BIG BANG

12-16 $\cdot 10^9$ years ago ~~our~~ Universe was concentrated in a tiny spatial region => ∞ density of energy and temperature.

The Universe then expanded and cools down untill it reached the conditions that characterize it now.

Let's have a look at the evolution of the Universe:

$t = 0$ Big Bang

$t \simeq 10^{-6}$ s Density so high that hadrons could not form => QUARK GLUON PLASMA (QGP) $\equiv$ state of matter where quark and gluons are deconfined

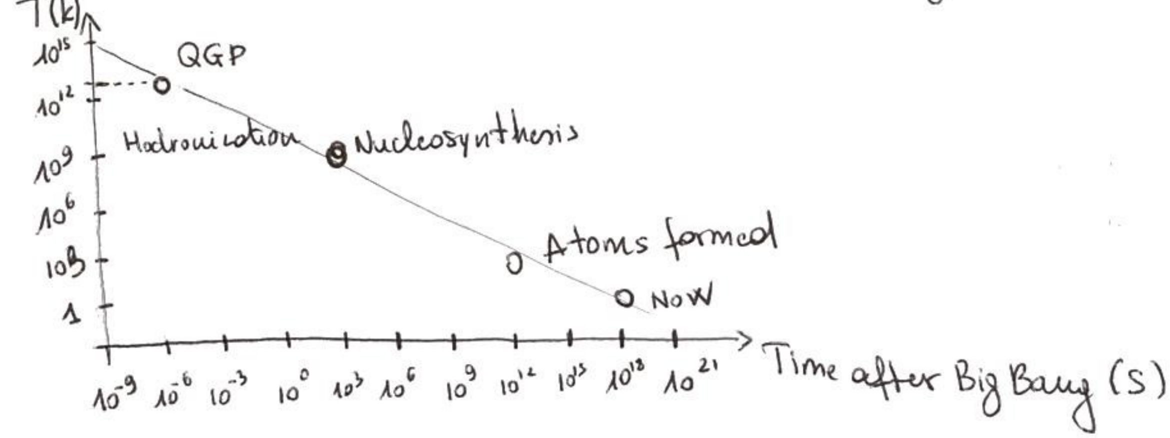
$t \simeq 10^{-4}$ s Energy density < Critical energy density ($\varepsilon_{cr} \simeq 1$ GeV / fm^3)
Temperature < Critical temperature ($T_{cr} \simeq 170$ MeV $\simeq 2 \cdot 10^{10}$ K)
The degree of freedom of the color charge are confined inside the neutral-color objects with a dimension of $\sim$fm
=> Hadron formatation.

$t \simeq 3$ min $T < 100$ keV ($\simeq 10^9$ K) => Formation of small atomic nuclei
-> CHEMICAL FREEZE-OUT:
- Instable adrons are decayed
- Anti-particle annihilated (=> Remaining ~~exces~~ of p, n, e$^-$)
- The "chemical composition" of the Universe will change only after hundred thousands of years with the stellar formation

The Universe is still ionized => opaque to the e.m. radiation

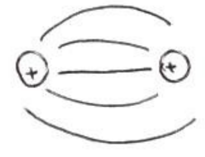
$t \simeq 300.000$ years $T < 3000$ K : electrons and nuclei can form the atoms
Now the e.m. radiation decouple with a black body spectrum with $T \simeq 3000$ K : this is called THERMAL FREEZE OUT
Due to the Universe expansion this black body radiation went to a red shift untill a $T \simeq 2,7$ K, which constitutes the cosmic microwave background.

$t = 600 \cdot 10^6 \div 10^9$ years : Formation of first galaxies

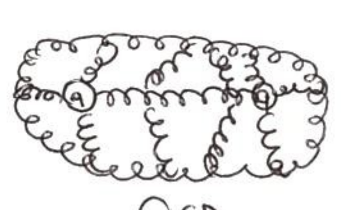


The opacity of the Universe makes it impossible to "see" what happened in the first 300.000 years after the Big Bang, so the QGP is hided behind the cosmic microwave background courtain.
If we want to study it we have to recrate it.

