Lezione 5 Marzo 2021

Presenti in aula A (su autodichiarazione) dalle 9 alle 11

sm 6000695

sm 6000694

sm 6000701

Calcolare lo sviluppo in serie di Taylor al secondo ordine della funzione

$$f(x) = \underbrace{\cos(e^x - 1)} \quad \text{in} \quad x_0 = 0.$$

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f ha per dominio $\mathbb{R}$.
ed è infinite volte derivabile in $\mathbb{R}$.

$$f(x) = \cos(e^x - 1) \qquad \boxed{f(0) = 1}$$

$$f'(x) = \underbrace{-\operatorname{sen}(e^x - 1)}\cdot e^x \qquad \boxed{f'(0) = 0}$$

Applicando la regola della derivazione di funzioni (derivabili) composte (Regola della catena)

$$f''(x) = \left[-\cos(e^x - 1)\cdot e^x\right]\cdot e^x + \left(-\operatorname{sen}(e^x - 1)\cdot e^x\right)$$

Regola di Leibniz e della catena

$$\boxed{f''(0) = -1}$$

$$f(x) = \cos(e^x - 1) =$$

$$= \underset{f(0)}{1} + \underset{\substack{\parallel \\ 0}}{f'(0)} \cdot x + \frac{\overset{\substack{-1 \\ \parallel}}{f''(0)}}{2!} x^2 + o(x^2)$$

$$= \underbrace{1 - \frac{1}{2} x^2}_{= T_2(x)} + o(x^2)$$

Oss

$$e^x - 1 = x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + o(x^n)$$

$$\cos t = 1 - \frac{1}{2} t^2 + \frac{1}{4!} t^4 + \dots + \frac{(-1)^k}{(2k)!} t^{2k} + o(t^{2k})$$

Se dunque pongo $\boxed{t = e^x - 1} = \underbrace{x + o(x)}$

$$\cos(e^x - 1) = \cos t = 1 - \frac{1}{2} \underbrace{(x + o(x))^2}_{=t} + o\left[\underbrace{(x + o(x))^2}_{=t^2}\right]$$

$$(x + o(x))^2 = (x + o(x)) \cdot (x + o(x)) = x^2 + \underbrace{2x \cdot o(x)} + \underbrace{o(x) \cdot o(x)}$$

Ricordando che

$$x \cdot \underbrace{\sigma(x)}_{} = \sigma(x^2)$$

$$\sigma(x) \cdot \sigma(x) = \sigma(x^2)$$

Pertanto

$$\left(x + \sigma(x)\right)^2 = x^2 + \sigma(x^2)$$

Inoltre $$\sigma\left(\underbrace{x^2 + \sigma(x^2)}_{}\right) = \sigma(x^2)$$

$$\cos\left(\underbrace{e^x - 1}\right) = 1 - \frac{1}{2}\underbrace{\left(x + o(x)\right)}^2 + o\left(\left(x + o(x)\right)^2\right)$$

$$= 1 - \frac{1}{2}\left(x^2 + o(x^2)\right) + o(x^2)$$

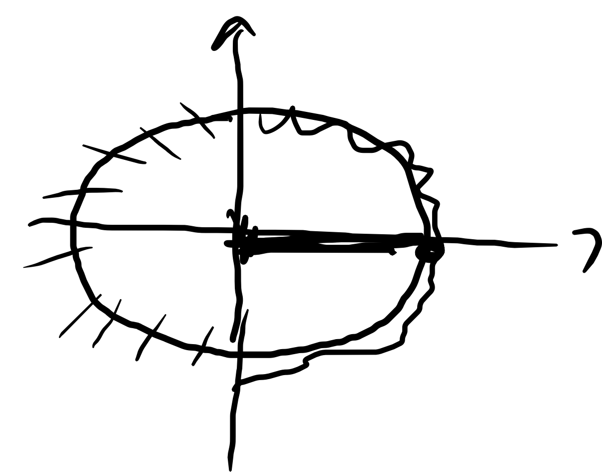
$$= \underbrace{1 - \frac{1}{2}x^2} \underbrace{- \frac{1}{2}o(x^2) + o(x^2)}_{= o(x^2)}$$

Calcolare il polinomio di Taylor della funzione

$$f(x) = \ln(\cos x)$$

di ordine 2 in $\underline{x_0 = 0}$.

$$\mathrm{Dom}(f) = \{ x \in \mathbb{R} : \cos x > 0 \}$$



$$-\frac{\pi}{2} + 2k\pi < x < \frac{\pi}{2} + 2k\pi \qquad = \bigcup_{k \in \mathbb{Z}} \left\{ -\frac{\pi}{2} + 2k\pi < x < \frac{\pi}{2} + 2k\pi \right\}$$

$$f'(x) = \frac{1}{\cos x} \cdot (-\operatorname{sen} x) = \underline{-\operatorname{tg} x}$$

$$f''(x) = -(1 + \operatorname{tg}^2 x)$$

$$f'''(x) = \ldots \quad -$$

$$f(x) = \ln(\cos x) \qquad f(0) = \ln(1) = \underline{0}$$

$$f'(x) = -\operatorname{tg} x \qquad f'(0) = \underline{\underline{0}}$$

$$f''(x) = -(1 + \operatorname{tg}^2 x) \qquad f''(0) = -1$$

$$T_2(x) = \underline{0} + \underline{\underline{0}} \cdot x - 1 \frac{1}{2!} x^2 = -\frac{x^2}{2}$$

$$\boxed{f(x) = -\frac{x^2}{2} + o(x^2)}$$

$$\ln(\cos x) = \ln\Big(\underbrace{1 - \frac{x^2}{2} + o(x^2)}_{= t}\Big)$$

$$\cos x = 1 - \frac{x^2}{2} + o(x^2)$$

$$\ln(1+t) = t - \frac{t^2}{2} + \frac{t^3}{3} + o(t^3) - \ldots$$

$$= \underbrace{-\frac{x^2}{2} + o(x^2)}_{t} + o\Big(\underbrace{\Big(-\frac{x^2}{2} + o(x^2)\Big)}_{t}\Big)$$

$$o(x)$$

$$\ln\left(1 \underbrace{- \frac{x^2}{2} + o(x^2)}_{t}\right) =$$

$$= -\frac{x^2}{2} + o(x^2) - \frac{\left(-\frac{x^2}{2} + o(x^2)\right)^2}{2} + o\left(\left(-\frac{x^2}{2} + o(x^2)\right)^2\right)$$

$$= t - \frac{t^2}{2} + o(t^2)$$

$$\left(-\frac{x^2}{2}+\sigma(x^2)\right)^2 = \underbrace{\frac{x^4}{4} - \underbrace{x^2\,\sigma(x^2)}_{=\,\sigma(x^4)} + \underbrace{\sigma(x^2)\cdot\sigma(x^2)}_{=\,\sigma(x^4)}}_{\sigma(x^4)}$$

$$\sigma\left(\left(-\frac{x^2}{2}+\sigma(x^2)\right)^2\right) = \sigma(x^4)$$

$$\ln(\cos x) = -\frac{x^2}{2} + o(x^2)$$

$$\ln(\cos x) \simeq -\frac{x^2}{2} + o(x^2)$$

ordine d'infinitesimo di $\ln(\cos x)$ in 0

è 2.