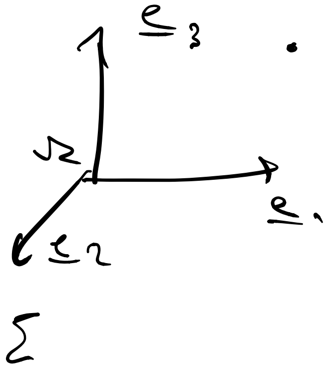
MECCANICA RAZIONALE

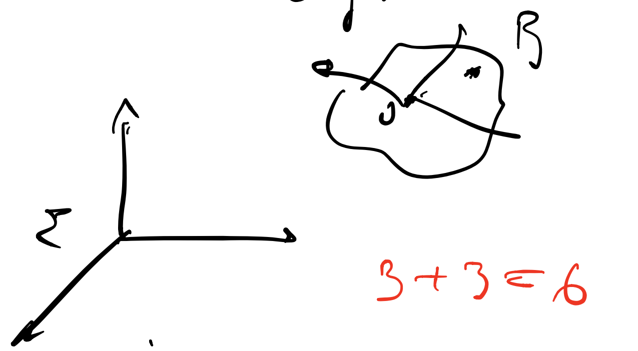
Ing Civile
Navale

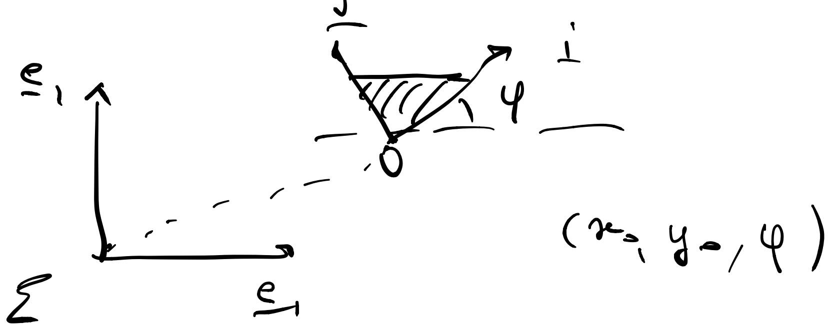
Lezione 3 marzo 2021

Corpo rigido

Studio del punto materiale → studio del corpo rigido

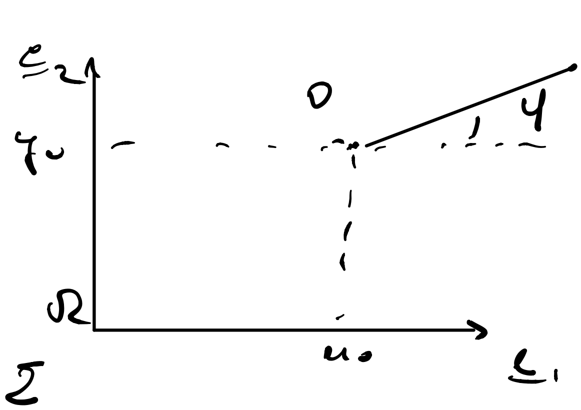


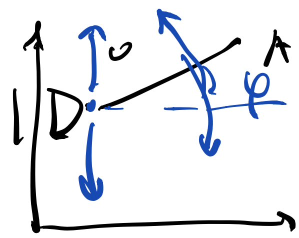




(x_0, y_0, φ)