HOW TO RECREATE QGP?
QGP is a state of thausands of quarks and gluons not baundend into nuclei: this can be done either heating or compressing matter
This have to happen because quarks and gluons are confined inside colorless objects (i.e. the hadrons): this characteristic is related to the QCD theory:
Quarks carry electric charge, color charge and other quantum numbers, and they interact strongly by exchange of colored gluons. Because of gluons are colored, QCD is very different from QED.
Since QCD is non Abelian field theory, when quarks are confined in hadrons and you try to pull them apart, the interaction becomes stronger

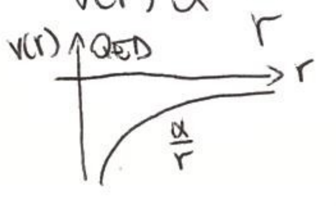


QED

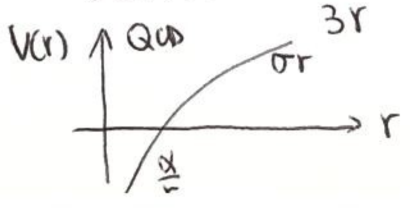


QCD

$V(r) \propto \frac{\alpha}{r}$

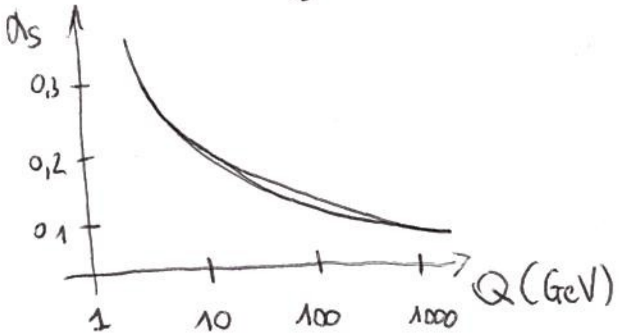


$V(r) \sim -\frac{4\alpha_s(r)}{3r} + \sigma r$ linear rise due to confinement



In QED vacuum polarization leads to increase of coupling constant with decreasing r (running slow $\alpha = 1/128$ at 58,5 GeV).

In QCD occurs the opposite: coulored gluons spread out color charge leading to anti-shielding decrease of coupling constant α_s with decreasing r or increasing the ~~transfer~~ transfer momentum Q



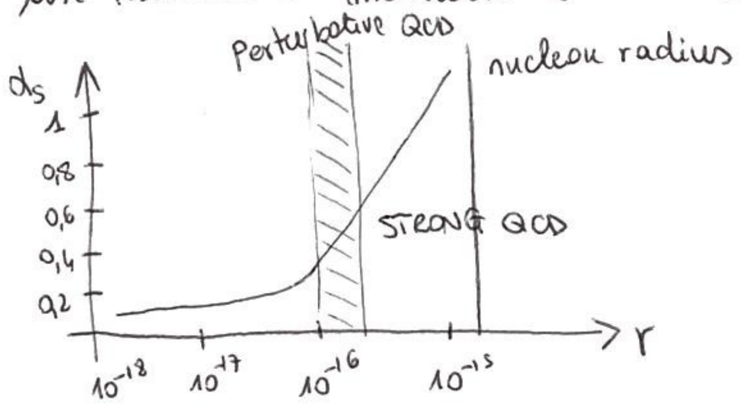
$$\alpha_s(Q^2) \propto \frac{1}{\ln(Q^2/\Lambda^2)}$$

scale constant

For large values of Q^2 ("hard" collisions we enter the perturbative region and we can use calculation methods

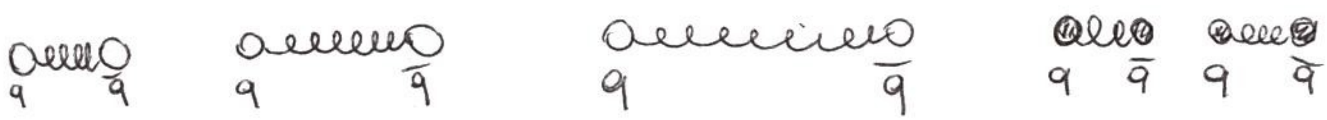
~~⇒ THE QCD HAS A PHASE DIAGRAM~~

developed for QED, while for small Q^2 ("soft" collisions) one can only use numerical methods (LATTICE QCD).



