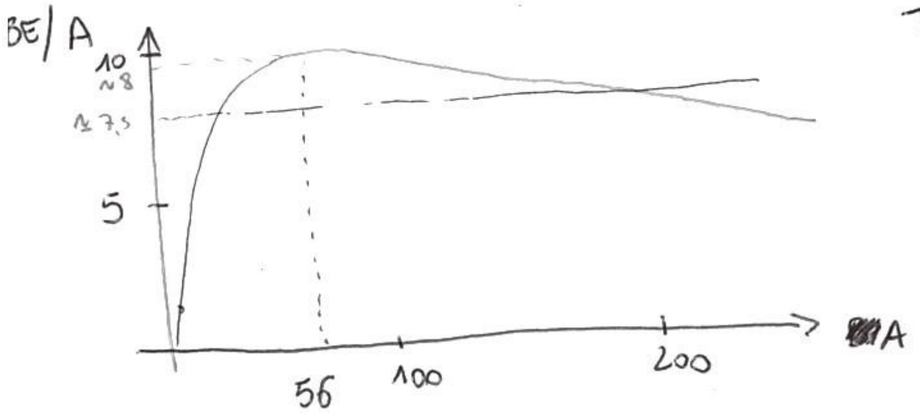
NUCLEAR FUSION

The behaviour of BE/A vs A suggests an other way to extract energy from nucleus. It is possible to climb the B.E. curve towards a more stable nucleus by begining from light nuclei rather than by a heavy nucleus.

If we combine 2 nuclei inta nucleus below A = 56 energy is released



This process is called NUCLEAR FUSION, because 2 light nuclei fuses into a heavier one

ADVANTAGES OF FUSION VS FISSION

- Light nuclei are easy to obtain
- Product of reaction are light and stable

DISVANTAGE OF FUSION:

- The Coulomb barrier must be overcome, while induced fission reaction has no Coulomb barrier and low-energy n can be used

Let's consider two ^{20}Ne nuclei which fuse into a ^{40}Ca.
The Q-value of reaction is 20,7 MeV* ($\simeq$ 0,5 / nucleon · comparable with that fra fission). However, before ^{40}Ne can interact, the Coulomb repulsion must be overcome.

$$^{20}Ne + ^{20}Ne \rightarrow ^{40}Ca + W$$

↳ γ-ray
neutron
light ptc

$$M(^{20}Ne)c^2 = (10m_p + 10m_n)c^2 - B.E.$$

↳ $\frac{BE}{A}(A=20)$

Rest mass before fusion:

$$(20 m_p + 20 m_p)c^2 - 40 \frac{BE}{A}(A=20)$$

After fusion

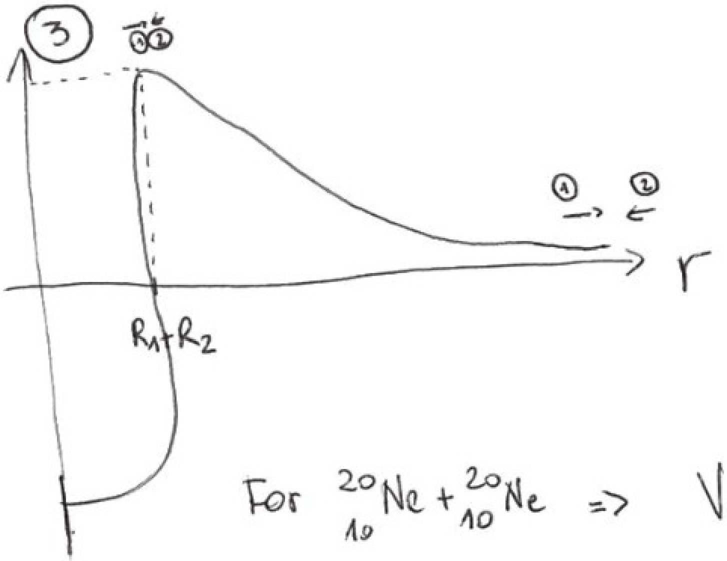
$$(20 m_p + 20 m_p)c^2 - 40 \frac{BE}{A}(A=40)$$

$$Q_{VALUE} = 2M(^{20}Ne)c^2 - M(^{40}Ca)c^2$$

$$= 40\left[\frac{BE}{A}(A=40) - \frac{BE}{A}(A=20)\right]$$

$$= 40\left[8,6 - 8,0\right] \simeq 24\ MeV$$

③



Coulomb height:

$$V_0 = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 R}$$

$$R = R_1 + R_2$$

For $^{20}_{10}Ne + ^{20}_{10}Ne \Rightarrow$ $$V_0 = \frac{10 \cdot 10 \cdot 1,44\ MeV \cdot fm}{2 \cdot 1,25 \cdot 10^{1/3}\ fm} \simeq 20\ MeV$$

3,6

The minimum Coulomb energy to overcome is that produced by 2 p:

$$V_0 = \frac{1 \cdot 1 \cdot 1,4 \cdot MeV \cdot fm}{2\ fm} = 0,7\ MeV$$

Range of nuclear force

→ It means that in all the cases we have to give some energy because of the reaction can start.

