$0 + 1 \longrightarrow 2 + 3$

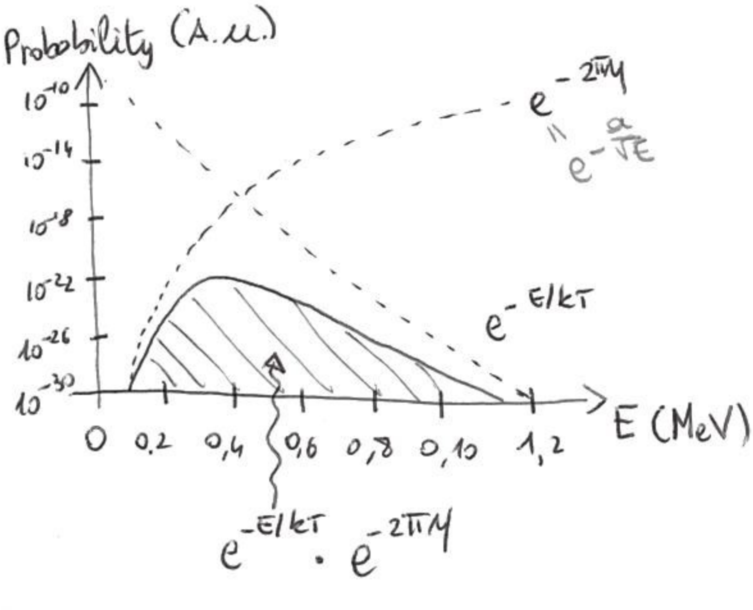
$$r_{01} = N_0 N_1 \int_0^\infty v\, P(v)\, \sigma(v)\, dv \equiv N_0 N_1 \langle \sigma v \rangle_{01}$$

$$\langle \sigma v \rangle_{01} = \left(\frac{8}{\pi m_{01}}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty E\, \sigma(E)\, e^{-E/kT}\, dE$$

$$N_A \langle \sigma v \rangle_{01} = \left(\frac{8}{\pi m_{01}}\right)^{1/2} \frac{N_A}{(kT)^{3/2}} \int_0^\infty E\, \underbrace{\sigma(E)}_{\sigma(E) \equiv \frac{1}{E} e^{-2\pi\eta} S(E)}\, e^{-E/kT}\, dE$$

$$= \left(\frac{8}{\pi m_{01}}\right)^{1/2} \frac{N_A}{(kT)^{3/2}} \int_0^\infty \underbrace{\overbrace{e^{-2\pi\eta}}^{e^{-\frac{a}{\sqrt{E}}}}\; e^{-E/kT}}_{\text{GAMOW PEAK}}\, dE$$

GAMOW PEAK
=
Energy range over which most nucleon reactions occur in the plasma



CONTROLLED FUSION REACTORS

In order to have controlled fusion reactions and reasonable energy, the thermonuclear fuel have to be heated up to 10^8K (mean particle $E_k \sim 10$keV) while it is kept to a high density long enough to generate desired power.
At these Temperature, atoms must become ionized (for H only 13,6eV is needed to strip the electron): the fuel is a hot mixture of positive ions and negative elettrons, but overall is electrically neutral.
This is called PLASMA. The electrostatic properties of a plasma determin a lenght scale called DEBYE LENGHT ≡ Measure of the charge carriers' net electrostatic effect in a solution & how far its electrostatic effect persist.

$$L_D = \left(\frac{4\pi\varepsilon_0}{e^2} \frac{k_B T}{4\pi n} \right)^{1/2}$$

where n is the average density of ion or elettrons.

Using solid densities (10^{28} ~~cm~~$^{-3}$), the Debye lenght for a 10 keV plasma is of the order of 10^{-8} m and the number of ptc in a volume of the plasme of dimension 1 Debye lenght is $\simeq 10^4$.

For a more rarified plasma ($\rho \simeq 10^{22} m^{-3}$), $L_D = 10^{-5}$ m and $n_{L_D} = 10^7$

In either cases, there are 2 basic properties: the physical size of the reacting plasma is far larger than Debye lenght in dimension, and there are many ptc in any characteristic volume.

These 2 properties permit to use plasme equations to describe thermonuclear fuel.

A major problem is to CONFINE the plasma: hot fuel exchanging energy with walls will simultaneously cool the fuel and melt the container.

2 schemes are used to confine thermonuclear fuel: a) magnetic confinement b) inertial confinement (solid pellet heated and compressed by being struck simultaneously from many directions with intense beams of γ or ptc)

Confinement of plasma in not absolute, since there are some ways ~~which~~ for the plasma to loose energy.

