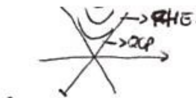


MULTIPLICITY OF DIFFERENT HADRONIC SPECIES

The measurement of particles ~~of~~ multiplicity of varius hadronic specie (i.e. how many pions, kaons, protons, ...), that is the chemical composition after hadronization, allows to answer some questions about the state of the system at the time of the chemical freeze-out.

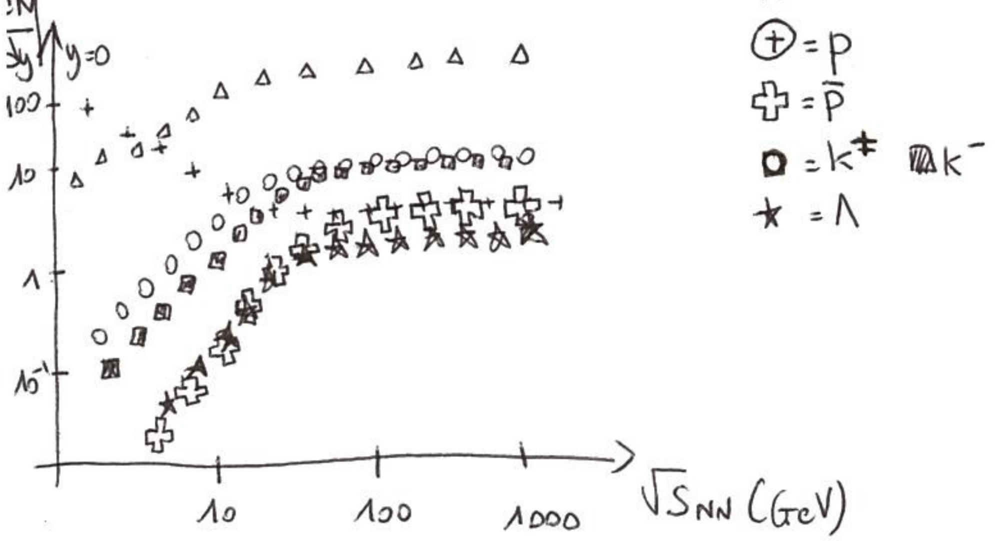
- Was the fire ball (i.e. the system created after the collision) in thermal and chemical equilibrium?
- What is the temperature of the sysmem at the chemical freeze-out?
- What was the baryon content of the fire ball?

Some notes have to be done

1. Thermal equilibrium: at the macroscopic level, the temperature T of the fireball is defined and uniform; at the microscopic level, particles velocities are described by a Maxwell-Bolzmann distribution, with only one free parameter, the Temperature T.

2. Chemical equilibrium: at macroscopic level, the densities of the varies species is uniform within the fireball; at microscopic level: particle multiplicity depends only on the masses of particles and on the temperature.

MULTIPLICITY OF IDENTIFIED PARTICLES



LOW $\sqrt{s_{NN}}$
- Fireball dominated by stopped particles
- High barion content
- The isospin of stopped quark is important

HIGH $\sqrt{s_{NN}}$
- Fireball dominated by produced particles
- Low baryon content
- Mass hierarchy ($N_{\pi} > N_{K} > N_{p}$ ...)

STATISTICAL HADRONIZATION MODELS

BASIC ASSUPTIONS

The system (fireball) created in a collision of heavy ions is in a thermal and chemical equilibrium at the moment of the freeze-out.
It is possible to write the partition function of the system and to use the statistical mechanics (using e.g. the grand canonical ensamble)
- The partition function is a quantity which describes the statistical properties of a system in equilibrium
- The production of hadrons in excited systems take place according to a purely statistic law
- The hadronic system is described as an ideal gas of hadrons and resonances (ideal gas ≡ non-interacting ptc) N.B. The hadron gas here already exists!

- The original idea come from Fermi (1950s) and Hagedorn (1960s)

NOTES

- Thermal and chemical equilibrium is POSTULATED as working hypotesis
  - ~~Chemical~~ freeze-out temperature is the SAME for ALL the particles specie
  - Comparing the expected multiplicies with measured one, it is possible to prove the validity of the equilibrium hypotesis.
- No asumptions on the presence or absence of a "partonic phase" are done
- Nothing about <u>when</u> and <u>how</u> the system reaches the chemical and thermal equilibrium is reached is said.
  No <u>kinetic</u> equilibrium is postulated.

USING THE GRAND-CANICAL ENSAMBLE: WHY?

