

The 2 colliding nuclei moves along the "light cone", until they collide in the center of the diagram. Nuclear fragments emerge from the collision again along the (forward) cone, while the matter between the fragmentation zone populate the central region.

This hot dense matter is believed to be the state of QGP. Interactions within it bring the system into local statistical equilibrium, and its further evolution can be described by relativistic hydrodynamics.

The surfaces of constant proper time, delineating various stages of this evolution, are approximately hyperbolae in this representation.

Let's now try to estimate the ~~maximal~~ energy density reached in relativistic heavy ion collision.

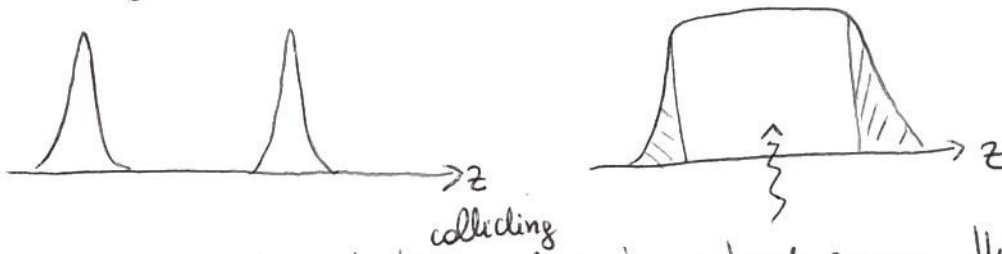
The concept on "energy density" is only apparently elementary, in fact we have to develop a precise definition which can be associated to an experimental measurement. Hence, something else should be applied: the so-called "Bjorken Regime."

The starting point of such Regime is:

- ① The collision have to be treated within the hydrodynamics model of Landau.

2) We observe ~~the~~ a central plateau in the inclusive particle production⁽¹⁷⁾ as a function of rapidity [Rapidity = see page 141 back; $y = \frac{1}{2} \ln \left(\frac{p_0 + p_z}{p_0 - p_z} \right)$ $v\beta$ for $\beta < 0.5$]

3) In the central rapidity region the net baryon number is null. (\Rightarrow Δ baryon n_B in the tails of distributions, while at mid-rapidity only "new" particles are produced)



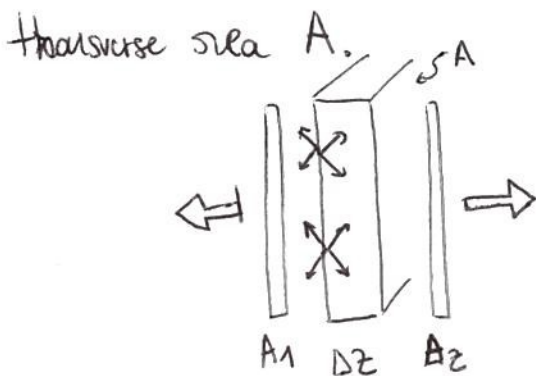
2) In the frame where both nuclei have high energy, they can be seen as thin disks which cross each other very rapidly

\Rightarrow "secondary" (i.e. new) particles are generated all together in an initial volume with a limited extension on the longitudinal axis.

There^{is} the fundamental Bjorken Condition.

Under all these conditions the central rapidity region is populated only by secondary particles. It is then possible to evaluate energy density of the formation time. (i.e. when ^{secondary} ptc are formed) (τ_f)

The region under study is the longitudinal slice of thickness Δz and



This region contains all the ptc with velocity β in the interval

$$0 \leq \beta_z \leq \frac{\Delta z}{\tau_f}$$

The number of particles in the region is:

$$\Delta N = \int_0^{\frac{\Delta z}{\tau_f}} \frac{dN}{d\beta_z} d\beta_z \approx \frac{dN}{d\beta_z} \frac{\Delta z}{\tau_f} = \frac{\Delta z}{\tau_f} \frac{dN}{dy}$$

$y \rightarrow \beta$ per $y \rightarrow 0$

\rightarrow This can be measured

$$E = m_T \cosh y \approx m_T \quad y \rightarrow 0$$

$$\langle E(\gamma_f) \rangle = \frac{\Delta N \langle m_T \rangle}{\Delta z A} = \frac{\Delta z}{\gamma_f} \frac{dN}{dy} \frac{\langle m_T \rangle}{\Delta z \cdot A} = \frac{1}{\gamma_f A} \langle m_T \rangle \frac{dN(\gamma_f)}{dy}$$

$$= \frac{1}{\gamma_f A} \cdot \frac{dE_T(\gamma_f)}{dy}$$

$$\boxed{\varepsilon_{Bj} = \frac{1}{\gamma_f A} \cdot \frac{dE_T(\gamma_f)}{dy}}$$

Energy density

Conditions for validity:

- ① γ_f have to be defined
- ② $\gamma_f \gg 2 \frac{R}{\lambda}$
- ③ Regime of nuclear transparency

HISTORICAL USE OF BJORKEN FORMULA:

- ① Transverse energy density of the final state
- ② Formation time assumed to be $1 \text{ fm}/c$

→ Formation time

From the indetermination principle a particle is formed at a time

$$t = \frac{\hbar}{m_T}$$

m_T can be estimated on the final states using the relation

$$\langle m_T \rangle = \frac{\frac{dE_T}{cY}}{\frac{dN}{dy}} \quad \text{in the final state}$$

At RHIC it has been possible to obtain the formation time $\gamma_{\text{form}} \approx 0,35 \text{ fm}/c$

Taking $dE_T/dy/dN_{ch}/dy = 0,85 \text{ GeV}$, after converting dN/dy a $\langle m_T \rangle \approx 0,57 \text{ GeV}$ is obtained. (13)

Assuming $\tau_{\text{form}} = \frac{\hbar}{m_T} \Rightarrow \tau_{\text{for}} \approx 0,55 \text{ fm/c}$

This value is $\ll 1 \text{ fm/c}$, but larger than $2R/8$

In the most central collisions at RHIC an energy density $\epsilon = 15 \text{ GeV/fm}^3$ relative to the particles formed at mid-rapidity.