Primarly mechanism is bremsstrahlung (since there are electrons accelerated in a magnetic field), in which Coulomb scattering of two ptc produces an acceleration in turn which give rise to emission of radiation.

The largest accelerations are suffered by lighter ptc, but since elettron and ions are in thermal equilibrium, any loss by elettrons is felt also by ions, which will be less energetic and less succesful in penetrating Coulomb barrier.

The power radiated by an electron experiencing acceleration a is: (120)

$$P = \frac{e^2 a^2}{6\pi \varepsilon_0 c^3}$$

If r is the distance between ion and e^-

$$a = \frac{F}{m_e} = \frac{Ze^2}{4\pi\varepsilon_0 m_e r^2}$$

If τ is the characteristic time in which ion and electron interacts, the number of ions encountered at a distance r is:

$n(v_e \tau)\, 2\pi r\, dr$, $n \equiv$ density of ions

$$\Rightarrow \quad dP = \frac{e^2 n}{6\pi\varepsilon_0 c^3} \frac{Z^2 e^4 (2\pi r\, dr) v_e \tau}{(4\pi\varepsilon_0)^2 m_e^2 r^4}$$

dP = contribution to the total power from electrons scattered at impact factor r and r + dr.

$$\tau = \frac{r}{v_e} \quad \Rightarrow \quad dP = \frac{4\pi e^6 n}{3(4\pi\varepsilon_0)^3 m_e^2 c^3} \frac{dr}{r^2}$$

Integrated from r_{min} and r_{max} dP, gives the power radiated by a single electron and multiplied by n_e gives the power per unit time radiated by plasma. $r_{max} \sim \infty$; $r_{min} \equiv$ distance of closest approach.

$\Delta p \Delta x = \hbar \quad \Delta x = \frac{\hbar c}{\Delta p c}$

For a 10 keV electrons; $r_{min} = 144 Z$ fm.

The Q.M. uncertainty for such electron, and taking $\Delta p \sim p \simeq 100\,keV/c$

$\Delta x \simeq 2000\,fm \left[\Delta p \Delta x = \hbar \quad \Delta x = \frac{\hbar}{\Delta p} = \frac{\hbar c}{\Delta p c} = \frac{200\,MeV \cdot fm}{100\,keV} \simeq \frac{2 \cdot 10^8}{10^5} \simeq 2000\,fm\right]$

$\Delta x = \frac{200\,MeV \cdot f}{10\,keV} = 2$

r_{min} cannot be as precise as $144 Z$ fm (which is $\ll \Delta x$) and r_{min} is better to be expressed as: $r_{min} = \frac{\hbar}{m_e v_e}$. After the intergration:

$$P = \int_{\frac{\hbar}{m_e v_e}}^{\infty} \frac{4\pi e^6 Z^2 n n_e}{3(4\pi\varepsilon_0)^3 m_e^2 c^3} \frac{dr}{r^2}$$

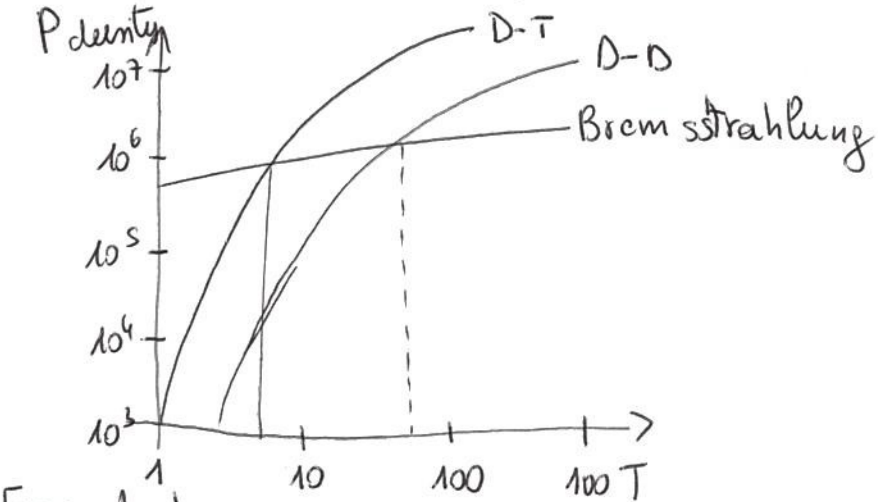
$$= \frac{4\pi n n_e Z^2 e^6 v_e}{3(4\pi\varepsilon_0)^3 m_e^2 c^3 \hbar}$$

If v is taken from Maxwell-Bolzmann distribution, $v_e = \sqrt{3k_B T/m_e}$

$$P_{Brem} = 0{,}5 \times 10^{-36} Z^2 n n_e (k_B T)^{1/2} \, W/m^3$$

where $k_B T$ is in keV.