With increasing mutual distance r, the attractive force (σr) increases and quarks cannot be separated. This is called "confinement" (→ no free quark cannot be observed)

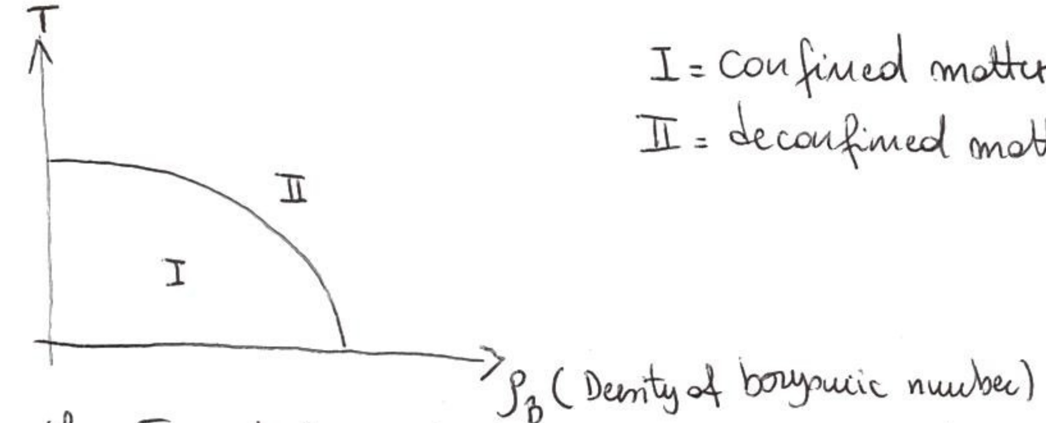
Moreover, one expects that at a point when energy of the streched string becomes bigger than 2 quark masses, the string would break and new quark-antiquark pair are created from the vacuum, forming a meson.



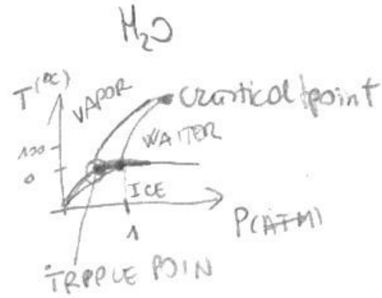
Another characteristic feature of QCD, beside the quark confinement is the asymtotic freedom, meaning that at very short distances quarks behave as free particles.

A possibility of quark liberation was noticed already in 1975 by Collins and Perry, and a "NEW PHASE OF MATTER" in which quarks are no longer confined was considered few months later by Cabibbo and Parisi.
The notation Quark Gluon Plasma was introduced in 1978 by Shuryak.
At high T and/or densities ρ a phase transition from hadronic matter to QGP should take place. Characteristics of this phase of strongly interaction matter can be obtained from QCD only by numerical simulations as they are related to low-energy, non-perturbative properties of the theory

PHASE DIAGRAM OF STRONGLY INTERACTING MATTER



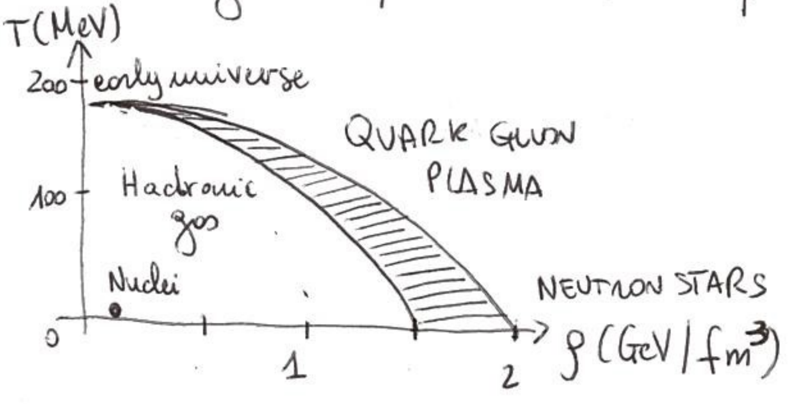
I = Confined matter
II = deconfined matter



- Low T and "normal" density: colored q and g bound in colorless hadrons -> confinement, chiral symmetry is spontaneously broken (generating e.g. the 99% of proton mass)

- High T and/or high ρ: quark and gluons free, new strongly interacting matter.