This value is a ~~procedural~~ estimation, since obtained from the measurement in the FINAL state.

In this phase of the collision, particles are not yet thermalized: at the time of thermalization the energy density would be enough to allow for the existence of QGP?

As long as the fireball expansion is mainly longitudinal, the Bjorken formula can be used, using time t instead of formation time τ_f .

For measurement it is possible to obtain a energy density of thermalization

$$5.4 \leq \epsilon_{\text{th}} \leq 9.0 \text{ GeV/fm}^3$$

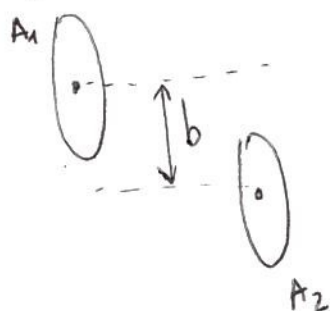
which is enough to create a QGP. It means that the system can evolve as a deconfined state.

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As we already seen the energy density can be evaluated using the number of particles which are produced ("multiplicity").

The multiplicity is related to the centrality of the collision.

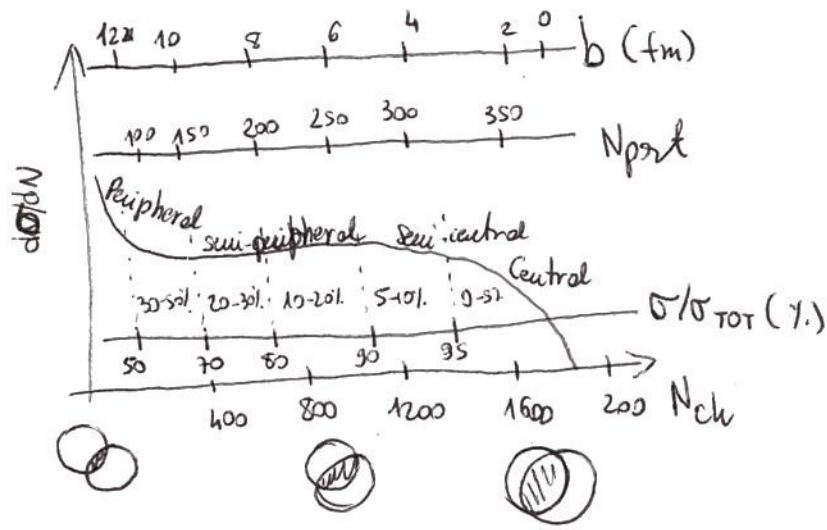
The centrality (~~inverse~~ [↔] of impact parameter) have to be defined since nuclei are extended objects. Moreover, it is possible to have some nucleus which collide:



to ~~the collisions~~ ($N_{\text{collision}} = N_{\text{coll}}$) and the number of participant ~~collisions~~ nucleus to the collision ($N_{\text{participant}} = N_{\text{part}}$).

Every collision / participant contributes to the particle production, so to the multiplicity.

Measuring the multiplicity it is, then, also possible to measure the $\langle N_{part} \rangle$ and b .



A brief excursus on "centrality".

The nucleus-nucleus interaction have to be described in terms of elementary collisions between nucleons.

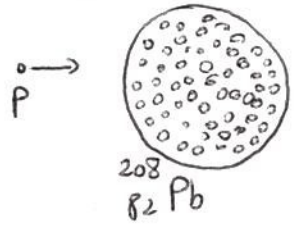
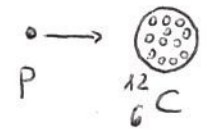
In pp collisions there are only 2 nucleons that participate to the collision: 1 projectile and 1 target which collide only 1 time among each other.

$$N_{collisions} = N_{coll} = 1$$

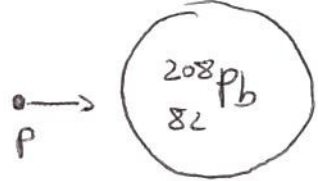
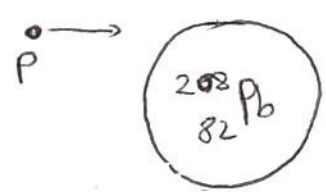
$$N_{participant} = N_{part} = 2$$

In p-A collisions there are more than two nucleons: the projectile p which collide several times with the nucleons (p, n) of the target A.