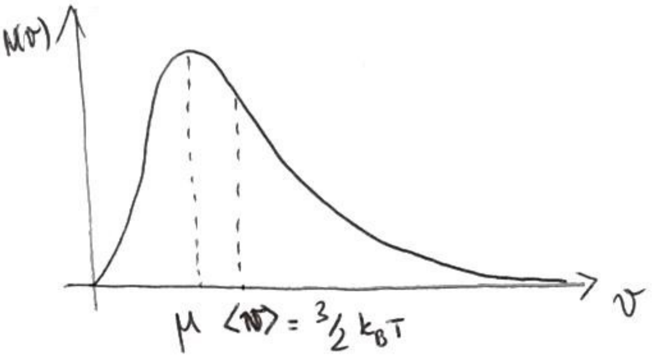
To do so we can accelerate the nuclei eg. ^{20}Ne beam as a

^{20}Ne target and this in not a big problem. But the energy needed to run the accelerator will be way larger than the produced energy.
Alternatively, one can heat a container of Ne gas until the 2 nuclei approaches and eventually collide.
Because of thermal energy is used to overcome Coulomb barrier, this process is called THERMONUCLEAR fusion.
How large is the T which is needed?
At room temperature (300 K) $K_B T = 0{,}027\ eV$

$$\frac{3}{2} k_B T = 150\ keV \Rightarrow T = \frac{150\ keV}{k_B} = \frac{150\ keV}{8{,}6 \cdot 10^{-5} \frac{eV}{K}} \cong 10^9 K$$

$T_{core_{SUN}} = 1{,}5 \cdot 10^7 K$ (Note that the minimum p $E \cong 700\ keV$!)

It means that a very high energy to reach such T is needed.
[In the sun the "low T" is compensated by the high density!]
In general we should find a way to express the velocity of the ptc in the plasma. The velocity distribution follows a Maxwell-Boltzmann distribution:



$$N(v) = N_0 \left(\frac{2}{\pi}\right)^{1/2} \left(\frac{m}{K_B T}\right)^{3/2} v^2 e^{-\frac{1}{2}\frac{mv^2}{k_B T}}$$

There are some particles that have a velocity which is much higher than the averaged velocity.
$\Rightarrow$ These ptc increases the probability of fusion!

The fusion reaction rate can be estimated as

$$R = N_x N_y \langle \sigma v \rangle$$

Let's see how we can estimate such rate:

If we have the reaction:

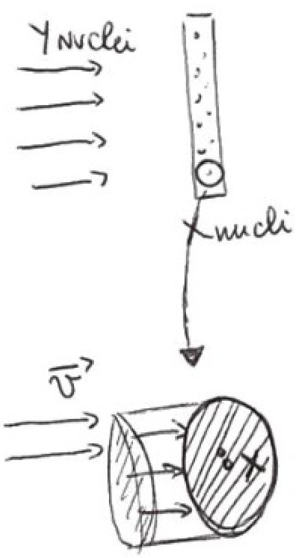
$$X + Y \rightarrow Z + w$$

n_x = n° of nuclei x per volume

n_y = " y "

σ = cross-section for fusion reaction

The probility to hit x from y is the cross-section!



How many scattering there will be per unit time?

$\vec{v}$ = relative speed between y and x

We can build a cylinder around the x nucleus:

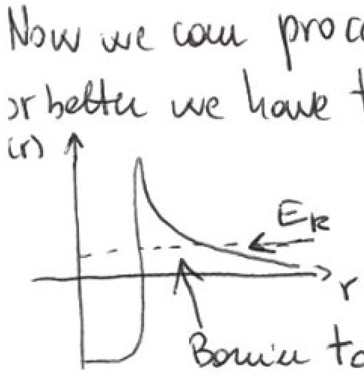
the "height" is $v \cdot dt$ the base is σ

$n_{\text{scattering}}$ in $dt = n \sigma v \, dt$ ≡ Volume of the cylinder

↳ probability MUST depend on v

This is valid for all the geometry of collisions, not only for "linear" geometry.