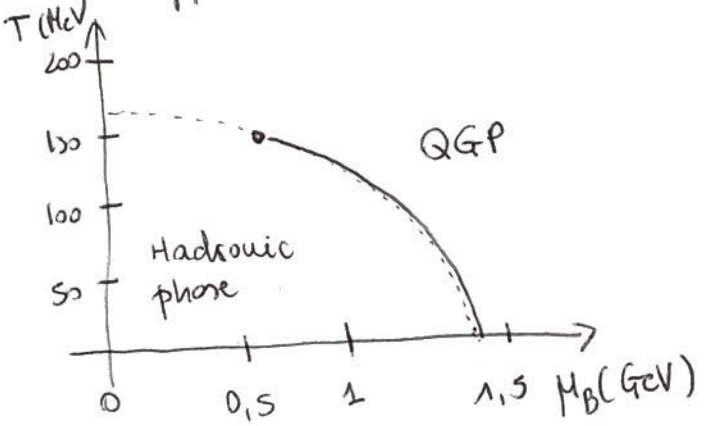
Including the quark masses the phase diagram becomes



The band reflect the uncertainties of the calculation, especially large w.r.t. the critical energy density.
The estimated values lyis between 1.5 and 2 GeV/fm^3 or about 10 times the matter density in nuclei ($\rho = 0{,}16$ GeV/f

The predicted values for critical temperature is $T_C \simeq (150-170)$ MeV, depending on the assumptions about the studied systems.

When the finite baryonic chemical potential is considered, the phase boundary and critical parameters are much better defined, and a critical point appears.



For μ_B < Critical point, the transition is of second-order or continuous. This situation is not favorable for experimental investigations as in such type of transitions no sharp change in the system parameters, indicative of a phase transition, can be expected.

ESTIMATION OF THE TRANSITION TEMPERATURE AND PRESSION

First of all let's set some work hypotesis:

1. The quark and gluon system is in thermal equilibrium in a volume V (QGP is a continuum).
   Thermal equilibrium is necessary to make any calculation. It means that:
   - Temperature does not evolve
   - In a microscopic level it means that the constituents interacted n times, with n very large.
2. Quark and gluon are massless ($\Rightarrow$ c, b, t, are neglected) and non interacting ($\Rightarrow$ no average potential) $\Rightarrow$ $\simeq$ Ideal gas.
3. Net baryon number $\equiv 0$ $\Rightarrow$ equal number of q and $\bar{q}$
   (At a sufficient high ~~temp~~ energy this is what happens)
4. We will evaluate the critical temperature T_c at which the pression of quarks and gluons is equal to that of the "bag" containing the quarks and gluons

Let's start from g_{TOT} ≡ total number of degrees of freedom relative to quarks and anti-quarks.
The contribution are:

$$\begin{cases} g_g = 8 \times 2 = 16 & \text{8 gluons} \times \text{2 possible polarizations} \\ g_q = g_{\bar{q}} = N_c \cdot N_s \cdot N_f = 3 \times 2 \times 2 = 12 & \text{3 colors} \times \text{2 spins} \times \text{2 flavor } (u,d) \end{cases}$$

$$g_{tot} = 37 \qquad = \qquad g_{TOT} = g_e + \frac{7}{8}(g_q + q_{\bar{q}})$$

The pressure of an ideal QGP is $\boxed{P = g_{TOT} \frac{\pi^2}{90} T^4}$ ≃ STEFAN BOLTZMANN LAW

$$P = 37 \cdot \frac{\pi^2}{90} T^4 \qquad [T] = \text{MeV} \; (k_B = 1)$$

The energy density at Temperature T is:

$$\underset{\substack{\uparrow \\ \text{Ideal gas} \\ \text{regime} = 3 \cdot \text{gld of space}}}{\varepsilon = 3P} = 37 \cdot \frac{\pi^2}{90} T^4 \implies \alpha T^4 \text{ as Stefan-Boltzmann Law.}$$