The number of collisions increases with the size of the target.



and with the impact parameter



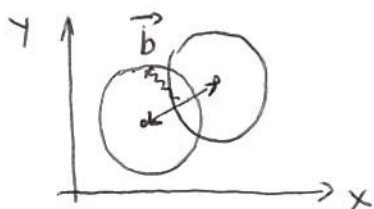
$$N_{coll} = N$$

$$N_{part} = N + 1$$

The impact parameter in the transverse plane is defined as the distance between the projectile and the center of the target nucleus



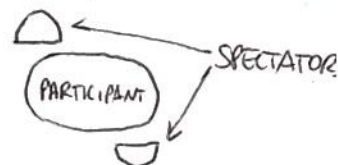
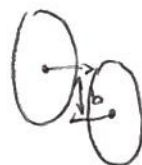
In AA collisions the impact parameter is defined as the vector in the transverse plane between the center of the 2 nuclei



The impact parameter determines the centrality of the collision:

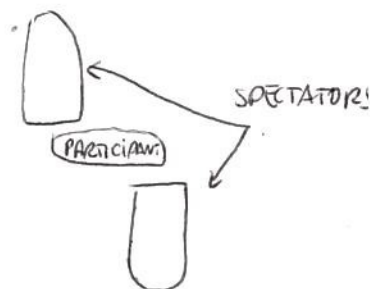
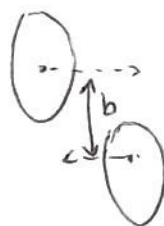
- Small impact parameter \equiv central collisions

- Many nucleons
- Many collisions
- Big interaction volume
- Many produced particles



- Large impact parameter \equiv peripheral collisions

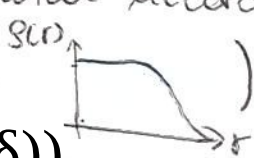
- Few nucleons
- Few collisions
- Small interaction volume
- Few produced particles



Impact parameter is, unfortunately, not measurable. There exists two complementary approaches to obtain an estimate of the centrality of a collision: one based on the count of non-interacting nuclei (i.e. Spectators), and the other uses the characteristics of produced particles (e.g. total multiplicity, transverse energy, ...)

The participant/spectators picture allows us for a simple calculation on the number of nucleons involved in the collision occurring with a given value of impact parameter b .

This is done using the "Glauber model", assuming that nucleons in each nucleus are hard sphere distributed according to the nuclear density function (e.g. Woods-Saxon) and they move along parallel

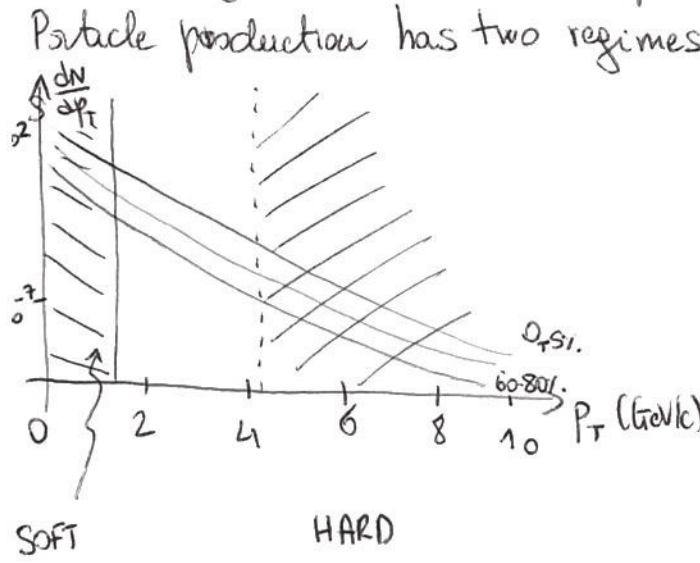


$$\rho = \rho_0 / (1 + \exp((r-r_0)/\delta))$$

straight lines, interacting with nucleons from the other nucleus with cross-sections known from elementary processes (i.e. p-p, p-n, n-n processes). Nucleons are treated as free particles and their internal motion and correlations are neglected.

When counting only the first collisions one obtains the number of nucleons participant N_{part} , counting also subsequent collisions one obtains the total number of binary collisions N_{coll} .

Let's now go back to particle production:



HARD PROCESSES: high transferred momentum ($p_T \gtrsim 4 \text{ GeV}/c$) \leftrightarrow small distances

- Parton-level interactions
- Rare processes (i.e. small cross-section) σ_{hard}

- They scale with nucleon-nucleon collisions $P_{AB}^{hard}(b) \propto \sigma_{hard} N_{part}$

"Second regime"

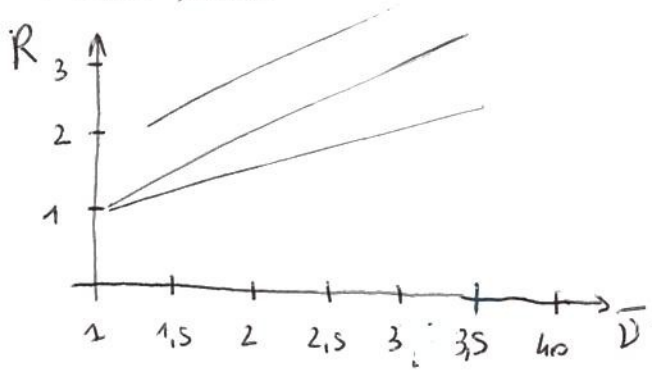
SOFT PROCESSES: The 99.5% of hadrons produced in a collision have $p_T \lesssim 1 \text{ GeV}$

