GAMMA DECAY

Most of α and β decays, and most of the nuclear reactions as well, leave the final nucleus in an excited state.
These excited states decay rapidly to the ground state through the emission of 1 or more γ rays. (i.e. photons)
γ rays are photons with energies typically in the range 0,1 ÷ 10 MeV, characteristic of the energy difference between nuclear states which corresponds to wavelenghts between 10^4 and 100 fm.
(λ are far shorter than visible light).

A COMPARISON OF α, β AND γ DECAY

Most naturally radioactive nuclei de-exite via α decay. The typical α-decay energy is 5 MeV and ranges between 5 and 10 MeV.
The "reduced" de Broglie wave lenght is:

$$\lambda\!\!\!^{-} = \frac{\hbar}{p} = \frac{\hbar c}{pc} = \frac{\hbar c}{\sqrt{E_k(E_k + 2mc_2)}} =$$

Thus, for α-ptc $\lambda\!\!\!^{-} \simeq 1$ fm with a range of about 0,72 ÷ 1,14 fm.
This dimension is "small" and this is why a semi-classical treatment of α decay is succesful. [α ~~can~~ be placed inside the nucleus and then the tunnelling is able to predict the "escape probability" and the halftime]

β decay involves any energy up to Q (typically 1 MeV), ranging from few keV to tens of MeV. $\lambda\!\!\!^{-}$ is ≃ 140 fm ($10 \lesssim \lambda\!\!\!^{-} \lesssim 1000$ fm). β has a large $\lambda\!\!\!^{-}$ compared to the nuclear size ⇒ QM has to be used.

γ decay has an energy ≃ 1 MeV and ranges from 0,1–10 MeV. $\lambda\!\!\!^{-}$ is ≃ 40 fm (ranges 20–2000 fm) ⇒ Only a QM approach has a chance of success. When QM was discovered, classical Electrodynamics was very mature ⇒ The wave mechanics of γ could mesh very easly with mechan of particles.

QUANTUM ELECTRODYNAMICS

The nuclear wave function can be written as the composition of single particle nucleons wave function:

$$\Psi_N = \prod_{a=1}^{A} \underbrace{C(l_a, s_a, t_a)}_{\text{COMBINATION FACTOR}} \psi_a(l_a, s_a, t_a)$$

COMBINATION FACTOR
of individual or
orbital angular momentum (l)
spin angular momentum (s)
and isospin (t)

[t is analogous to ~~spin~~ intrinsic spin. The strong force is almost independent of nuclear type ⇒ p e n are "degenerated" like spin state of atomic physics. ⇒ We can associate a quantum number (the isospin) $t_p = \frac{1}{2}$ $t_n = -\frac{1}{2}$ (by convention)]