Let's introduce dimensional constants. For T=200 MeV

$$\varepsilon = 37 \cdot \frac{\pi^2}{30} T^4 \cdot \frac{1}{(\hbar c)^3} = \frac{37\pi^2}{30} \cdot \frac{0{,}2^4}{0{,}197^3} \simeq 2{,}547 \text{ GeV/fm}^3$$

$T^4 = K^4$ $\quad K_B^4 = eV \cdot K^{-1}$
$K_B T = eV$

$\frac{37 \cdot 3 \cdot 3}{30} \cdot \frac{2}{10} \quad \frac{100}{10} \cdot \frac{2}{10}$

$P =$

The T at which the pressure of QGP is equal to that of the bag is called "critical temperature"

$$P = 37 \frac{\pi^2}{90} T^4 \equiv B \implies T_c = \sqrt[4]{\frac{90B}{37\pi^2}} \simeq 145 \text{ MeV}$$

$$\left[\sqrt[4]{B} = 206 \text{ MeV}\right]$$

⇒ Matter inside bag is heated up to ⇒ T_{crical}
⇒ Se la materia all'interno della sacca viene riscaldata ad una $T > T_c$

la sacca mai srà più in grado di confinarle ⇒ QGP

⇒ Anche se il modello è semplice l'ordine di grandezza è corretto.

EVALUATION OF THE PRESSURE

Let's start from the hypotesis:

1) Each state occupy in the phase space a volume $(2\pi\hbar)^3$

2) For a volume V the number of possible states with a momentum between p and p+dp is:

$$\frac{4\pi p^2 dp\, V}{(2\pi)^3}$$

$(\hbar = c = 1)$

V = Volume of the phase space

$2\pi^3$ = Volume of 1 cell

3) The number of quarks in volume V with a momentum between p and p+dp is given by Fermi-Dirac statistics

$$dN_q = \frac{g_q V 4\pi p^2 dp}{(2\pi)^3}\left[\frac{1}{1+e^{(p-\mu_q)/T}}\right]$$

μ_q = chemical potential

$g_q = N_c N_S N_f$ is the degree of degeneration of quarks

4) The presence of anti-quark corresponds to the absence of quark in the states at negative energy.

5) The density of anti-quark is by hypotesis equal to the quark one:

It is:

$$n_{\bar{q}} = \frac{g_q V 4\pi}{(2\pi)^3}\int_{-\infty}^{0} p_0^2 dp_0 \left[1 - \frac{1}{1+\exp(p_0-\mu_q)/T}\right]$$

...

$$n_{\bar{q}} = \frac{g_q 4\pi}{(2\pi)^3}\int_{0}^{\infty} p^2 dp \left[\frac{1}{1+\exp(p+\mu_q)/T}\right]$$

$n_{\bar{q}} = n_q \Rightarrow \mu_q = 0$

. For $\mu_q = 0$ we have the average energy of quarks

$$E_q = \frac{g_q V}{2\pi^2} \int_0^\infty \frac{p^3 dp}{1+e^{p/T}} \Rightarrow E_q = \frac{g_q V}{2\pi^2} T^4 \int_0^\infty \frac{z^3 dz}{1+e^z} \quad \text{with } z = \frac{p}{T}$$

$$E_q = \frac{g_q V}{2\pi^2} T^4 \Gamma(4)(1-2^{-3}) \zeta(4) = \frac{g_q V}{2\pi^2} T^4 \times 3! \times \frac{7}{8} \times \frac{\pi^4}{90}$$

$$\varepsilon_q = \frac{E_q}{V} = \frac{7}{8} \frac{g_q \pi^2}{30} \pi^4 \Rightarrow P_q = \frac{\varepsilon_q}{3} = \frac{7}{8} \frac{g_q \pi^2}{90} T^4 = P_{\bar{q}}$$

$$\zeta(x) = \sum_{n=1}^{\infty} \frac{1}{n^x}$$

-> We still miss gluons

$$E_g = \frac{g_g V}{2\pi^2} \int_0^\infty \frac{p^3 dp}{e^{p/T} - 1} \longrightarrow E_g = \frac{g_g V}{2\pi^2} T^4 \int_0^\infty \frac{z^3 dz}{e^z - 1} . \quad z = \frac{p}{T}$$

$$\varepsilon_g = \frac{E_g}{V} = \frac{g_g \pi^2}{30} T^4 \Rightarrow P_g = \frac{g_g \pi^2}{90} T^4$$

$$P = P_{\bar{q}} + P_q + P_g = \left[\frac{7}{8}(g_q + g_{\bar{q}}) + g_g \right] \frac{\pi^2}{90} T^4$$