- The multiplicity scales with the number of participant ("colpiti" Modello di nuclei colpiti)
- (Wounded nuclear model)

The wounded nuclear model is based on the experimental observation that the measured multiplicities in p-A collisions scale like:

$$R = \frac{N_{ch}^{pA}}{N_{ch}^{pp}} \approx \frac{1}{2} + \frac{1}{2} \bar{v}$$

\bar{v} is the average number of elementary collisions between nuclei ($= N_{coll}$)



$$R = \frac{N_{ch}^{PA}}{N_{ch}^{PP}} \approx \frac{1}{2} + \frac{1}{2} N_{coll}^{PA} = \frac{N_{coll}^{PA} + 1}{2} = \frac{N_{part}^{PP}}{N_{part}^{PP}}$$

Recalling that in (139)
 -pp : $N_{part} = 2$
 -pA : $N_{part} = N_{coll} + 1$

Why does this happen?

"Soft" multiplicity scales with N_{part} because the soft particle production happens like this:

- ① The hit nucleon become excited, with a long averaged lifetime
- ② ~~There~~ Any subsequent collisions do not significantly alter the formed "baryon-like" object
- ③ The long averaged lifetime and Lorentzian expansion of the time means that the baryon-like object traverses the entire nucleus before "decay".
 \Rightarrow formation time $\tau_f = \hbar/E$ of "soft" particles is long enough that their materialization takes place outside the nucleus.

Soft particle production

- ① Happen outside the colliding nuclei
- ② Is independent from the number of collisions ~~which~~ suffered by each nucleus
- ③ It only depends on the number of nucleons that have undergone at least one collision that excited it \Rightarrow It scales with N_{part} .

MEASURING MULTIPLICITY

Experimentally, it is possible to measure the multiplicity of charged (ionizing) particles, which can be detected (i.e. in region covered by detectors).

It is hence difficult to compare results from different experiments. That's why multiplicity are expressed in terms of particle density in a certain polar interval.

Usually the number of ptc in a region of (pseudo)rapidity around midrapidity are used ($N_{ch}(|\eta| < 0,5)$ or $N_{ch}(|\eta| < 0,5)$).

Such distributions contain other information on the dynamic of the interaction. Pseudo-rapidity is ~~more~~ more easily accessible because it requires to measure only one quantity (i.e. the angle) and does not require particle identification.

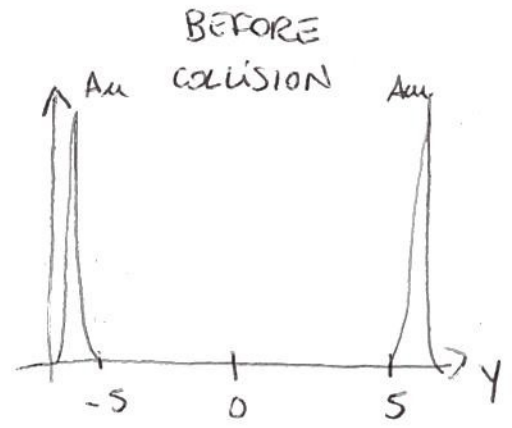
MULTIPLICITY DISTRIBUTION

Rapidity distribution in a collider for pp

E.g. $P_{\text{BEAM}} = 100 \text{ GeV}/c$ x nucleon

$$E_{\text{BEAM}} = \sqrt{m_p^2 + P_{\text{BEAM}}^2} = 100,0044 \text{ x nucleon}$$

$$\beta = 0,999956 \quad \gamma_{\text{BEAM}} \approx 100$$

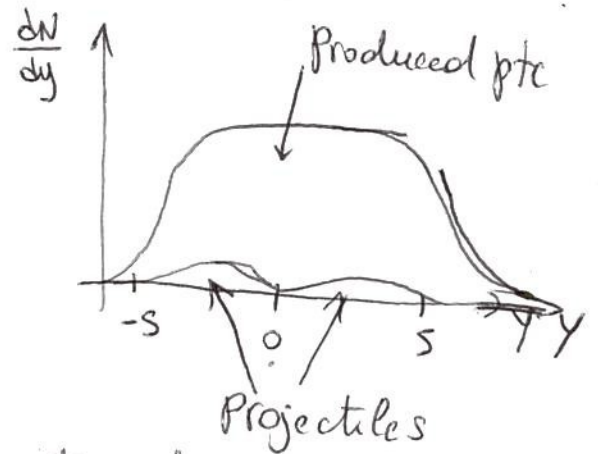


$$Y_{\text{PROJ}} = -Y_{\text{TARGET}} = \frac{1}{2} \ln \frac{E_{\text{BEAM}} + P_{\text{BEAM}}}{E_{\text{BEAM}} - P_{\text{BEAM}}} = \frac{1}{2} \ln \frac{1+\beta}{1-\beta} = 5,36$$

After the collision

The colliding nuclei are slowed down and γ, β, γ are lower

The produced particles are distributed in the ~~region~~ kinematic region between initial and final rapidity



$$Y_{\text{MID}} = \frac{Y_{\text{PROJ}} + Y_{\text{TARG}}}{2} = 0 \Rightarrow \text{The largest fraction of ptc are produced at mid-rapidity}$$