$2+1=3$

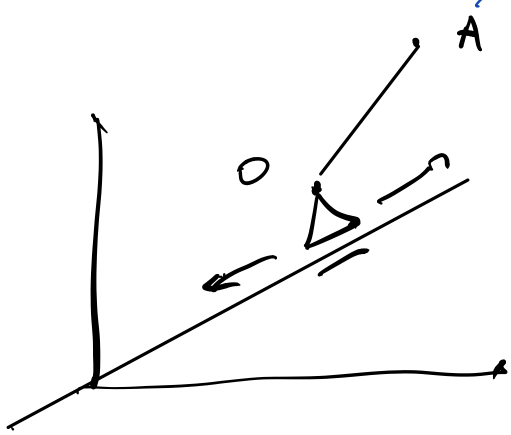




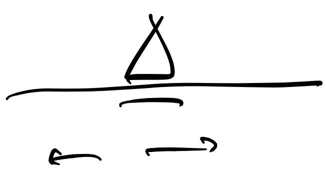
$x_0 = 0$

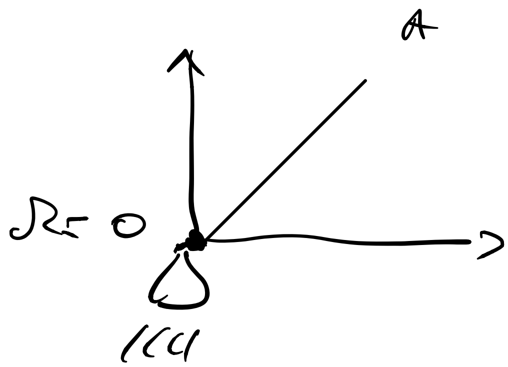
x_0

$(x_0, y_0, \varphi) \rightarrow (\cancel{x_0}, y_0, \varphi)$



φ





$$\begin{cases} x_0 = 0 \\ y_0 = 0 \\ \varphi = \alpha \end{cases}$$

Vincoli semplici: Tolgono 1 grado di libertà → principio di sovrapposizione dei vincoli

S sistema materiale

n gradi di libertà

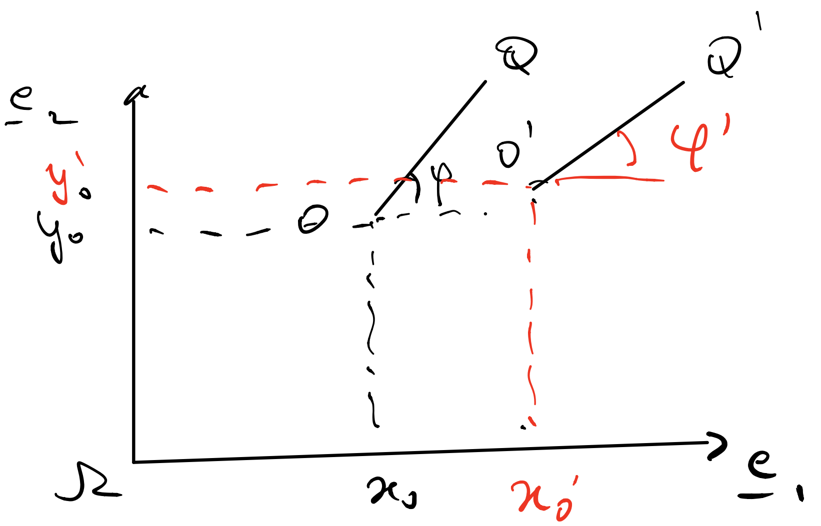
m vincoli

$m < n$ IPOSTATICO

$m = n$ ISOSTATICO

$m > n$ IPERSTATICO

Sistemi articolati



OQ

(x_0, y_0, φ)

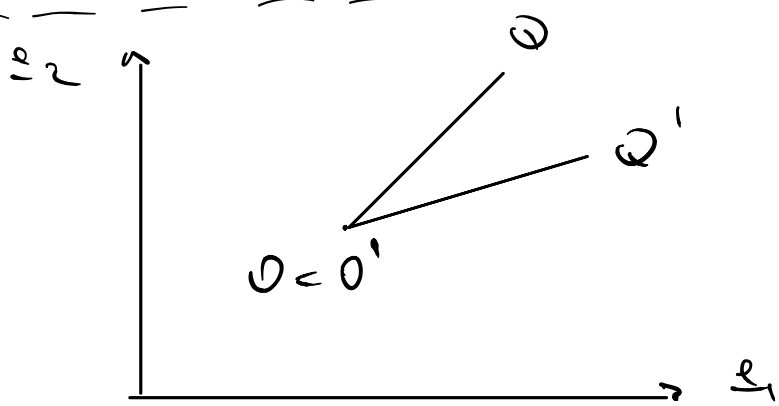
$O'Q'$

(x'_0, y'_0, φ')