The reaction rate for fusion is $n_1 n_2 \langle \sigma v \rangle$, if there is only one kind of ions, $n_1 n_2 = \frac{1}{2} n^2$. If one plot the power density of the reaction with the Bremsstrahlung:



FOR D-T reaction the temperature at which fusion exceed bremsstrahlung loss is of the order of $\simeq 4$ keV. For D-D it is about 40 ke

$\Rightarrow$ D-T is a superior fuel!

Energy loss by Bremsstrahlung is $\propto Z^2$; each nucleus different from H have a large bremsstrahlung and a smaller reaction rate.

Other radiation losses, including synchroton radiation from charged partic orbiting about magnetic field lines can be neglected.

Fusion reactor will have a net ENERGY GAIN if the energy released in reactions exceeds the radiation loss and the original energy invested in heating the plasma.

If reactor operates at T>4 keV, at 10 keV, the D-T fusion gain is greater than the radiation loss.

The energy released per unit volume from fusion reaction in plasma is

$$E_f = \frac{1}{4} n^2 \langle \sigma v \rangle Q \tau$$

where $\rho_D = \rho_T = \frac{1}{2} n$, $n = n_e$, Q is the energy released in the reaction ($Q = 17,6$ MeV for D-T), and τ is the lenght of time the plasma is confined.

The thermal energy per unit volume needed to raise the ions and e^- to temperature T is $\frac{3}{2} n k_B T$ (for ions) and $\frac{3}{2} n_e k_B T$ (for e^-). With $n = n_e$

$$E_{th} = 3 n k_B T$$

Reactor has an energy gain if $E_f > E_{th}$

$$\frac{1}{4} n^{\cancel{2}} \langle \sigma v \rangle Q \tau \geq 3 \cancel{n} k_B T$$

$$n\tau \geq \frac{12 k_B T}{\langle \sigma v \rangle Q} \quad \equiv \text{Lawson criterion}$$

For $T \simeq 10$ keV (D-T), $\langle \sigma v \rangle_{DT} \simeq 10^{-22}\ m^3/s$, $n\tau > 10^{10} \frac{s}{m^3}$

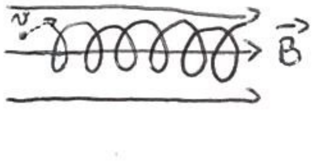
FOR D-D $k_B T = 100$ keV $\langle \sigma v \rangle_{DD} \simeq 0,5 \cdot 10^{-22}\ m^3/s$, $n\tau > 10^{22}\ s/m^3$

$\Rightarrow$ For D-D we have that: to increase the density of ions 10^{12} times to get the same gain energy.

BASIC REACTOR TYPES AND HOW THEY MEET LAWSON CRITERION

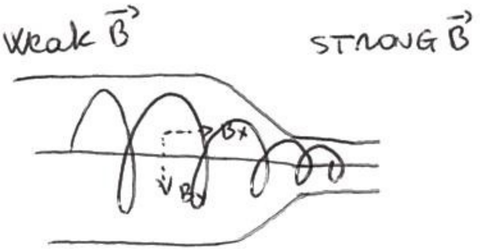
Ⓐ UNIFORM MAGNETIC FIELD

The simplest magnetic confinement is a uniform magnetic field



If a ptc moves in the field in can be confined by a simple field. At a certain point the wire will end. To overcome such problem we can think to use a non-uniform magnetic field

B) NON-UNIFOR MAGNETIC FIELD



$$\vec{v} = -v_z \hat{k} + v_x \hat{i}$$

$$\vec{B} = B_x \hat{i} - B_y \hat{j}$$

$-v_z \vec{k} \times B_y \hat{i} - v_y \hat{x} \cdot B_y \hat{j}$

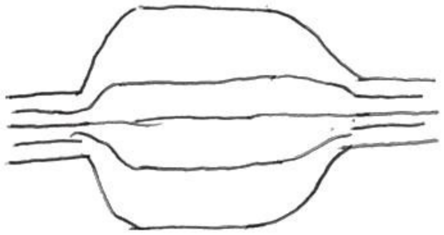
$-v_z \vec{k} \times B_y j = -v_z B_y \hat{j} - v_z B_y \hat{i} - v_y B_y \hat{z}$

$$\vec{F} = q \vec{v} \times \vec{B} = q\left[-v_z \hat{k} + v_x \hat{i}\right] \times \left[B_x \hat{i} - B_y \hat{j}\right] =$$

$$= q\left[\underbrace{\left(-v_z B_y\right)}\hat{i} + \left(-v_z B_x\right)\hat{j} + \left(-v_x B_y\right)\hat{z}\right]$$