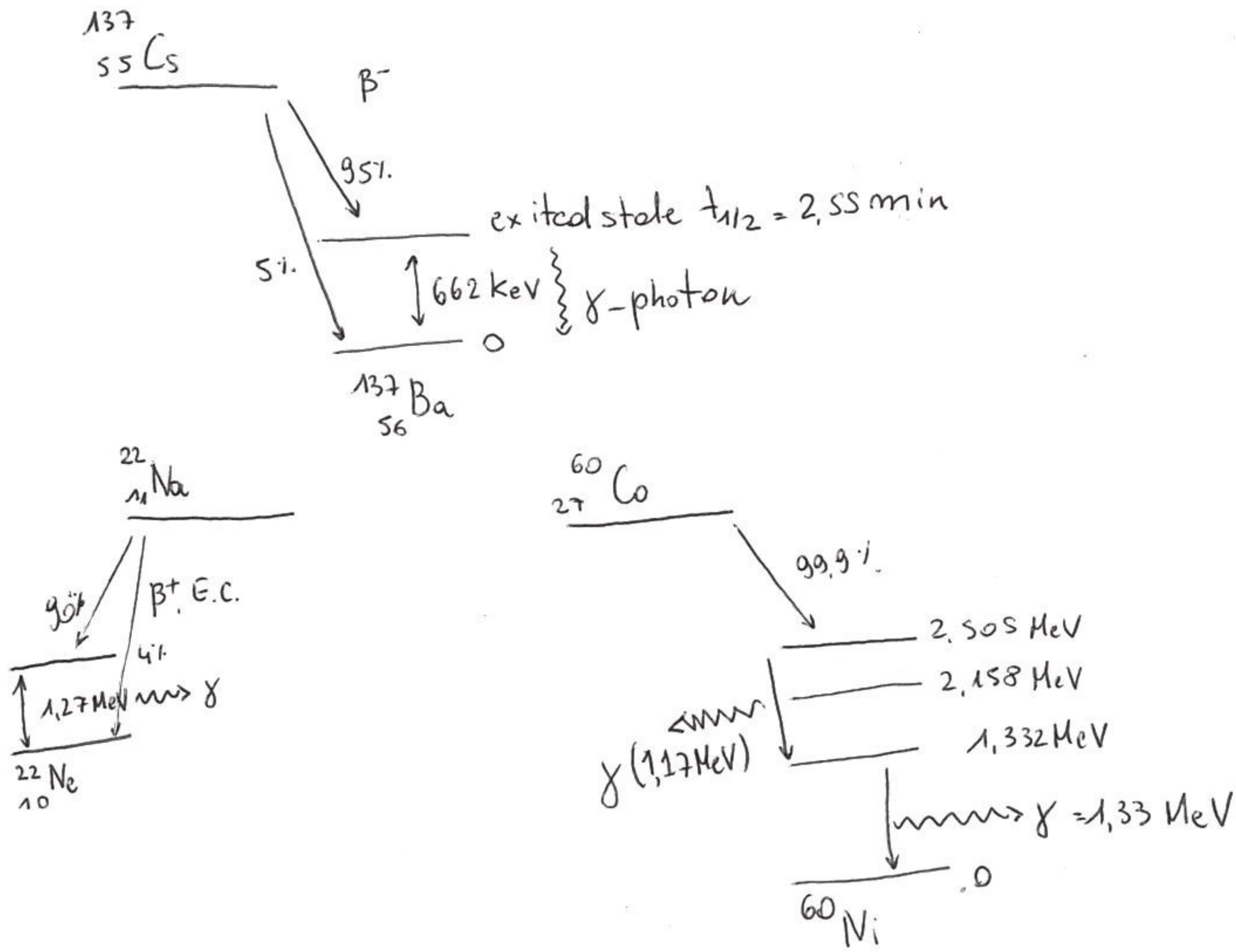
The transition rate between ψ_i e ψ_f, resulting from an e.m. decay producing a photon with energy E_γ can be described by Fermi's Golden Rule

$$\lambda = \frac{2\pi}{\hbar} \left| \langle \psi_i \,|\, O_{em} \,|\, \psi_f^* \rangle \right|^2 \frac{dn_\gamma}{dE_\gamma}$$

O_{em} is the e.m. transition operator. dn_γ / dE_γ is the density of final states. Since ψ_γ e O_{em} are well known, the measurements of λ gives detailed knoledge of nuclear structure.

Comparing results with theory gives an indication of how good the models are, to describe ψ_N (i.e. the nuclear structure)

EXAMPLE OF γ DECAY AND "TIME" OF THE DECAY



A γ decay lifetime is typically 10^{-12} s. The typical time from a nucleon to cross a nucleus is $\simeq 4 \cdot 10^{-22}$ s.

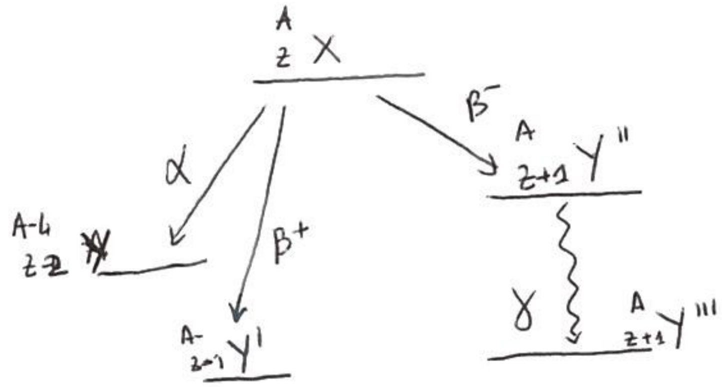
Why does γ decay take so long?