↓

coordinate indipendenti (libere)

$$\overline{OQ} = \overline{O'Q'} = \ell$$



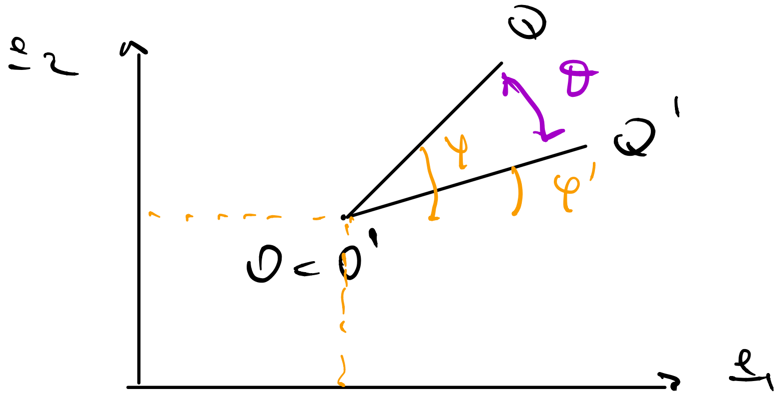
cerniera mobile interna

$$\begin{cases} x_0 = x'_0 \\ y_0 = y'_0 \end{cases}$$

sovrapposizione di due vincoli semplici

$(x_0, x'_0, y_0, y'_0, \varphi, \varphi')$

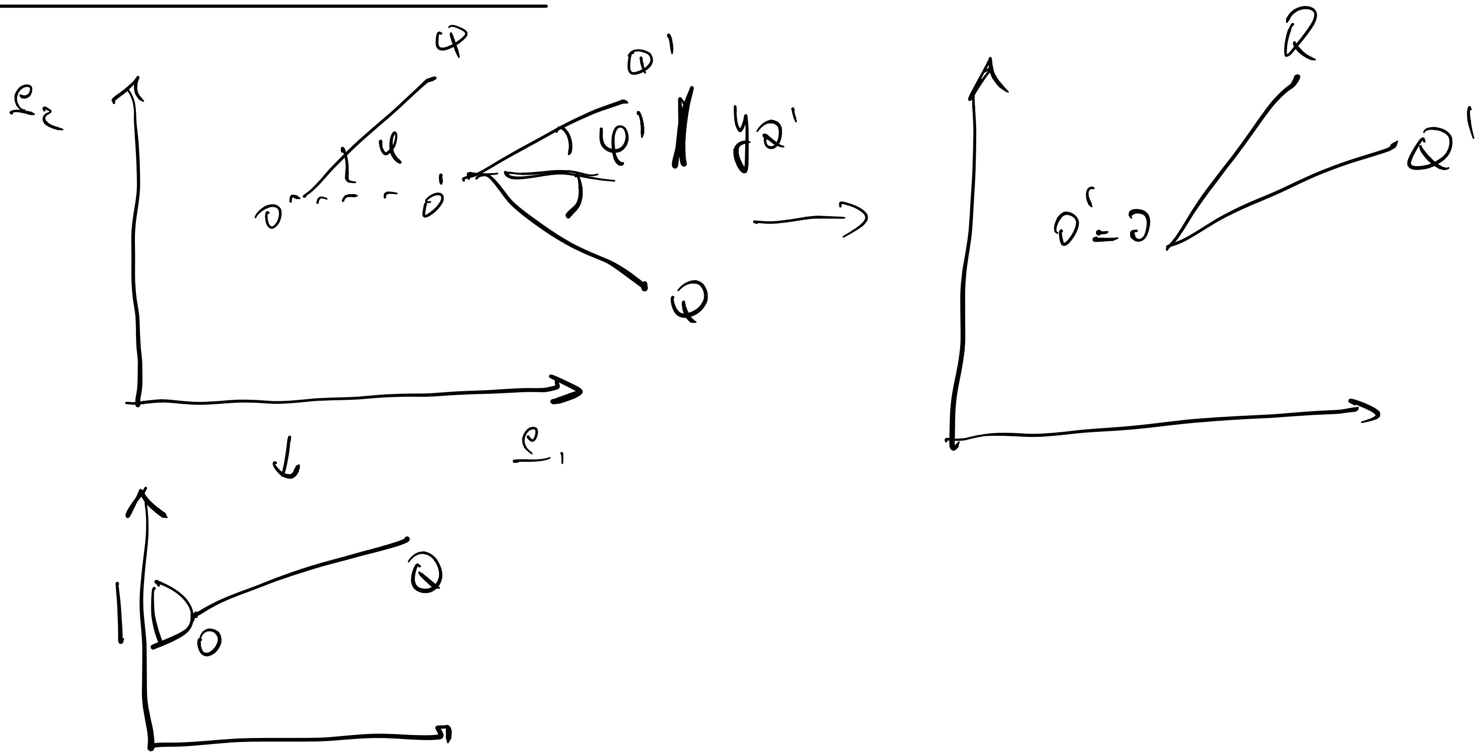
$6 \longrightarrow 4$



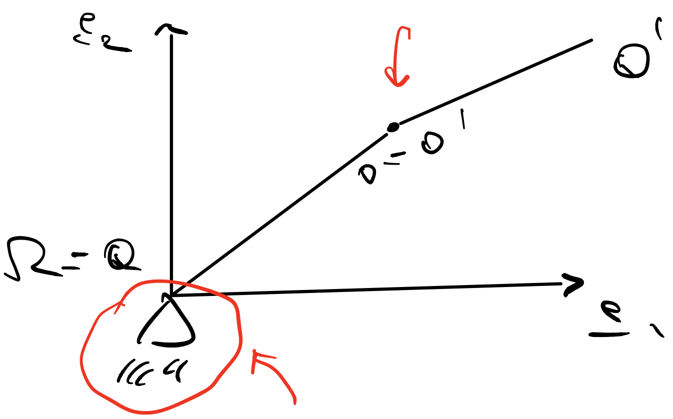
$(x_0, y_0, \varphi, \varphi')$