In the grand-canonical ensamble, energy and charges are conserved "on average" on a large volume (⇒ not exactly and locally as in the canonical system)
⇒ Easier calculations

The grand canonical ensamble describes a system that exchanges energy and particles with an external "tank"

- For small systems (low energy collisions, peripheral collisions, pp or e^+e^- collisions) it have to be used either the canonical ensemble (where energy ~~and~~ is conserved on average, while charges are conserved exactly and locally) or the micro canonical ensemble, where both energy and charges are conserved exactly.
- The model uses the particle production from a hadron-resonance-gas. Why? Usually what is used is a non-interacting gas of hadrons and resonances which contains contributions of:

- all the mesons ~~with~~ $m < 1{,}8\,GeV$
- all the baryons with $m < 2\,GeV$

$\Rightarrow$ All the contributions from c and b are excluded.

The limits on the masses, put a limit also to the model validity for $T < 190\,MeV$. For larger T the contribution for heavy resonance is not negligible, but ~~these~~ for temperature above $T > 160 \div 200\,MeV$ it does not make any sense to speak about a hadron gas.
The hadron resonance gas is an effective model of a strongly interacting model, consistent with the Equation of State which results from lattice QCD below the critical temperature.
It is hence possible to start with the statistical model using the grand-canonical ensemble.
The partition function for a non-interacting gas is given by the product of the partition function of the different hadronic specie:

$$Z^{GC}(T,V,\mu) = \prod_i Z_i^{GC}(T,V,\mu_i)$$

where i it indicates the hadronic specie ($\pi, k, p, \ldots$), T is the system temperature and V its volume. μ_i is the chemical potential which guarantee the averaged conservation of the charges.

Moving to logaritms:

$$\ln Z^{GC}(T,V,\mu) = \sum_i \ln(z_i^{GC}(T,V,\mu_i))$$

chemical potential is the product that guarantees the conservation of different charges on average and is defined as

$$\mu = \sum_j \mu_{Q_j} Q_j$$

Q_j are charges (quantum numbers) which are conserved

μ_{Q_j} are chemical potentials

$\mu \equiv$ energy needed to add to the system a ptc with quantum numbers Q_j

In a hadron gas (i.e. governed by strong interactions) with $m < 1{,}8\,GeV$ ($\to$ no c/b hadrons) there are 3 conserved charged:

- Q : electric charge (or 3rd component of the isospin)
- B : Baryon number
- S : strangeness

$$\Rightarrow \qquad \mu_i = \mu_{I_3} I_{3i} + \mu_B B_i + \mu_S S_i$$

The partition function for the i-th hadronic specie is that of an ideal gas (Bose-Einstein o Fermi gas) in the macroscopic limit:

$$\ln Z_i^{GC}(T,V,\mu_i) = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln\left[1 \pm e^{-\beta(E-\mu_i)}\right] =$$

$$= \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln\left[1 \pm \lambda_i e^{-\beta E}\right]$$

where $\oplus$ is for fermion and $\ominus$ for bosons

$g_i = 2S+1$ is the degeneration due to spin

λ_i is the fugacity defined as: $\lambda_i = e^{\beta\mu_i}$

The multiplicity is obtained by integrating the partition function, by developing the logarithm by using Taylor's serie and integrating by parts. After some variable changes, the particle density can be obtained and is:

$$n_i(T,\mu_i) = \frac{g_i \cdot T}{2\pi^2} \sum_{k=1}^{\infty} \frac{(\pm 1)^k}{k} \lambda_i^k \, m_i^2 \, K_2\left(\frac{k m_i}{T}\right)$$

which can be implemented / evaluated by knowing K_2 (Bessel function)

The possible corrections that have to be implemented are:

1) Decay chains: If I want to counts π, i'm counting also the resonances which decays into π

$$\langle N_i \rangle^{MEAS}(T,V,\mu_i) = \langle N_i \rangle^{therm}(T,V,\mu_i) + \sum_j BR_{j \to i} \langle N_j \rangle (T,V,\mu_j)^{THERM}$$

At high temperature and/or μ_B, multiplicity of light hadrons is dominated by the contribution of the decay of resonances.

2) REpulsive interactions: For high particle density (i.e. high T and/or μ_B) low distance repulsive interaction are observed between hadrons.

A "hard core" Van der Waals repulsion have to be assigned

$$V_{eigen} = 4 \cdot \frac{4}{3} \pi R^3 \qquad \text{"Excluded volume correction"}$$

The radius R is $\simeq 0{,}3$ fm ($\Rightarrow$ I cannot put too many hadrons in a small space)

3) Width of Resonances: Resonances have a widht that have to be taken into account

$$n_i(T,\mu_i) = \frac{g_i}{2\pi^2} \frac{1}{N_{BW}} \int dm \int_0^{\infty} \frac{\Gamma_i^2}{(m-m_i)^2 + \Gamma_i^2/4} \, \frac{p^2 \, dp}{e^{\beta(E_i(m)-\mu_i)} \pm 1}$$

) Strangeness suppression $\gamma_s < 1$

At low energy, strange quark should not be thermalized due to its large mass w.r.t u and d quarks

For RHIC and LHC, $\gamma_s = 1$, for lower energy or smaller collision systems (pp, pA) it might be needed to have $\gamma_s < 1$.