There are several reason:

1) The photon wave lenght is many times the size of the nucleus $\Rightarrow$ There is a little overlap between the γ and the nucleus wave function (And is the nucleon that is responsible for emission of the photon)
2) The photon carries at least one unit of angular momentum $\Rightarrow$ the transition involves some degree of re-orientation.
3) The e.m. force is relatively weak compared to strong force ($\simeq$ a factor 100 weaker)

ENERGETICS OF γ DECAY

A typical γ decay scheme ~~cou~~ be represented as:



$${}^{A}_{\cancel{Z}}X^{*}_{\cancel{N}} \longrightarrow {}^{A}_{Z}X'_{N} + \gamma$$

$$\left[m_{X^*} - m_{X'}\right] c^2 = T_{X'} + E_\gamma$$

$$Q = T_{X'} + E_\gamma$$

The Q-energy is distributed between the recoil energy of the daught nucleus and the energy of the photon. m_{X^*} and $m_{X'}$ are nuclear masses.

It is easier to use relativistic formalism to determine the share of E_K.

$$E_\gamma = Q\left[\frac{1 + Q/2m_{X'}c^2}{1 + Q/\ m_{X'}c^2}\right] \qquad (A)$$

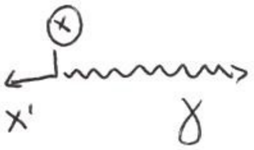
$$T_{X'} = \frac{Q}{2}\left[\frac{Q/m_{X'}c^2}{(1 + Q/m_{X'}c^2)}\right] \qquad (B)$$

Since $Q \ll m_{X'}c \quad (10^{-4}/A) < (Q/m_{X'}c^2) < (10^{-2}/A)$

$$E_\gamma \simeq Q - \frac{Q^2}{2m_{X'}c^2}$$

$$T_{X'} \simeq \frac{Q^2}{2m_{X'}c}$$

$T_{X'}$ range is "small", and ranges from $5 - 50 \cdot 10^3$ eV/A, but excep' very low energy they are strong enought to overwhelm atomic bonds and destroy the crystal.



$M = M_X C^2$

$m = M_{X'} C^2$

$\boxed{k = m(\gamma_x - 1)}$ x

$M = m\gamma + E$ (1)

$M = m(\gamma - 1) + m + E$

$\boxed{Q = k + E}$ x (2)

$\boxed{m\beta\gamma = E}$ (3)

$m^2\beta^2\gamma^2 = E^2 \Longrightarrow \quad \gamma^2 = \frac{1}{1-\beta^2} \Rightarrow \gamma^2(1-\beta^2) = 1$

$\gamma^2\beta^2 = \gamma^2 - 1 = (\gamma-1)(\gamma+1)$

$\downarrow$

$m^2(\gamma-1)(\gamma+1) = E^2$

$\underbrace{m(\gamma-1)}_{k} m(\gamma+1) = E^2$

$E^2 = k(k+2m)$

$k = m(\gamma - 1) \qquad m(\gamma+1) = m(\gamma - 1 + 2)$

$K = k + 2m$

$\boxed{k(k+2m) = E^2}$ (4) $\quad (2 \Rightarrow 4)$

$\underbrace{m(\gamma-1)}_{K} \underbrace{m(\gamma+1)}_{} = E^2$

$m(\gamma+1) = m(\gamma-1+2)$

$=$

$k(k+2m) = (Q-k)^2$

$\cancel{k^2} + 2km = Q^2 - 2Qk + \cancel{k^2}$

$2k(m+Q) = Q^2$ (5)