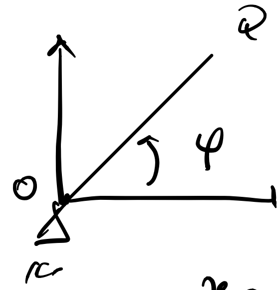
$(x_0, y_0, \varphi, \theta)$

Seconda parte



Ad esempio





$x_O = 0$

$y_O = 0$

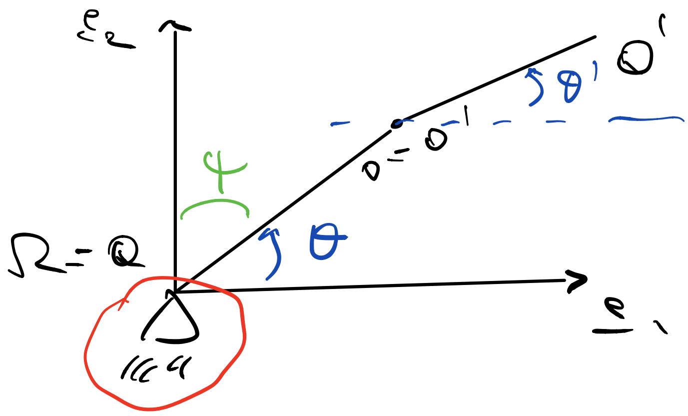
Partiamo da $(x_O, y_O, \varphi, x_O', y_O', \varphi')$ 6 gradi di libertà

cerniera interna

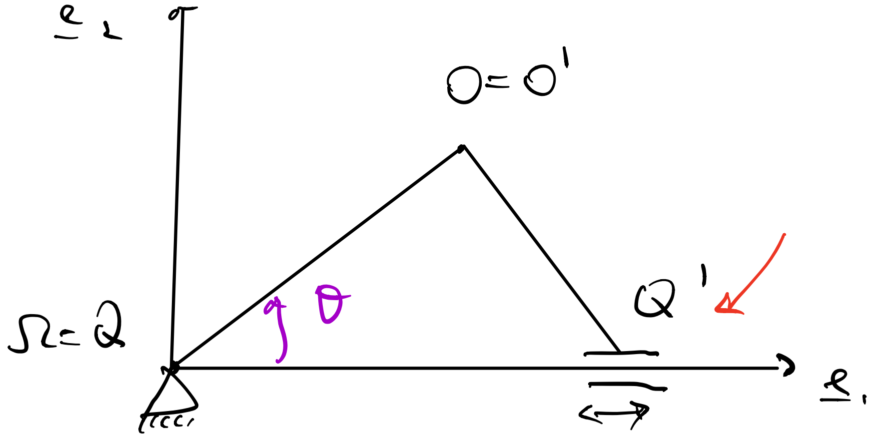
$$\begin{cases} x_O = x_O' \\ y_O = y_O' \end{cases}$$

cerniera esterna

$$\begin{cases} x_Q = 0 = x_O + l\cos\varphi \\ y_Q = 0 = y_O + l\sin\varphi \end{cases}$$