Now we can proceed. As already mentioned, we have to "beat" Coulomb or better we have to set a probability for barrier penetration



This probability is $\propto e^{-2G}$

$$2G = \frac{Z_1 Z_2 e^2}{4\pi \varepsilon_0} \frac{2\pi}{\hbar v}$$ → Valid in gueral for small energies

↓

We can use a semi-clasical picture:

b is the "impact prameter

The angular momentum is

$$m v b = l\hbar$$

$$b_l = \frac{l\hbar}{m v}$$

Eeach nucleus with a angular mementum below $(l+1)\hbar$ will enters into $l\hbar$ and hence it is quantified $\left(b_{l+1} = \frac{(l+1)\hbar}{m v}\right)$

So we can ~~evaluate~~ in which "area" we have to define the σ

The area is:

$$\pi b_{l+1}^2 - \pi b_l^2 = \pi \left\{ \left[\frac{(l+1)\hbar}{m v}\right]^2 - \left[\frac{l\hbar}{m v}\right]^2 \right\} = \frac{\pi \hbar^2}{m^2 v^2}\left[2l+1\right]$$

↑ reduced mass

$$\frac{\hbar^2}{m^2 v^2}\left((l+1)^2 - l^2\right)$$

$m v R = l_{max} \hbar$ R = nuclear interaction range:

$\Sigma(2l+1)$ = ARITHMETIC PROGRESSIONS

$$\sigma = \sum_{l=0}^{l_{max}} \frac{\pi \hbar^2}{m^2 v^2}(2l+1) = \frac{\pi \hbar^2}{m^2 v^2}\left(\underbrace{\frac{l_{max}+1}{2}}_{\text{TOT. NUM of Term}}\left(1 + 2 l_{max} + 1\right)\right)$$

last term

first term

TOTAL AREA

$$= \frac{\pi \hbar^2}{m^2 v^2}\left(l_{max}+1\right)^2 = \frac{\pi \hbar^2}{m^2 v^2}\left[\frac{m v R}{\hbar} + 1\right]^2$$

$$= \pi \left[R + \frac{\hbar}{m v}\right]^2$$

How large is it? For a typical plasma: m_{red} for $2d \simeq 1 m_p$

$$\left(\frac{\hbar}{m v}\right)^2 = \frac{\hbar^2}{m^2 v^2} = \frac{\hbar^2 c^2}{m(m v^2) c^2} = \frac{200 \times 200 \times MeV^2 \times fm^2}{1000\,MeV \cdot 1\,keV} = 4000\,fm$$

$$\frac{\hbar}{m v} = 200\,fm$$

The range of the nuclear force is $\simeq 2\,\text{fm} \Rightarrow \frac{\hbar}{mv} \gg R$

So $$\sigma = \pi \frac{\hbar^2}{(mv)^2} \propto \frac{1}{v^2}$$

The σ_{FUS} at low energy is $\propto \frac{1}{v^2}$

Let's go back to the reaction rate. The fraction of ptc with speed between v and $v+dv$ is

$$p(v)\,dv = \left(\frac{2}{\pi}\right)^{1/2} \left(\frac{m}{k_B T}\right)^{3/2} v^2 e^{-\frac{1}{2}\frac{mv^2}{k_B T}}\,dv$$

$$\langle \sigma v \rangle = \int_0^\infty \frac{1}{T^{3/2}} \frac{C}{\cancel{v^2}} e^{-\frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{2\pi}{\hbar v}} \cdot v \cdot \cancel{v^2} e^{-\frac{1}{2}\frac{mv^2}{k_B T}}\,dv$$

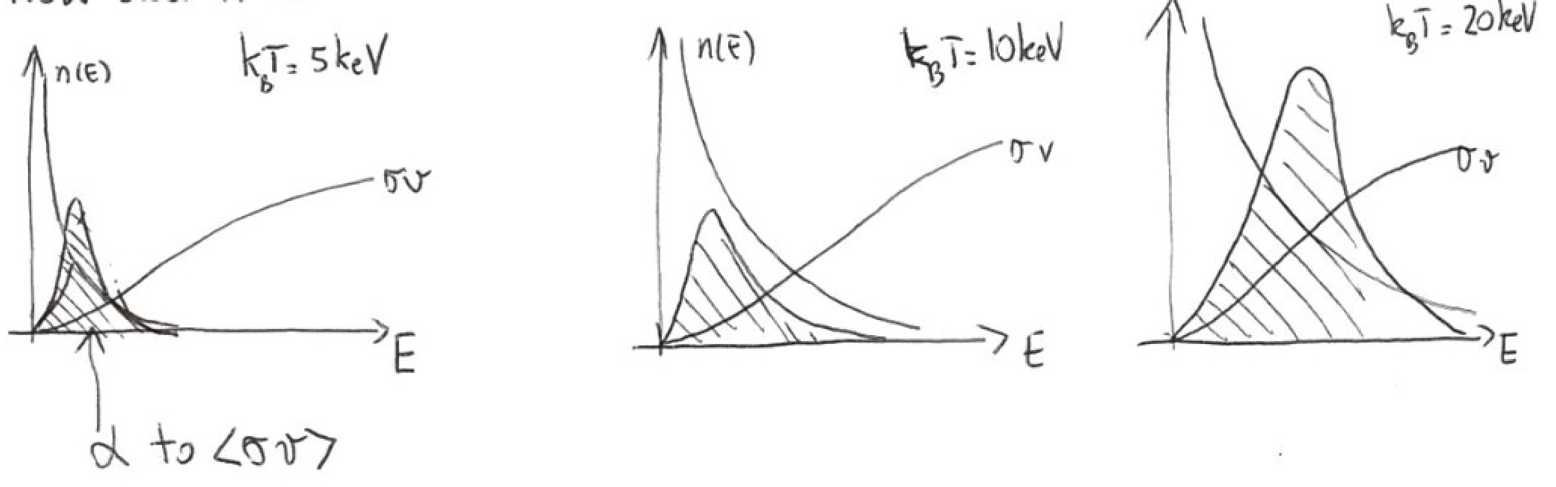
$$E = \frac{1}{2} m v^2 \qquad dE = mv\,dv \qquad v\,dv = \frac{dE}{m}$$