$k = \frac{Q^2}{2}\frac{1}{(m+Q)} = \frac{Q^2/m}{2}\left[\frac{1}{1+\frac{Q}{m}}\right]$

$$\boxed{T_X{}' = \frac{Q}{2}\left[\frac{Q/m_{x'}C^2}{1+Q/m_{x'}C^2}\right]}$$

$E = Q - k = Q - \frac{Q^2}{2m\left(1+\frac{Q}{m}\right)}$ (From (2) & (5))

$= Q\left[\frac{2m\left(1+\frac{Q}{m}\right) - Q}{2m\left(1+\frac{Q}{m}\right)}\right] = Q\left[\frac{2m+2Q-Q}{2m+2Q}\right] = Q\left[\frac{1+\frac{Q}{2m}}{1+\frac{Q}{m}}\right]$

$$\boxed{E = Q\left[\frac{1 + Q/2m_x' c^2}{1 + Q/m_x' c^2}\right]}$$

A

CLASSICAL ELECTROMAGNETIC RADIATION

MULTIPOLE EXPANSION

- Electric multipoles: The electric multipole expansion starts by considering the potential due to a static charge distribution:

$$V(\vec{x}) = \frac{1}{4\pi\varepsilon_0} \int d\vec{x} \, \frac{\rho(\vec{x})}{|\vec{x} - \vec{x}_i'|}$$

Only the systems that conserve charge will be considered.
Assuming the charge to be localized $V(\vec{x})$ can be expanded in $\vec{x}$

$$V(\vec{x}) = \frac{1}{4\pi\varepsilon_0} \left[\frac{Q_0}{|\vec{x}|} + \frac{Q_1}{|\vec{x}|^2} + \frac{Q_2}{|\vec{x}|^3} \cdots \right]$$

$$Q_0 = \int d\vec{x} \, \rho(\vec{x}, t) \qquad \equiv \text{TOTAL CHARGE} = \text{MONOPOLE}$$

$$Q_1 = \int d\vec{x} \, z' \rho(\vec{x}) \qquad = \text{DIPOLE MOMENT}$$

$$Q_2 = \frac{1}{2} \int d\vec{x} \, (3z' - r'^2) \, \rho(\vec{x}) \qquad = \text{QUADRUPOLE MOMENT}$$

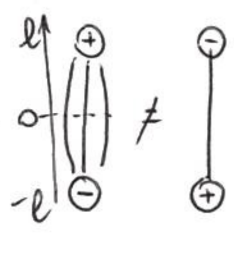
...

In general $\quad V(\vec{x}, t) = \frac{1}{4\pi\varepsilon_0} \frac{1}{|\vec{x}|} \sum_n \frac{Q_n}{|\vec{x}|^n}$

In classical E.M. the higher n's diminish in influence as $|\vec{x}|$ grows.
In quantum E.M. theory, the higher n's are associated with transitions that become weaker with increased n.

ELECTRIC DIPOLES AND QUADRUPOLES

ELECTRIC DIPOLE



Parity of electric dipole $= -1$

q is locatet at $z = \ell$, $-q$ is located at $z = -\ell$: This is a pure electric dipole:

$$\rho(\vec{x}) = q\left[\delta(x)\,\delta(y)\,\delta(z-\ell) - \delta(x)\,\delta(y)\,\delta(z+\ell)\right]$$

$$Q_0 = 0$$
$$Q_1 = q\ell + (-q)(-\ell) = 2q\ell$$
$$Q_{n\geq 2} = 0$$

Under a parity operation, $\vec{x} \to -\vec{x}$ the configuration is opposite from the original configuration $\Rightarrow$ the parity of electric dipole radiation $\Pi(E^1) = -1$.