$6 - 4 = 2$ gradi di libertà



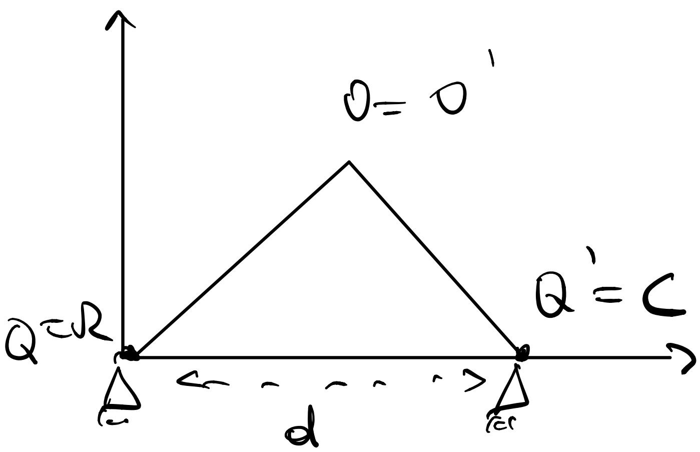
$\rightarrow y_{Q'} = 0$

$= y_O' + l\sin\varphi'$

$6 - 5 = 1$ grado di libertà

$6 - 4 - 1 = 6 - 5 = 1$ θ

$6 - 6 = 0$ sistema isostatico



$x_{O'} = d$

$= x'_O + l \cos\varphi'$

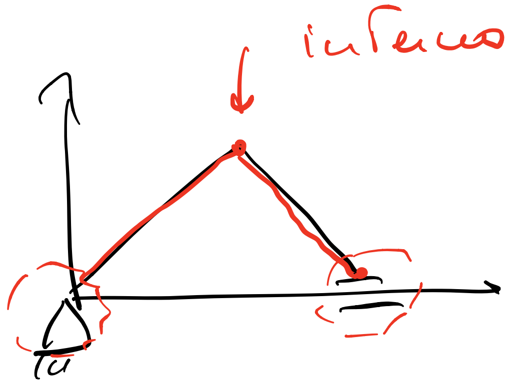
↑ cerniera fissa

$\overline{OQ} = \overline{O'Q'} = l$

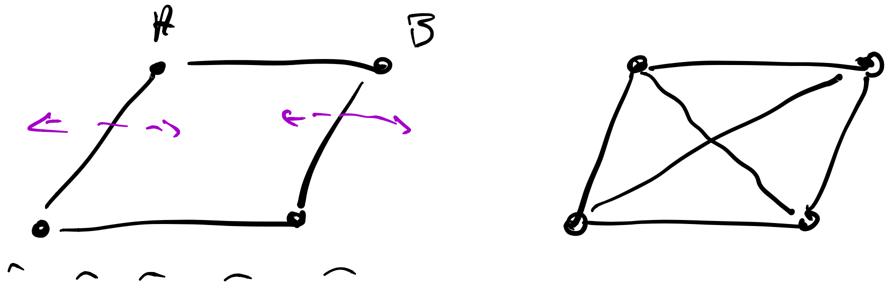
→ principio di sovrapposizione dei vincoli

spazio delle configurazioni è (x_0, y_0, φ)