Rapidity distribution in a fixed target collider

Before collision

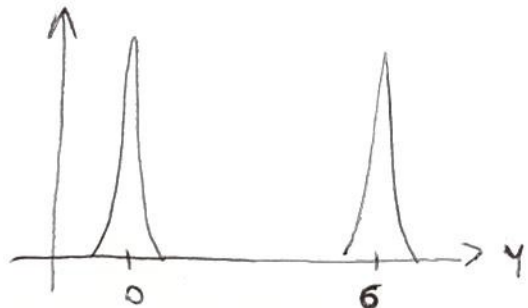
E.g. $P_{\text{BEAM}} = 158 \text{ GeV}/c$ $\beta_{\text{BEAM}} = 0,999982$

$$P_{\text{TARGET}} = \beta_{\text{TARGET}} = 0$$

$$Y_{\text{PROJ}} = \frac{1}{2} \ln \frac{E_{\text{BEAM}} + P_{\text{BEAM}}}{E_{\text{BEAM}} - P_{\text{BEAM}}} = \frac{1}{2} \ln \frac{1+\beta}{1-\beta} = 5,82$$

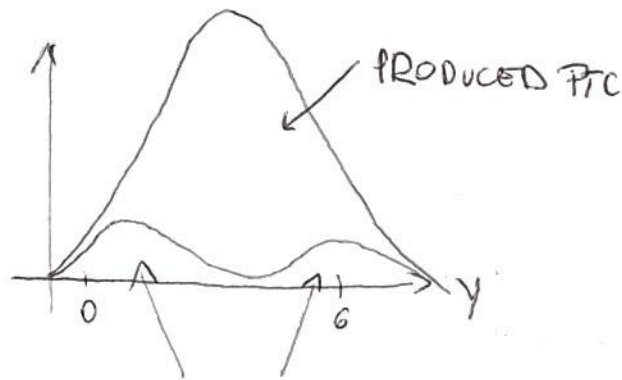
$$Y_{\text{TARGET}} = \frac{1}{2} \ln 0 = 0$$

$$\Delta Y = Y_{\text{PROJECTILE}} - Y_{\text{TARGET}} = 5,82$$



MID-RAPIDITY

$$Y_{MID} = \frac{Y_{proj}}{2} = 2,91$$

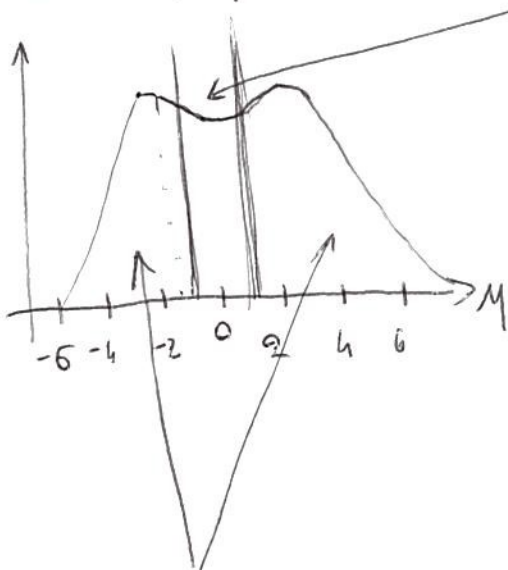


TARGET + Projectile

$\frac{dN}{dy}$ in the center of mass frame can be obtained from the one measured

in the laboratory center-of-mass $y' = y - Y_{MID}$ ($\frac{dN}{dy}$ on the other hand cannot!)

PSEUDO-RAPIDITY



MID-RAPIDITY REGION is populated by

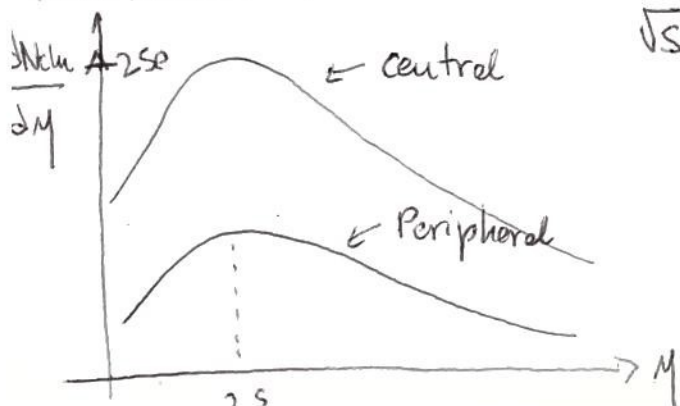
- ptc with $p_T > p_L$ produced at angles $\approx 90^\circ$
- The Bjorken formula can be used to estimate energy density when there is a plateau of mid-rapidity, which is invariant for Lorentz-boost

$$E_{Bj} = \frac{\langle M_T \rangle}{AC \gamma_f} \left(\frac{dN}{dy} \right)_{y=0}$$

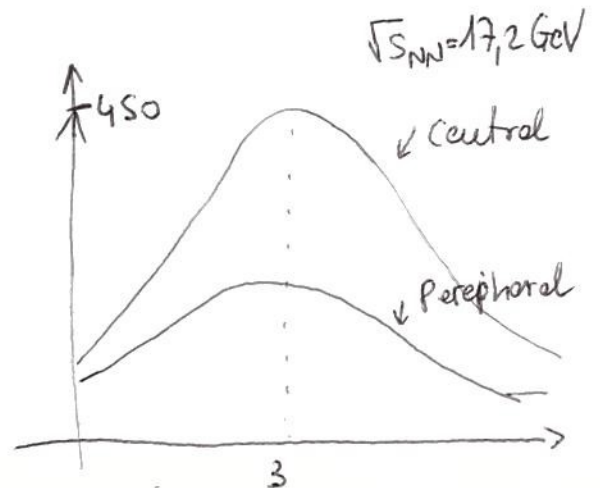
FRAGMENTATION REGION is populated by

- ptc with $p_L \gg p_T$ produced in the fragmentation of ^{colliding} nuclei ~~in~~ in the region around 0° and 180° .