A similar argument for an electric quadrupole

In general, the parity of an electric multipole is:

$$\Pi(EL) = (-1)^L$$

<u>Magnetic multipoles</u>: We will focus on the magnetic dipole momets

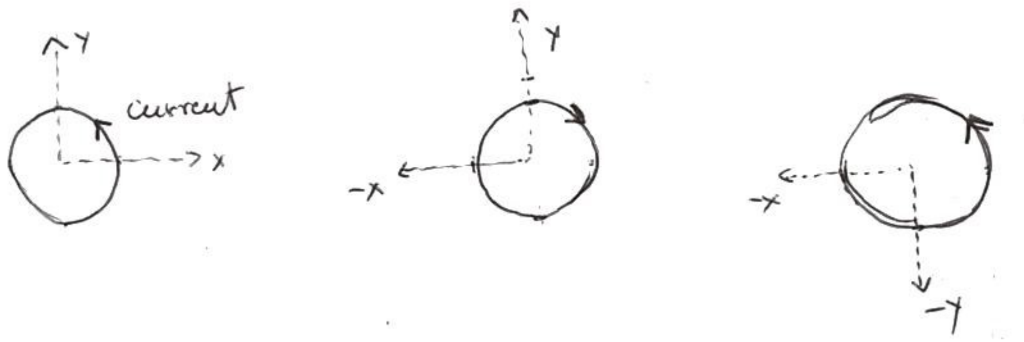
$$B(\vec{x}) = \frac{\mu_0}{4\pi}\left[\frac{3\vec{n}(\vec{n}\cdot\vec{m}) - \vec{m}}{|\vec{x}|^3}\right]$$

$\vec{n}$ = unit vector

$\vec{x}$ = direction

$\vec{m}$ = magnetic moment $\qquad \vec{m} = \frac{1}{2}\int d\vec{x}'\left[\vec{x}' \times \vec{J}(\vec{x}')\right]$

$\vec{J}$ = current density



Under a parity change, the magnetic dipole is unchanged, so in this case $\Pi(B^1) = +1$. In general

$$\Pi(ML) = -(-1)^L = (-1)^{L+1}$$

A MORE SOPHISTICATED TREATMENT OF CLASSICAL MULTIPOLE FIELD

In order to make the transition to QM a bit more transparent, it is possible to use "advanced electromagnetism".

If

$$\rho(\vec{x},t) = \rho(\vec{x})e^{i\omega t}$$
$$\vec{J}(\vec{x},t) = \vec{J}(\vec{x})e^{i\omega t}$$
$$\vec{m}(\vec{x},t) = \vec{m}(\vec{x})e^{i\omega t}$$

are time dependent charge density (= proton density), current density (protons with orbital angular momentum) and magnetic moment density (= pen intrinsic spin), the radiation fields are characterized by the electric, $Q_{\ell m}$, and magnetic, $M_{\ell m}$, multipoles

$$Q_{\ell m} = \int d\vec{x}\,|\vec{x}|^{\ell}\, Y_{\ell}^{m}(\theta,\varphi)\left[\rho(\vec{x}) - \frac{i\omega}{(\ell+1)c^2}\vec{\nabla}\left[\vec{x}\times\vec{m}(\vec{x})\right]\right]$$

$$M_{\ell m} = -\int d\vec{x}\,|\vec{x}|^{\ell}\, Y_{\ell}^{m}(\theta,\varphi)\left[\vec{\nabla}\,\vec{m}(\vec{x}) + \frac{1}{(\ell+1)}\vec{\nabla}\left[\vec{x}\times\vec{J}(\vec{x})\right]\right]$$

The power, dP, radiated into a solid angle $d\Omega$, by mode (ℓ, m) is

$$\frac{dP}{d\Omega}\left(\ell, m, \begin{bmatrix} E \\ M \end{bmatrix}\right) = \frac{2(\ell+1)c}{\varepsilon_0 \ell(2\ell+1)[(2\ell+1)!!]^2}\left(\frac{\omega}{c}\right)^{2\ell+2}\left|\begin{matrix} Q_{\ell m} \\ M_{\ell m}\end{matrix}\right|^2 |X_{m\ell}(\theta,\varphi)|^2$$

$$|X_{\ell m}|^2 = \frac{\frac{1}{2}(\ell-m)(\ell+m+1)|Y_\ell^{m_\ell+1}|^2 + \frac{1}{2}(\ell+m)(\ell-m+1)|Y_\ell^{m_\ell-1}|^2 + m^2|Y_\ell^{m_\ell}|^2}{\ell(\ell+1)}$$