For a 200 MeV QGP

$$\begin{cases} T = 200 \text{ MeV} \\ \varepsilon = 2{,}5 \text{ GeV/fm}^3 \\ n_q = n_{\bar{q}} = 1{,}7 \text{ fm}^{-3} \\ n_g = 2 \text{ fm}^3 \end{cases}$$

How to Reach SUCH HIGH TEMPERATURE?

Heavy-ion collisions!
Collision of heavy atomic nuclei is used to bring in as much energy as possible to spread the energy over a large volume and on many ptc.
Time scale for experiments

1984: First step: 1-2 GeV/c per nucleon beams from SuperHILAC into Bevalac at Berkeley

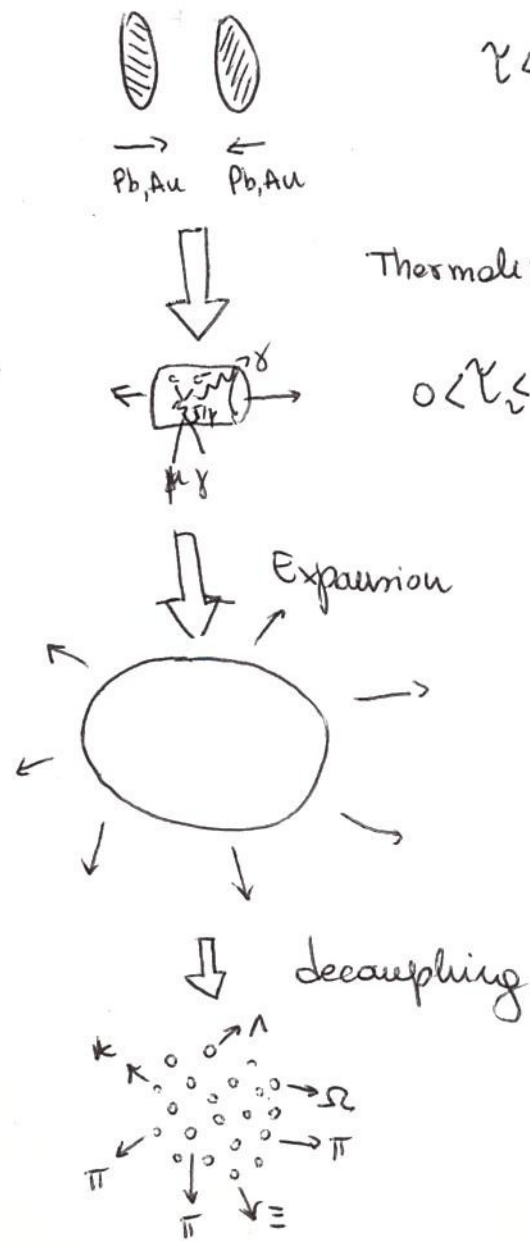
1986: Beams of O, Si, S (Sulfur) in Brookhaven AGS and CERN SPS

1992-94: Beams of Gold / Lead

2000-Today: Gold collisions in RHIC $\sqrt{S_{NN}} = 200$ GeV

2010-Today: Lead collisions in LHC $\sqrt{S_{NN}} = 5000$ GeV

THE COLLISION PROCESS



$\tau < 0$: kinetic transport theory
Parton cascade (bottomonium, ...)
Quantum transport

$1\,fm/c = 3 \times 10^{-24}\,s$

Thermalization

$0 < \tau \lesssim 1\,fm/c$ "hard scattering"

hydrodinamics QCD equation of state
transport coefficient

$1\,fm/c \lesssim \tau \lesssim 15\,fm/c$

Cooper-Frye freeze out
Hadronic cascade
resonance decays

$\tau \gtrsim 10-15\,fm/c$ "Freeze-out"