MULTIPLICITY DISTRIBUTION: FIXED TARGET



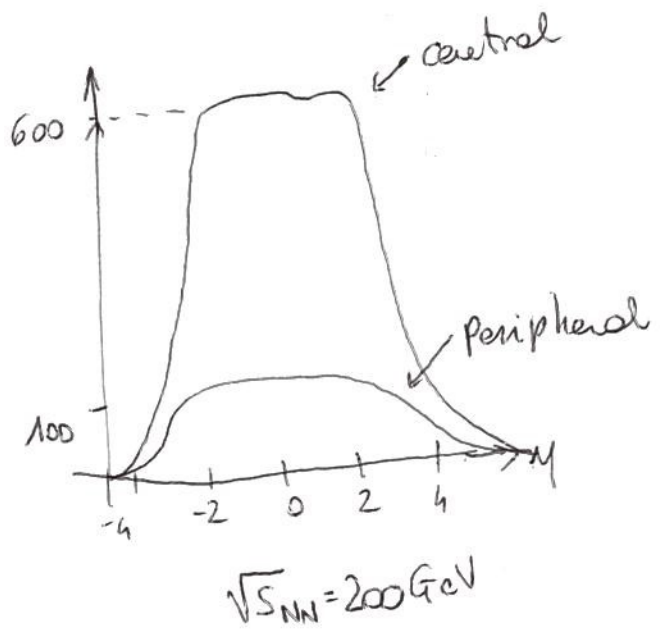
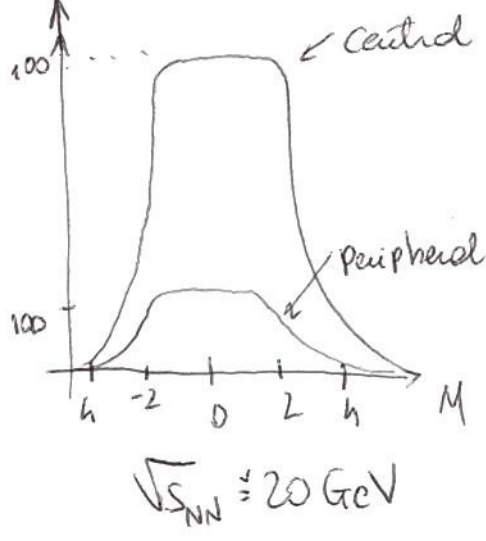
$\sqrt{s_{NN}} = 8,77 \text{ GeV}$



$\sqrt{s_{NN}} = 17,2 \text{ GeV}$

- Particle density increases with Energy increase
 - The peak position moves ($\gamma_{HID} = \frac{\gamma_{Beam}}{2}$)

MULTIPLICITY DISTRIBUTION @ COLLIDER



At LHC in the 0-5%.

$$\langle dN_{ch}/dy \rangle = 1943 \pm 54$$

$$N_{ch, |M| \leq \gamma_{Beam}} = 21,400 \pm 1300$$

Energy density at the RHIC ($\sqrt{s_{NN}} = 200 \text{ GeV}$)

$$E_{Bj} = \frac{\langle M_r \rangle}{A C \gamma_f} \left(\frac{dN}{dy} \right)_{y=0} = \frac{0,6 \text{ GeV}/c^2}{145 \text{ fm}^2 \times C \times \gamma_0} \left(700 \times \frac{3}{2} \times 4,1 \right) \begin{cases} \rightarrow 15 \text{ GeV}/c^2 \quad \gamma_0 = 0,55 \text{ fm}/c \\ \rightarrow 5 \text{ GeV}/c^2 \quad \gamma_0 = 1 \text{ fm}/c \end{cases}$$

$\rightarrow \gg$ the critical density $E_c \approx 0,5 - 1 \text{ GeV}/\text{fm}^3$ predicted by Lattice QCD

At the LHC for $\sqrt{s_{NN}} = 2,76 \text{ TeV}$

$$E_{Bj} \gamma \approx 15 \text{ GeV}/(\text{fm}^2 c) \approx 3 E_{Bj}^{RHIC}$$

In fact

$$\begin{pmatrix} E' \\ P_z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} E \\ P_z \end{pmatrix}$$

$$C_0' = \gamma C_0 - \gamma\beta C_z = \gamma(C_0 - \beta C_z)$$

$$C_z' = \gamma(C_z - \beta C_0)$$

$$C_+' = C_0' + C_z' = \gamma(1-\beta)(C_0 + C_z) = \gamma(1-\beta)C_+$$

$$X_+' = \frac{C_0' + C_z'}{b_0' + b_z'} = \frac{\gamma(1-\beta)(C_0 + C_z)}{\gamma(1-\beta)(b_0 + b_z)} = X_+$$

For large energies X_+ is the fraction of longitudinal momentum of C w.r.t. b

RAPIDITY

Rapidity is defined as:

$$y = \frac{1}{2} \ln \left(\frac{P_0 + P_z}{P_0 - P_z} \right)$$

Rapidity is an adimensional quantity.