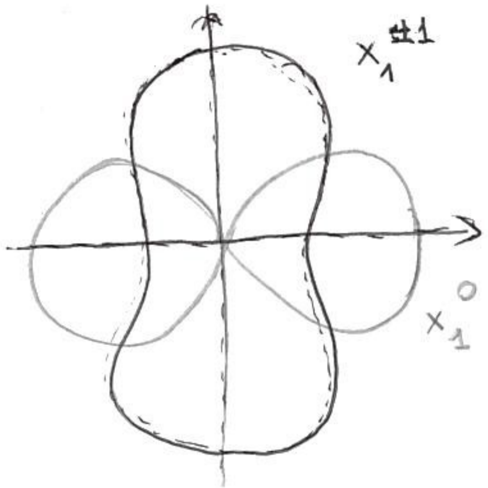
If all m's contribute equally

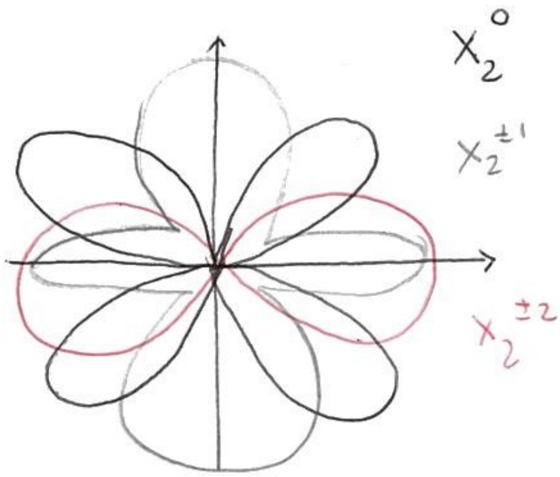
$$\sum_{m=-\ell}^{\ell} |X_\ell^{m_\ell}(\theta,\varphi)|^2 = \frac{2\ell+1}{4}$$

In case of isotropic radiation.

In case of non-isotropic distributions, different contributions related to spherical harmonics have to be considered.

E.g.





The angular distributions indicates how a measurement of the angular distribution map ~~the~~ of the radiation field can identify the multipole of the radiation.

HOW MEASURE PARITY? Secondary scattering experiment which is sensitive to to direction of the electric field vector.

TRANSITION TO QUANTUM MECHANICS

The most of the hard work has been already done! In Q.M.

$$\underbrace{\frac{d\lambda}{d\Omega}\left(l, m, \begin{bmatrix} E \\ M \end{bmatrix}\right)}_{\text{Transition rate per photon}} = \underbrace{\frac{1}{\hbar\omega}}_{\text{Energy per photon}} \underbrace{\frac{dP}{d\Omega}\left(l, m, \begin{bmatrix} E \\ M \end{bmatrix}\right)}_{\text{Power}}$$

The expression on the right hand side is equal to the classical expression, except that in this case, the charge, current and intrinsic magnetization densities are replaced by probability densities.

$$Q_{me}^{i\to f} = e\int d\vec{x}\,|\vec{x}|^{l}\, Y_m^{m_e}(\theta,\varphi)\left[\sum_{i=1}^{Z}(\psi_i^*)_f(\psi_i)_i\right]$$

$$M_{me}^{i\to f} = -\frac{1}{(l+1)}\frac{e\hbar}{m_p}\int d\vec{x}\,|\vec{x}|^{l}\, Y_m^{m_e}(\theta,\varphi)\left[\vec{\nabla}\cdot\left[\sum_{i=1}^{Z}(\psi_i^*)_f\,|\vec{L}|\,(\psi_i)_i\right]\right]$$

WEISSKOPF ESTIMATES

A detailed investigation of $Q_{m_e}^{i\to f}$ and M_{me}^{i-f} requires detailed knowledge of nuclear w.f. in order to pin down the absolute decay rates of various transition types.

The detailed knowledge of w.f. is unknown, however. But it might be useful to have a "ball park" estimate to get relative transition rates.

This approximation was done by Weisskopf using the following strategy.

