CONTROLLED FISSION REACTIONS AND REACTORS

i.e. How can we use the fission to produce energy in a controlled way.

Let's consider a infinitely large mass of Uranium, which for now we assume to be of natural isotopic composition (0,72% ^{235}U, 99,28 ^{238}U)

A single fission will produce 2,5 neutrons on average.

Eeach "second-generation" neutron is capable to produce other fissions producing each time more neutrons. This is called "chain reaction"

Each fission releases $\simeq 200$ MeV in form of kinetic energy and radiation.

It can be convenient to define a neutron reproduction factor k_∞ (for infinite medium => ignoring the loss of neutron at the surface)

Each thermal neutron produces k_∞ new thermal neutron.

For a reaction chain to continue we must have $k_\infty \geq 1$.

Althought for each reaction there 2.5 n, there are fast neutrons

Neutron lose energy in elastic collisions with nuclei. A popular choice for moderator is Carbon in form of graphite blocks.
(Why? C is solid, inexpensive and easy to handle).
A lattice of blocks of U alternated with graphite is called "chain-react pile, and the first one was constructed by Fermi in 1942.

If $k = 1 \Rightarrow$ The pile is critical $\longrightarrow$ Steady release of energy
$k < 1 \Rightarrow$ The pile is subcritical
$k > 1 \Rightarrow$ " supercritical

Let's assume we have N thermal neutrons at the present generation.
Even if we will not have νN fast fission neutrons immediately available
Some will be absorbed by many processes, mainly (n,γ) in both ^{235}U and ^{238}U.
η is defined as mean number of fission neutron produced per original thermal neutron. $\eta < \nu$ in generale
If σ_f is the fission cross-section, and σ_a the cross-section for absorptive processes the relative probability for a neutron to cause fission is

$$p = \frac{\sigma_f}{\sigma_f + \sigma_a}$$

and then
$$\eta = \nu \frac{\sigma_f}{\sigma_f + \sigma_a}$$

For ^{235}U $\sigma_f = 584\,b$ $\sigma_a = 97\,b$ $\Rightarrow \eta = 2{,}08$ fast neutrons are produced per thermal neutron

^{238}U is not fissionable so $\sigma_f = 0$ and $\sigma_a = 2{,}75\,b$. For a natural mixture of Uranium:

$$\sigma_f = \frac{0{,}72}{100}\sigma_f(235) + \frac{99{,}28}{100}\sigma_f(238) = 4{,}20\,b$$

$$\sigma_a = \frac{0{,}72}{100}\sigma_a(235) + \frac{99{,}28}{100}\sigma_a(238) = 3{,}45\,b$$

$$\Rightarrow \eta_{NAT} = 1{,}33$$

Very close to 1, so we should minimize the neutrons to be lost in order to obtain a critical reactor.
If enriched Uranium is used ($^{235}U = 3\%$) then $\eta = 1{,}84$, allowing more neutron to be lost by other means to maintain the criticality conditions.
At this point the N thermal neutrons have been partially absorbed and now there are ηN neutrons to slow down to thermal energies.
These fast neutron can fission with ^{238}U ($\sigma \simeq 1\,b$), increasing the number of neutrons -> A factor ε (fast fission factor) has to be taken into account -> $\eta \varepsilon N$ neutrons are present ($\varepsilon = 1{,}03$ for natural Uranium)
Moderation of n is accomplish by light moderator.
It takes $\simeq$ 100 collissions in carbon to moderate a MeV neutron down to thermal energy.
But, in the process, n must pass through the resonance region with high σ (up to 1000b $\gg \sigma_{fission}$) => If we want thermal neutrons we have to avoid resonance capture at all.
If C and U are mixed in powder this is impossible to achieve, but if there is enough C n can scatter inside it avoiding resonance region
About 20 cm of C are needed to moderate MeV n down to 10^{-3}eV.
Nevertheless there will be a probability p that some n are captured.
$p \simeq 0{,}9$. The # of "usable" n now is $n_{neut} = \eta \varepsilon p N$
Now that n are thermalized they must interact with ^{235}U.
Note that n can still interact with C ($\sigma_{fission} = 0{,}0034\,b$), because there i

a lot of it. ⇒ An additional term must be included.