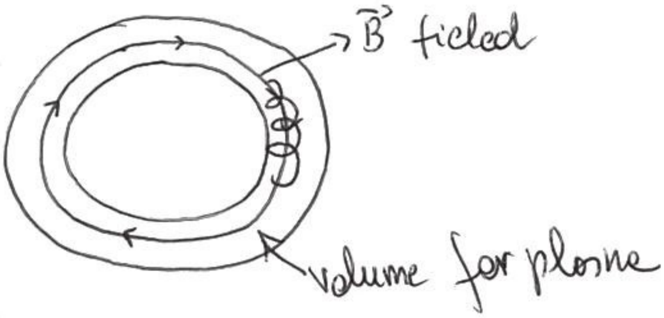
⇒ For a negative x the ptc is pushed back!

Similarly it is possible to build a magnetic bottle



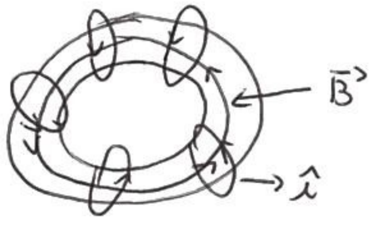
And ptc can go back and forth.

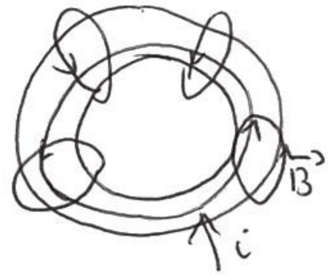
The idea which is resulting to be the best come from Russia (1968)



Like that there is a problem, only with a toroidal field: the problem is that the magnetic field is not constant with the radius

=> Combination of 2 magnetic field 1 toroidal and 1 poloidal





The combination of the 2 is at the basic of the so called TOKAMAK. (TOroidalnaja kAmera MAguitnaja Katushka)
Toroidal Camera Magnetic coil

At present exists some "Experimental" tokamak ~~...~~ accelerators such as ASDEX (Auxillary Simmetric Divertor Experiment) in Max-Planck-Institute für Plasmaphysics in Garching exist.
In 2035 it is foreseen the start of an international program called ITER (see slides).
A second idea to confine the plasma which ~~...~~ has been used is called "Inertial confinement fusion".
It is possible for example to have a tiny pellet containging deuterium and tritium suddenly struck with an intense laser pulse that both heats the pellet and compresses it to high density.
The goal of this tecnique is to achieve densities and temperatures that are high enough that fusion occur before the pellet expands and blows apart.

We can make a crude estimate for such devices, considering that the time necessary for the compressed pellet to blow apart will be determined by the speed of propagation of mechanical waves in the medium.
This velocity is $\simeq$ to the thermal speed of ptc in the medium.
($\rightarrow$ i.e. for solid at room temperature $v \simeq 10^3$ m/s)

At $kT \simeq 10\,keV$ the mean thermal speed is $\simeq 10^6\,m/s$. If the pellet has a diameter of $0{,}1 \div 1\,mm$, the pellet blow apart in $10^{-9} \div 10^{-10}\,s$. Applying Lawson's criterion for D-T, we should need a density of $\simeq 10^{29} - 10^{30}/m^3$ which is 2 order of magnitude greater than the ordinary liquid.

To heat a spherical pellet of 1mm diameter to a mean thermal energy of 10 keV, the energy that must be supplied is

$$E_{th} \simeq \frac{4}{3}\pi (0{,}5\,mm)^3 \times 10^{29}\,m^{-3} \times 10^4\,eV \simeq 10^5\,J$$

$\Rightarrow$ We must have an energy of $\simeq 10^5\,J$ for $10^{-9}\,s \Rightarrow 10^{14}\,W$!

This estimate does not take into account that a portion of energy will be absorbed by the surface and that the energy needed for laser energy is not all converted to radiation energy. These 2 contributions led to a power of $10^{17}\,W$ which can be compared with e.g. the entire electrical capacity of the USA which is of the order of $10^{12}\,W$!

This means that to run an inertial confinement reactor the Lawson law should be exceeded considerably.

To do so the laser-driven fusion should follow these steps:

1. A pellet is injected into the machine and struck from many direction by an intense laser pulse.
2. The outer layer of the solid pellet is vaporized and form a plasma which absorb the laser radiation
3. the plasma is unconfined and blow-off: by Newton's 3rd law a compressional shock wave hits the remaining core of the pellet
4. An additional thermonuclear reaction can occur inside the core near the center of the pellet.