Let's consider a ptc which travels along z with velocity β :

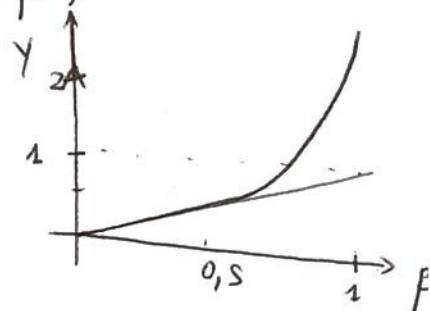
$$P_0 = \gamma m$$

$$P_z = \gamma\beta m$$

$$\rightarrow y = \frac{1}{2} \ln \left(\frac{1+\beta}{1-\beta} \right) = \frac{1}{2} \ln \left(1 + \frac{2\beta}{1-\beta} \right)$$

$$y \approx \frac{1}{2} \left(\frac{2\beta}{1-\beta} - \frac{1}{2} \left(\frac{2\beta}{1-\beta} \right)^2 \right) \approx \beta$$

For small β



Rapidity is not a Lorentz invariant variable, but its transformation law is very simple.

In the system frame F'

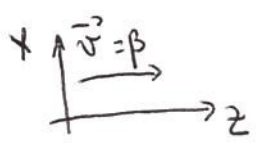
$$y' = \frac{1}{2} \ln \left(\frac{p_0' + p_z'}{p_0' - p_z'} \right)$$

$$\begin{cases} p_0' = \gamma (p_0 - \beta p_z) \\ p_z' = \gamma (p_z - \beta p_0) \end{cases}$$

$$y' = \frac{1}{2} \ln \left(\frac{\gamma(1-\beta)(p_0 + p_z)}{\gamma(1+\beta)(p_0 - p_z)} \right) = y - \frac{1}{2} \ln \left(\frac{1+\beta}{1-\beta} \right)$$

$$y' = y - y_B$$

y_B is the rapidity of a particle in system F' when it travels with velocity β of the system in movement



This quantity is

$$\begin{aligned} p_0 &= \gamma m \\ p_z &= \gamma \beta m \end{aligned} \rightarrow y' = \frac{1}{2} \ln \left(\frac{\gamma m (1+\beta)}{\gamma m (1-\beta)} \right) = \frac{1}{2} \ln \left(\frac{1+\beta}{1-\beta} \right) = y_B$$

For a free ptc (on-shell condition)

$$p^2 = p^\mu p_\mu = p_0^2 - \vec{p}^2 = m^2$$

In this relation the number of degree of freedom transverse and longitudinal are not separated. ~~The~~ ^{The relation} can be rewritten as:

$$p_0^2 - p_z^2 = m^2 + p_T^2 = m_T^2$$

Where $p_T = \sqrt{p_x^2 + p_y^2}$ Transverse momentum $m_T = \sqrt{m^2 + p_T^2}$ Transverse mass

\Rightarrow The four momentum of a ptc on the on-shell has only 3 degree of freedom.

Usually these NDF are y and p_T (which are obtained by integrating on azimuthal variables)

What is the relation between (y, p_T) and (p_0, p_z) ?

From the definition of rapidity

$$y = \frac{1}{2} \ln \left(\frac{p_0 + p_z}{p_0 - p_z} \right)$$

$$e^y = \sqrt{\frac{p_0 + p_z}{p_0 - p_z}} \quad e^{-y} = \sqrt{\frac{p_0 - p_z}{p_0 + p_z}}$$

$$e^y + e^{-y} = 2 \cosh y = \sqrt{\frac{p_0^2 - p_z^2}{(p_0 - p_z)^2}} + \sqrt{\frac{p_0^2 - p_z^2}{(p_0 + p_z)^2}} = \sqrt{p_0^2 - p_z^2} \left(\frac{1}{p_0 - p_z} + \frac{1}{p_0 + p_z} \right) = m_T \frac{2p_0}{M_T^2}$$

$$p_0 = m_T \cosh y \quad \text{and} \quad p_z = m_T \sinh y$$

PSEUDORAPIDITY

Experimentally, the measurement of the rapidity require the identification of the particle, which is not always easy, or the measurement of two momenta pairs.

In many cases it is possible to measure ~~rather~~ the angle of emitted ptc quite easily. It is then possible to define the pseudorapidity η as

$$\eta = -\ln \left(\tan \left(\frac{\theta}{2} \right) \right)$$

The same variable can be written as a function of the momentum as:

$$\eta = \frac{1}{2} \ln \left(\frac{|\vec{p}| + p_z}{|\vec{p}| - p_z} \right) \quad \text{recalling that} \quad \tan \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$$