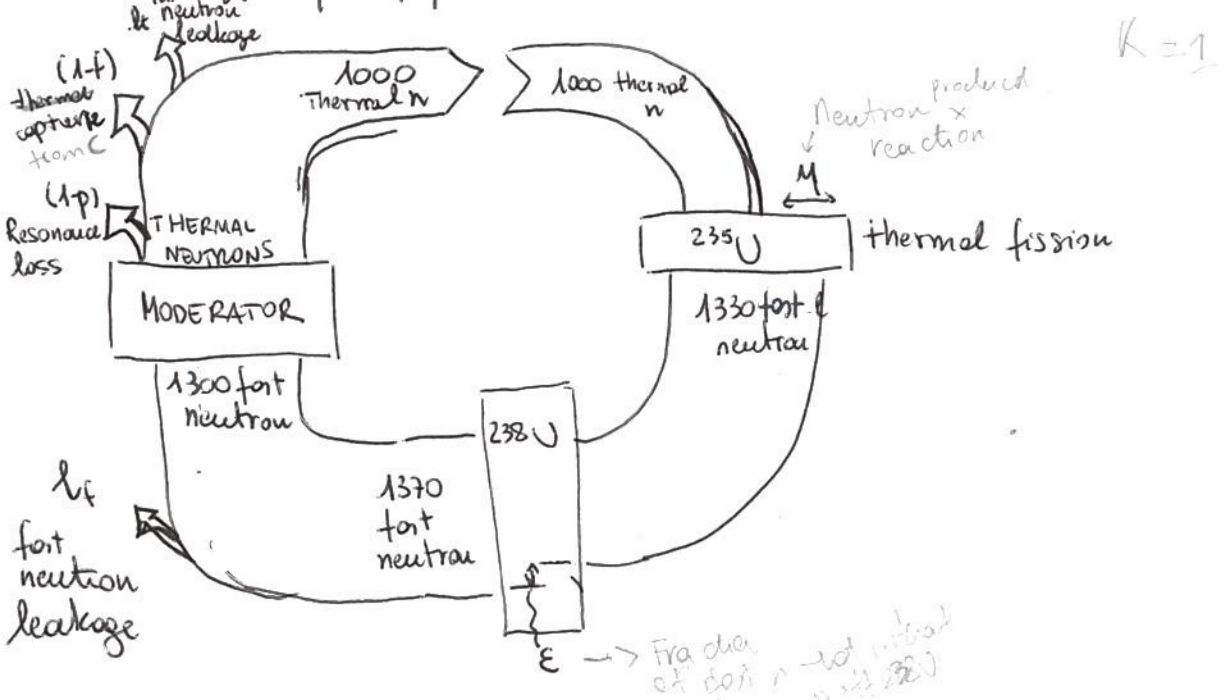
f = thermal utilization factor ≡ fraction of thermal neutron available to ^{235}U and ^{238}U. $f \simeq 0,9$

Find number of n $n_{neut} = \eta \varepsilon p f N$ wheter this is > or < than N determines the criticality of reactor.

The reproduction factor is

$k_\infty = \eta \varepsilon f p$ AKA "Four-factor formula"

$\eta = 1,33$
$\varepsilon = 1,03$
$p = 0,9$
$f = 0,9$
$\simeq 1,11$



$k_\infty \simeq 1,11$ → which is still not appropriate because we ingnored leakage of n at the surface ⇒ l_f and l_t are fraction of leakage of fast and thermal neutrons

$$k_\infty = \eta \varepsilon p f (1-l_f)(1-l_t)$$

The larger the pile, the smaller is Surface/Volume ratio ⇒ the smaller is the fraction of neutrons that leak

If l_f, l_t are small → $k_\infty - k \simeq k(l_f + l_t)$

the total leake decreases as the surface increases.

Leakage will increase if neutron have to travel for a long distance, called MIGRATION LENGHT M.

M includes 2 contributes diffusion lenght L_d for thermal neutrons and the slowing distance L_s

$$M = (L_d^2 + L_s^2)^{1/2}$$

For graphite $L_S = 18{,}7$ cm $L_d = 50{,}8$ cm

If the pile has a dimension R (radius or side lenght)

$$(k_\infty - k) \propto R^{-2} \quad \text{and} \quad k_\infty - k \propto M$$

Are reasonable assumptions

$$\rightarrow k_\infty - k \propto \frac{M^2}{R^2}$$

So there will be a critical size R_c which corresponds to $k=1$

$$R_c \propto \frac{M}{\sqrt{k_\infty - 1}}$$

From spherical arrangement

$$R_c = \frac{\pi M}{\sqrt{k_\infty - 1}}$$

For neutral uranium-graphite reactor $R_c = 5$m. For $R = 5$m the "reactor" goes critical. How to decrease the size? Surround the pile with a reflector material for neutron.

Let's now evaluate now the time needed for the processes.

Neutrons are characterized by a time constant τ which includes the time to moderate ($\simeq 10^{-6}$s) and a diffusion time ($\simeq 10^{-3}$s).

If the reproducing factor is k and there are N neutrons at time t, on average there are kN neutrons at time $t+\tau$ and k^2N at time $t+2\tau$

In a short interval dt

$$dN = (kN - N)\frac{dt}{\tau}$$

$$\Rightarrow N(t) = N_0 e^{(k-1)t/\tau}$$

If $k = 1 \Rightarrow N =$ constant. If ~~k~~ $k<1$ the number of neutrons decreases (115)
exponentially. If $k>1$ N increases exponentially, with a time constant
$\tau/k-1$.
Control. the number of neutron is essential and can be achieved by ~~by~~ interting ~~the~~ some bars of materials able to absorb neutrons (like Cadmium).

BASICS OF NUCLEAR REACTORS

We will cover only the general properties ~~of~~ and categories of reactors.