$y_0 = 0$



<u>Commento</u> Travature: aste collegate da cerniere



Formalismo simile a quello sviluppato per i vincoli

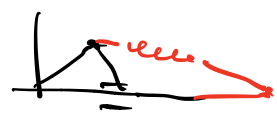
Nodi → punti materiali

Aste → vincoli

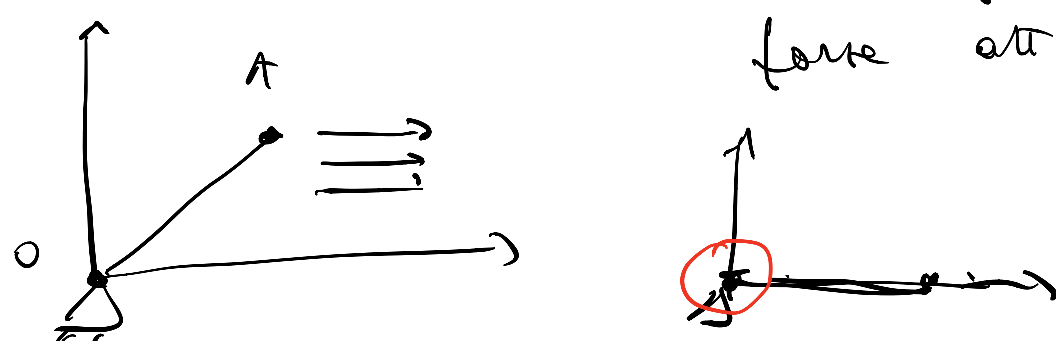
Terza parte

Classificazione delle forze agenti su un sistema

1. Forze attive: forza peso, forza elastica



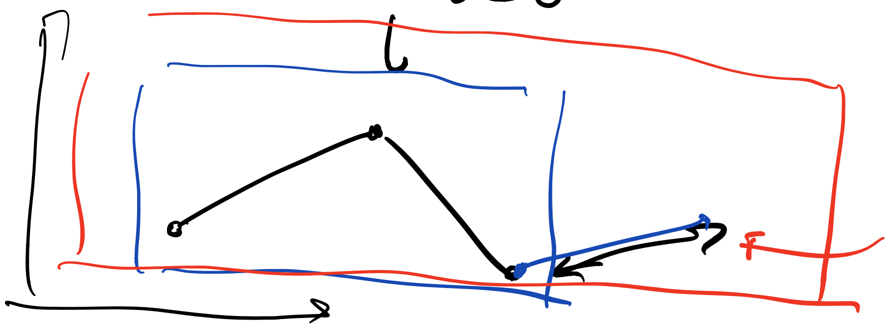
reattive: forze dovute ai vincoli e di risposta alle forze attive

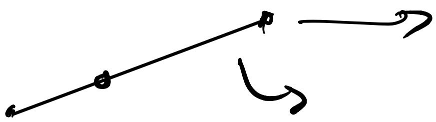


2. forze · esterne : esercitate da agenti esterni

· interne : esercitate da una parte del sistema sull'altra

$O = O'$





(sforzi interni)

3. forze concentrate

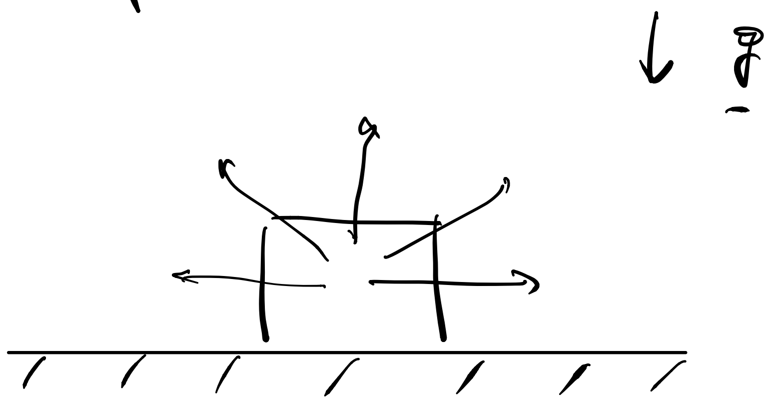
distribuite

PRINCIPIO DEI LAVORI VIRTUALI

Forze attive

(vincoli → limitazioni geometriche)