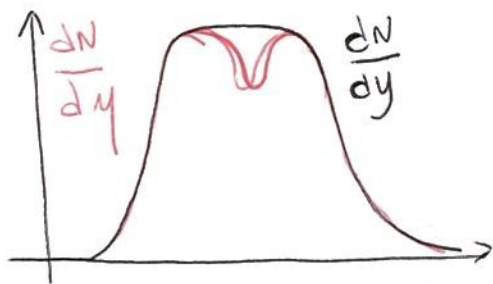
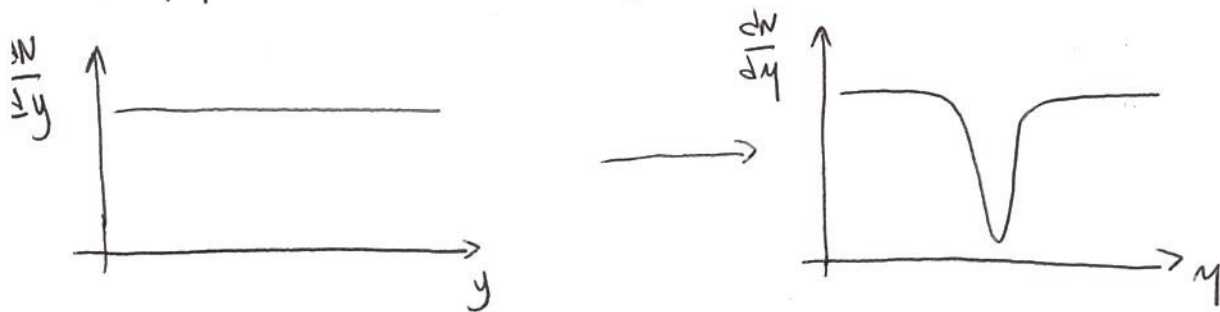
$$\eta = -\ln \left(\sqrt{\frac{1 - \frac{p_z}{|\vec{p}|}}{1 + \frac{p_z}{|\vec{p}|}}} \right) \Rightarrow \eta = -\ln \left(\tan \left(\frac{\theta}{2} \right) \right)$$

For relativistic particles

$$|\vec{p}| \simeq |\vec{p}| \Rightarrow \eta \simeq y$$

η and y are NOT the same thing because a distortion is introduced. (143)

$$\frac{d^2 N}{dy dp_T} = \sqrt{1 - \frac{m^2}{m_T^2 \cosh^2 y}} \frac{d^2 N}{dy dp_T}$$



CRITICAL ENERGY DENSITY

Temperature and energy density are ~~linearly~~ related by the Stefan-Boltzmann law.

For a 2 flavour QGP

$$E_c = \left\{ \underbrace{2f \cdot 2s \cdot 2q \cdot 3c \cdot \frac{7}{8} + 2g \cdot 8}_{\text{quark and gluons N.D.F.}} \right\} \frac{\pi^2}{30} \cdot \frac{4}{c} \sigma T^4$$

Stefan-Boltzmann constant
 $\sigma = 5,67 \cdot 10^8 \text{ W m}^{-2} \text{ K}^{-4}$

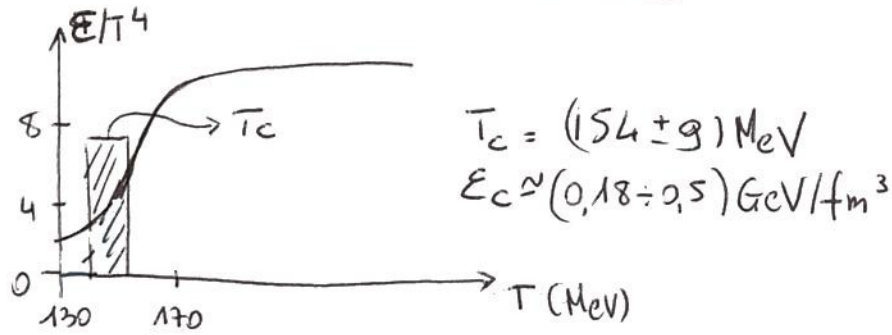
$$T = \frac{1}{k_B} \cdot 170 \text{ MeV} = \frac{1}{8,617 \cdot 10^{-5} \text{ eV K}^{-1}} \cdot 170 \cdot 10^6 \text{ eV} \approx 2 \cdot 10^{10} \text{ K}$$

$$E_c \approx 1,5 \cdot 10^{35} \frac{\text{J}}{\text{m}^3} = 1,5 \cdot 10^{35} \frac{6,24 \cdot 10^9 \text{ GeV}}{10^{45} \text{ fm}^3} \approx 0,9 \frac{\text{GeV}}{\text{fm}^3}$$

$$E_c \approx 0,9 \frac{\text{GeV}}{\text{fm}^3}$$

Energy density for the transition.

CALCULATIONS FROM LATTICE QCD



BIG BANG VS H.I. COLLISION NUCLEOSYNTHESIS

SIMILARITIES:

- Inelastic nucleonic reactions freeze-out before nuclei formation
- Isentropic expansion of boson-dominated matter (photons in BBN vs pions in HIC). Baryon/Boson ratio:

$$\mu_{\text{BBN}} \approx 10^{-10} ; \mu_{\text{HIC}} \approx 0.05$$

- STRONG NUCLEAR FORMATION AND REGENERATION REACTIONS

DIFFERENCES

- Time scale: 1-100 s BBN vs 10^{-22} s in HIC
- Temperature: $T_{\text{BBN}} < 1 \text{ MeV}$ vs $T_{\text{HIC}} \sim 100 \text{ MeV}$
- Binding energies, proton-nucleon mass difference and neutron lifetime are important for BBN, less for HIC
- $\mu_B \approx 0$ at LHC ; $\mu_B \neq 0$ in BBN
- Resonance feeddown important at LHC, irrelevant in BBN