FREE PARAMETERS OF THE MODEL

In the model we have

$$\langle N_i (T, V, \mu_i) \rangle = V \cdot n_i (T, V, \mu_i) = \frac{V g_i T}{2\pi^2} \sum_{k=0}^{\infty} \frac{(\pm 1)^k}{k} \lambda_i^k m_i^2 K_2\left(\frac{k m_i}{T}\right)$$

$$\lambda_i = e^{\mu_i / T} \qquad \mu_i = B_i \mu_B + S_i \mu_S + I_{3i} \mu_{I_3}$$

$\Rightarrow$ In total there are 5 free parameters: $T, \mu_B, \mu_S, \mu_{I_3}$ and V.

Knowing the electric charge Q, barion number B and strangeness S of the INITIAL state ($= Z_S$ protons and N_S neutrons $\boxed{\text{STOPPED}}$) it is possible to fix the volume of the fireball V and the chemical potentials μ_S and μ_{I3}

$$V \sum_i n_i I_{3_i} = \frac{Z_S - N_S}{2} \qquad V \sum_i n_i B_i = Z_S + N_S \qquad V \sum_i n_i S_i = 0$$

So 5 parameters + 3 initial conditions $\Rightarrow$ 2 free parameters T and μ_B

The same 2 parameters of the nuclear phase diagrams

FIT TO THE MEASURED PARTICLES

Sometimes, the ratios of particles are used. This is done because:

1) Some systematic uncertainities cancels out with the ratio
2) The Volume dependence is removed

It is possible to obtain T and μ_β by minimizing the χ^2

$$\chi^2 = \sum_i \frac{\left(R_i^{exp} - R_i^{model}\right)^2}{\sigma_i^2}$$

i: Particle Species

σ_i^2 → Statistical and Systematic uncertainies of the experimental data

R_i^{exp}: Measured Ratio

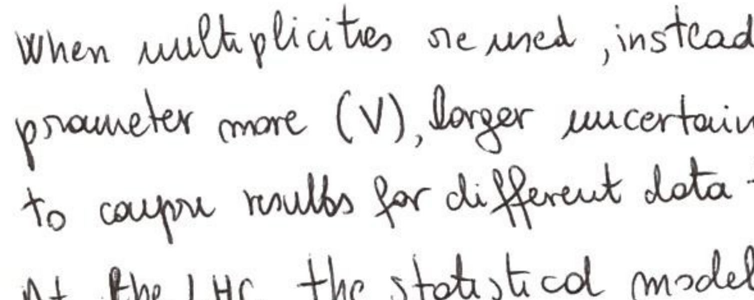
R_i^{model}: Model Ratio

When multiplicities are used, instead of particle ratios, there is a free parameter more (V), larger uncertainties and a worse χ^2, but its easier to compare results for different data tacking.

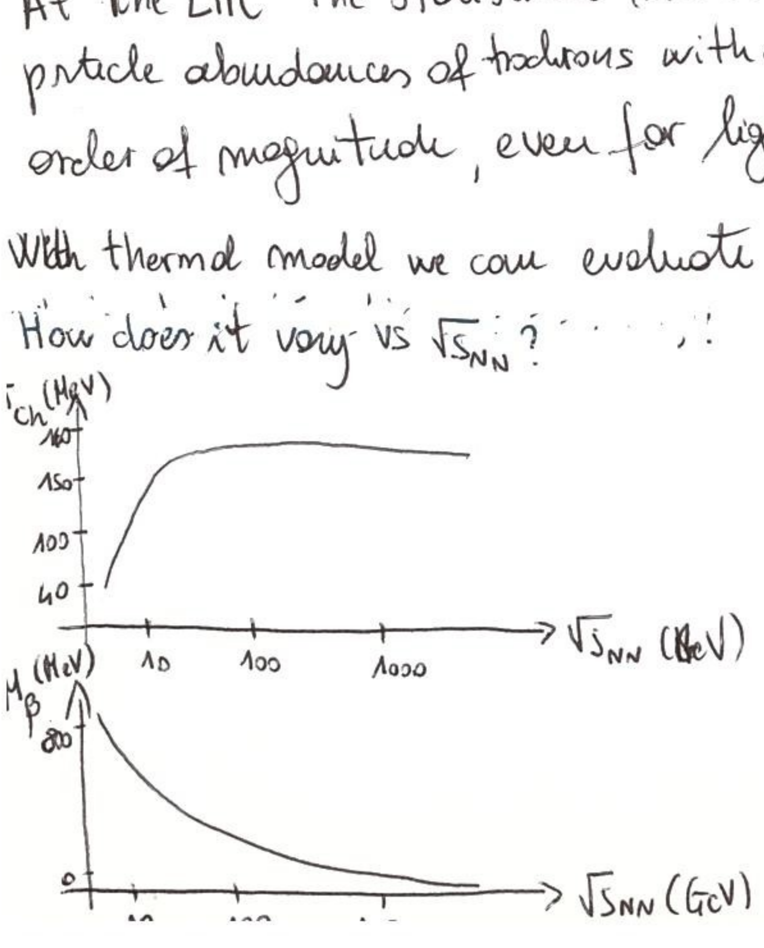
At the LHC the statistical model is able to describe fairly well particle abundances of hadrons with a yield which ranges over 9 order of magnitude, even for light nuclei and hypernuclei.