Let the radial part of both initial and final state w.f. be:

$$R(r) = \theta(\underbrace{R_N}_{R_0 A^{1/3}} - r)\Big/\sqrt{\int_0^{R_N} dr\, r^2}$$

$$\int_0^{R_N} r^2\, dr = 1$$

The radial part of $Q_{\ell m}^{i-f}$ is given by:

$$Q_{\ell m}^{i-f} = e\int_0^{\infty} r^2 r^\ell |R(r)|^2 dr = e R_N^\ell \frac{3}{\ell+3}$$

So

$$\lambda(E\ell) = \frac{8\pi(\ell+1)}{\ell[(2\ell+1)!!]^2} \underbrace{\left\{\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right\}}_{1/137} \left(\frac{E_\gamma R_N}{\hbar c}\right)^{2\ell+1} \left(\frac{3}{\ell+3}\right)^2 \underbrace{\frac{c}{R_N}}_{\text{Time for a photon to cross the nuclear diameter}}$$

for electric ℓ-pole transition

similarly

$$\lambda(M\ell) = \frac{8\pi(\ell+1)}{\ell[(2\ell+1)!!]^2} \underbrace{\left(\mu_p - \frac{1}{\ell+1}\right)^2}_{\text{Nuclear magneton}} \left(\frac{\hbar c}{m_p c^2 R_N}\right) \left\{\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right\} \left(\frac{E_\gamma R_N}{\hbar c}\right)^{2\ell+1} \left(\frac{3}{\ell+2}\right) \frac{c}{R_N}$$

$$\lambda(E_1) = 1 \times 10^{14} A^{2/3} E_\gamma^3$$
$$\lambda(E_2) = 7{,}3 \times 10^{7} A^{4/3} E_\gamma^5$$
$$\lambda(E_3) = 3{,}3 \times 10^{1} A^{2} E_\gamma^7$$
$$\lambda(E_4) = 1{,}1 \times 10^{-5} A^{8/3} E_\gamma^9$$

$$\lambda(M_1) = 5{,}6 \times 10^{13} E_\gamma^3$$
$$\lambda(M_2) = 3{,}5 \times 10^{7} A^{2/3} E_\gamma^5$$
$$\lambda(M_3) = 1{,}6 \times 10^{1} A^{4/3} E_\gamma^7$$
$$\lambda(M_4) = 4{,}5 \times 10^{-6} A^{2} E_\gamma^9$$

Some conclusions:

- SAME ORDER, DIFFERENT TYPE : $\dfrac{\lambda(El)}{\lambda(M_l)} \cong 2A^{2/3}$

  - For a given l, electric transition always dominates, with difference getting larger with A

- NERBY ORDER, SAME PARITY: $$\frac{\lambda(E(l+1))}{\lambda(Ml)} \simeq 10^{-6} A^{4/3} E_\gamma^2$$

  - For large A and E, E2 can compete with M1, E_3 with M_2 and so on

- SAME TYPE, DIFFERENT ORDER: $$\frac{\lambda(\sigma(l+1))}{\lambda(\sigma l)} \simeq 10^{-8} A^{2/3} E_\gamma^2$$

  where σ is either M or E.
  - The factor $A^{2/3} E^2 \leq 10^4$ so, as l increases the λ decreases drammatically!

ANGULAR MOMENTUM AND PARITY SELECTION RULES

Conservation of total angular momentum and parity dictates that:

$$\vec{I}_f = \vec{I}_i + \vec{l}$$

$$\pi_f = (-1)^l \pi_i \qquad \text{E-Type}$$

$$\pi_f = (-1)^{l+1} \pi_i \qquad \text{M-Type}$$

In addition, the quantization of angular momentum imply that

$$\Delta I = |I_f - I_i| \leq l \leq I_f + I_i \quad \text{in steps of 1.}$$

The emission of e.m. decay photon CANNOT be associated by $0 \to 0$ transition (this can occur via internal conversion). The parity selection rules can be stated as:

$$\Delta\pi = \text{no} \Rightarrow \text{Even E / Odd M}$$

$$\Delta\pi = \text{yes} \Rightarrow \text{Odd E / Even M}$$

Some examples

$\frac{3}{2}^{\pi} \longrightarrow \frac{5}{2}^{\pi}$ $\quad 1 \leq \Delta M \leq 4$