Esempio

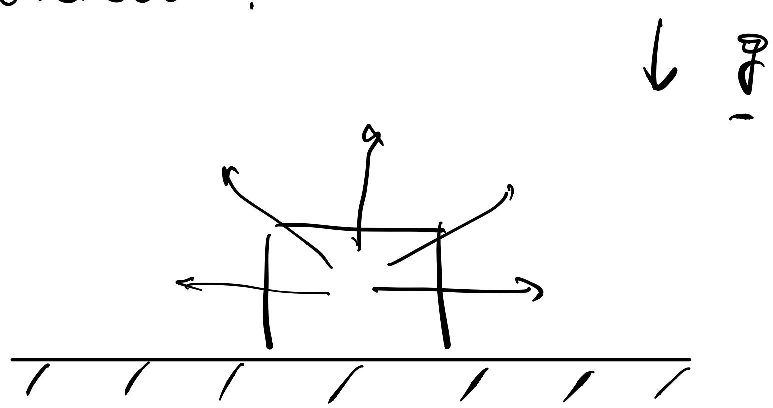


Forza peso $m\underline{g}$

forze di reazione

Idea: consideriamo le forze attive

→ lavoro svolto da queste forze se pensiamo di spostare il sistema materiale in modo compatibile con i vincoli.

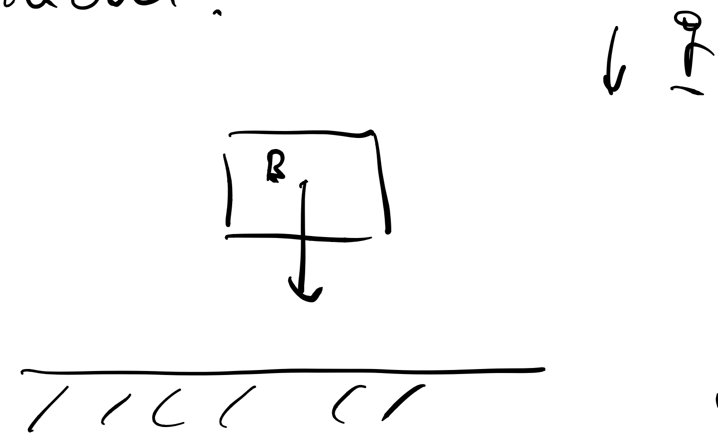


Spostamenti virtuali

$\delta \underline{x}_B$

<u>Vedremo</u>: All'equilibrio il lavoro virtuale delle forze attive è minore o uguale a zero per ogni spostamento virtuale consentito dai

vincoli:

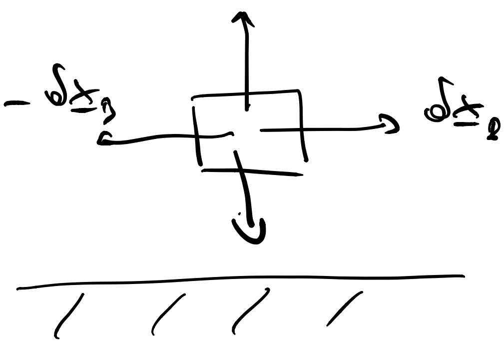


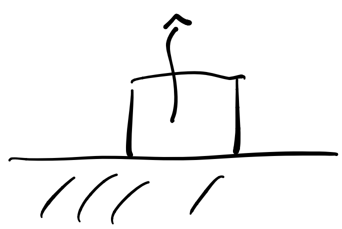
$$\underset{\downarrow}{\delta \underline{x}_B} \cdot \underset{m\underline{g}}{\underline{F}_{peso}} > 0$$

Spostamenti virtuali:

- invertibile se:

$\delta \underline{x}_B$ è uno spostamento virtuale $\Rightarrow$ $-\delta \underline{x}_B$ lo è





Lavoro virtuale $L.V. = \underline{F}_B \cdot \delta \underline{x}_B$

forza attiva nel punto B

spostamento virtuale del punto B

All'eq. $\underline{F}_B \cdot \delta \underline{x}_B \leq 0$

Se $\delta \underline{x}_B$ è invertibile, allora $-\delta \underline{x}_B$

è spostamento virtuale e quindi

all' eq. $\underline{F}_B \cdot (-\delta \underline{x}_B) \leq 0$

Se $\delta \underline{x}_B$ invertibile, all' eq.

$$\left.\begin{array}{l} \underline{F}_B \cdot \delta \underline{x}_B \leq 0 \\ \underline{F}_B \cdot (-\delta \underline{x}_B) \leq 0 \end{array}\right\} \Rightarrow \underline{F}_B \cdot \delta \underline{x}_B = 0$$