$$\frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{2\pi}{\hbar v} = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 \hbar}\frac{2\pi\sqrt{m}}{\sqrt{2}\sqrt{E}} = \frac{a}{\sqrt{E}} \qquad a = \frac{Z_1 Z_2 e^2\, 2\pi\sqrt{m}}{4\pi\varepsilon_0 \hbar\sqrt{2}}$$

$$v = \sqrt{\frac{2E}{m}}$$

$$\langle\sigma v\rangle = \frac{C}{T^{3/2}} \int_0^\infty e^{-\frac{a}{\sqrt{E}}} e^{-\frac{E}{k_B T}}\,dE = \frac{C}{T^{3/2}} \int_0^\infty e^{-\left(\frac{a}{\sqrt{E}} + \frac{E}{k_B T}\right)}\,dE$$

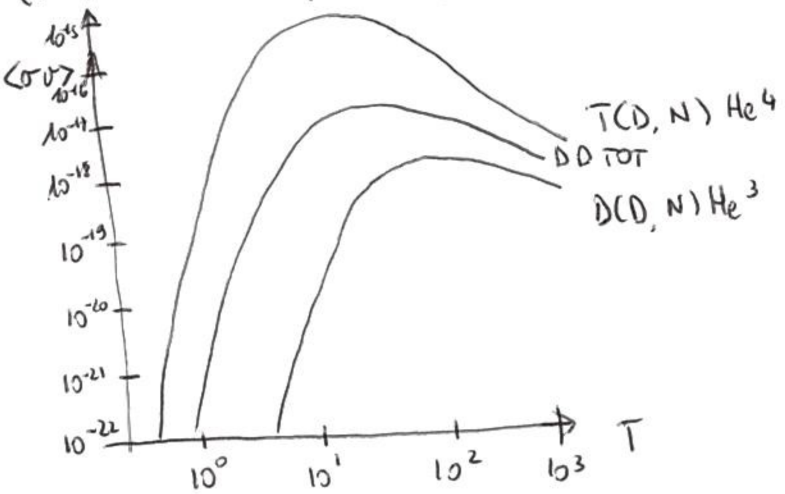
How does it behave?



At low temperature there is a little overlap between $n(E)$ and σv (10)
and the average is small. At very high T, the area of Boltzmann distribution becomes small and again the average value of σv is small.

At intermediate temperature $\langle \sigma v \rangle$ rises to a maximum.

At extremely high temperature ($T \sim 10^{10}$ K $\Rightarrow$ MeV energies) D-T reaction may become less favorable than others, but in temperature region that is likely to be achievable in a termonuclear fusion reactor (~ 10 keV $\Rightarrow T \sim 10^7 \div 10^8$ K) D-T reaction is ~~more~~ is favorable.



CONTROLLED FUSION REACTORS

In order to have controlled fusion reactions and reasonable energy, the thermonuclear fuel have to be heated up to 10^8 K (mean particle $E_k \sim 10$ keV) while it is kept to a high density long enough to generate desired power.

At these Temperature, atoms must become ionized (for H only 13,6 eV is needed to strip the electron): the fuel is a hot mixture of positive ions and negative elettrons, but overall is electrically neutral. This is called PLASMA. The electrostatic properties of a plasma determine a lenght scale called DEBYE LENGHT

$$L_D = \left(\frac{4\pi\varepsilon_0}{e^2} \frac{k_B T}{4\pi n} \right)^{1/2}$$

where n is the average density of ion or elettrons.