$\Delta\pi$ = no $\quad \boxed{M1}$ E2, M3, E4 $\quad$ M1 Most likely

$\Delta\pi$ = Yes $\quad \boxed{E1}$ M2, E2, M4 $\quad$ E_1 Most likely

$4^{\pi} \longrightarrow 0^{\pi}$ $\quad \Delta M = 4$

$\Delta\pi$ = no $\quad$ E4

$\Delta\pi$ = yes $\quad$ M4

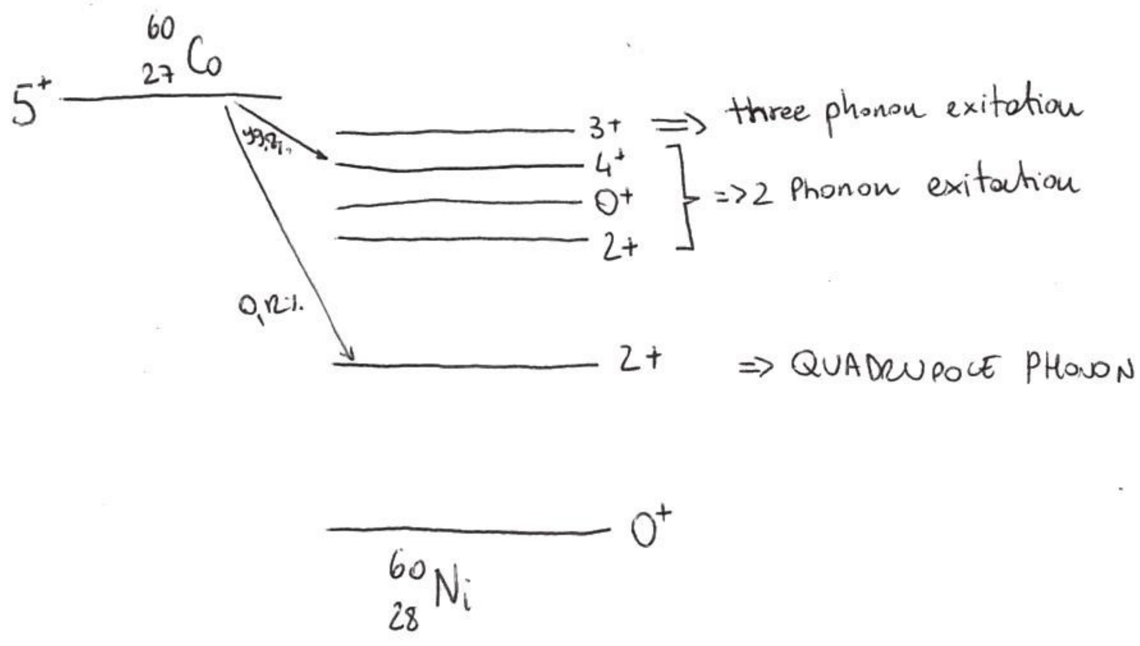
$2^{+} \longrightarrow 0^{+}$ $\quad \Delta M = 2$ $\quad \Delta\pi$ = no $\quad$ E2 only

"Real life example"

One of the most useful decay ~~of~~ γ decay is the decay of ^{60}Co. It has been used for radiotherapy purpose and today for industrial radiation processes, sterilization, food processing, material transformation, ...

^{60}Co does not occur naturally, as it has a $t_{1/2} = 5{,}27\,y$. It is produced by neutron activation of stable ^{59}Co.

$$^{59}_{27}Co_{32} + n \rightarrow {}^{60}_{27}Co_{33} \rightarrow {}^{60}_{28}Ni^{*}_{32} + \beta^{-} + \bar{\nu}_e \; (5{,}27\,y) \Rightarrow {}^{60}_{28}Ni_{32} \rightarrow n\gamma$$



Comparing the results from measurement with Weisskopf estimate, a very good agreement is obtained.