With thermal model we can evaluate CHEMICAL freeze-out temperature.

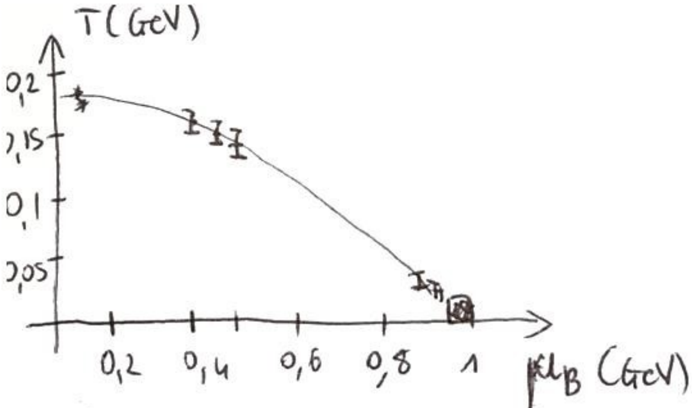
How does it vary vs $\sqrt{s_{NN}}$?



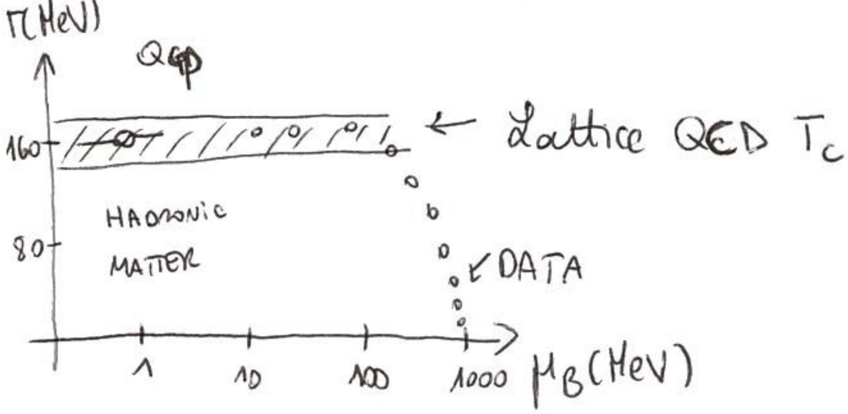
- Temperature T increases rapidly with $\sqrt{s_{NN}}$ up to $\simeq 160$ MeV ($\equiv$ Critical temperature) for $\sqrt{s_{NN}} \gtrsim 7$-8 GeV



- Chemical potential μ_β decreases with increasing $\sqrt{s_{NN}}$ in the whole range of energies explorated. At the LHC $\mu_B \equiv 0$



The extracted values for different $\sqrt{s_{NN}}$ are aligned along a freeze-out curve



For $\sqrt{s_{NN}} \gtrsim 100$ GeV, where $\mu_B < 300$ MeV, the chemical freeze-out is very close to the phase transition predicted from Lattice QCD

Is the STATISTICAL MODEL A "UNIVERSAL" MODEL FOR HADRONIZATION?
The same model can be used to explain hadronization in pp and e^+e^- collisions. Also in this case one has to ensure thermal and chemical equilibrium, using the CANONICAL formulation of the partition function (i.e. exact conservations of charges)
In this case there ~~is also~~ are 3 fit parameters, T, V and μ_s (to account for the incomplete equilibrium for the s-quark).
The temperature extracted from the fit is $\simeq 170$ MeV, independent from the collision energy, and similar to the T obtained from A-A collisions. (This may be indicates a limit-temperature for a hadron gas, or a phase boundary).
To conclude, the statistical models for the hadronization allow to determine T and μ_B of the fireball at the chemical freeze-out starting from the ratios of particles which are measured.
The Temperature reached the critical temperature obtained by lattice QCD temperature.

Finally, there exists an "universality" of freeze-out temperature for pp, e^+e^- and AA collisions.
If this observation indicates a QGP phase transition or something specific ~~for~~ the hadron gas is yet not fully understood.