SEISMOLOGY I

Laurea Magistralis in Physics of the Earth and of the Environment

Wave propagation

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Wave propagation

Wave propagation is ruled by:

SUPERPOSITION PRINCIPLE

REFLECTION

REFRACTION

DIFFRACTION

DOPPLER EFFECT

GEOMETRICAL SPREADING

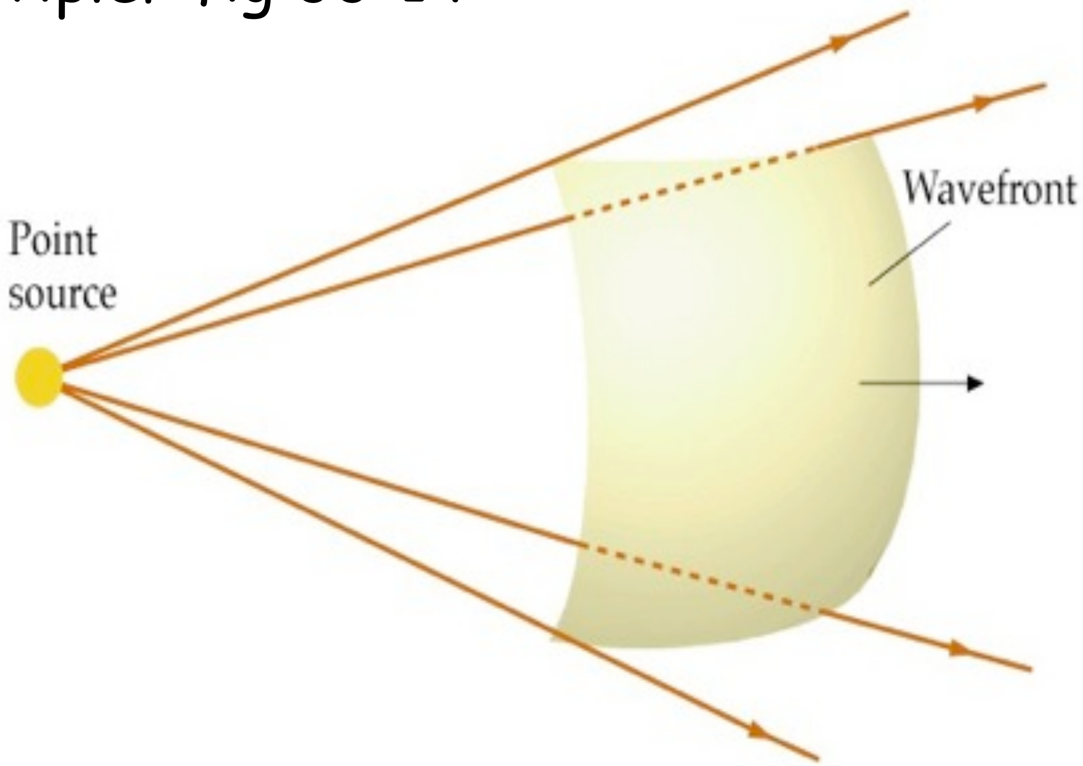
Before Maxwell's equations were developed Huygens and Fermat could describe the propagation of light using their empirically determined principles.

Huygens Principle

All points on a given wavefront are taken as point sources for the production of spherical secondary waves, called wavelets, which propagate outwards with speeds characteristic of waves in that medium.

After some time has elapsed the new position of the wavefront is the surface tangent to the wavelets.

Tipler fig 33-14

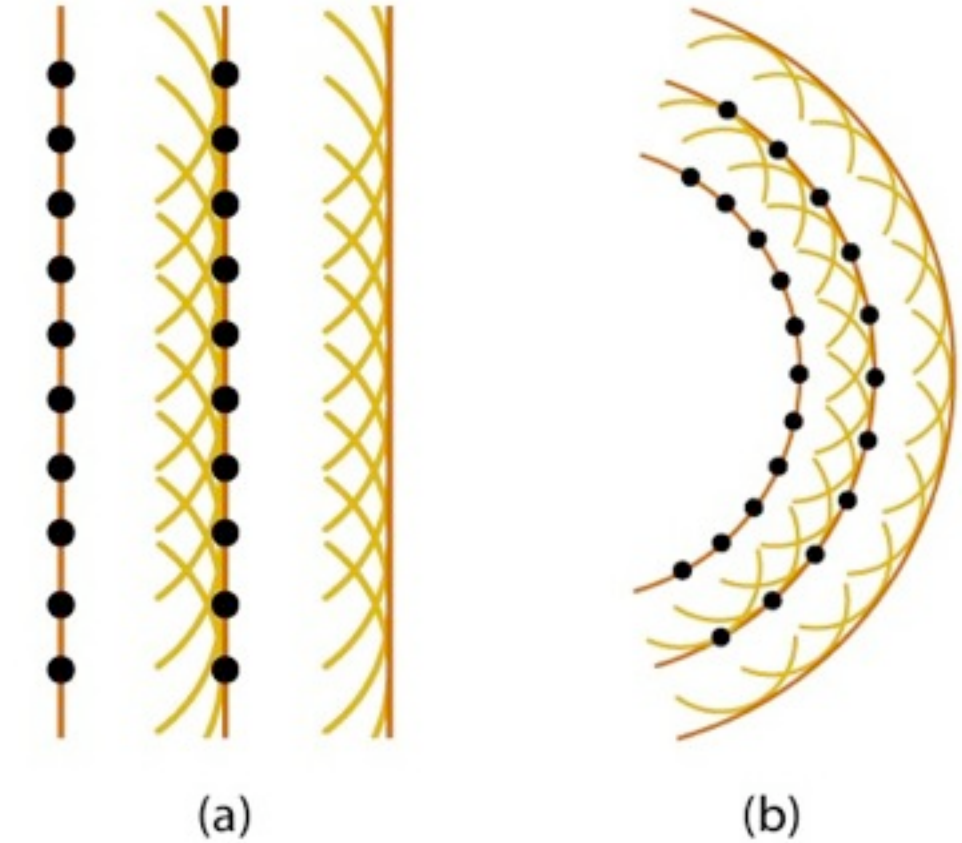


Spherical wavefront from a point source

Huygens construction for

plane wave

spherical wave



Tipler fig 33-16

The propagation of light - Fermat's Principle

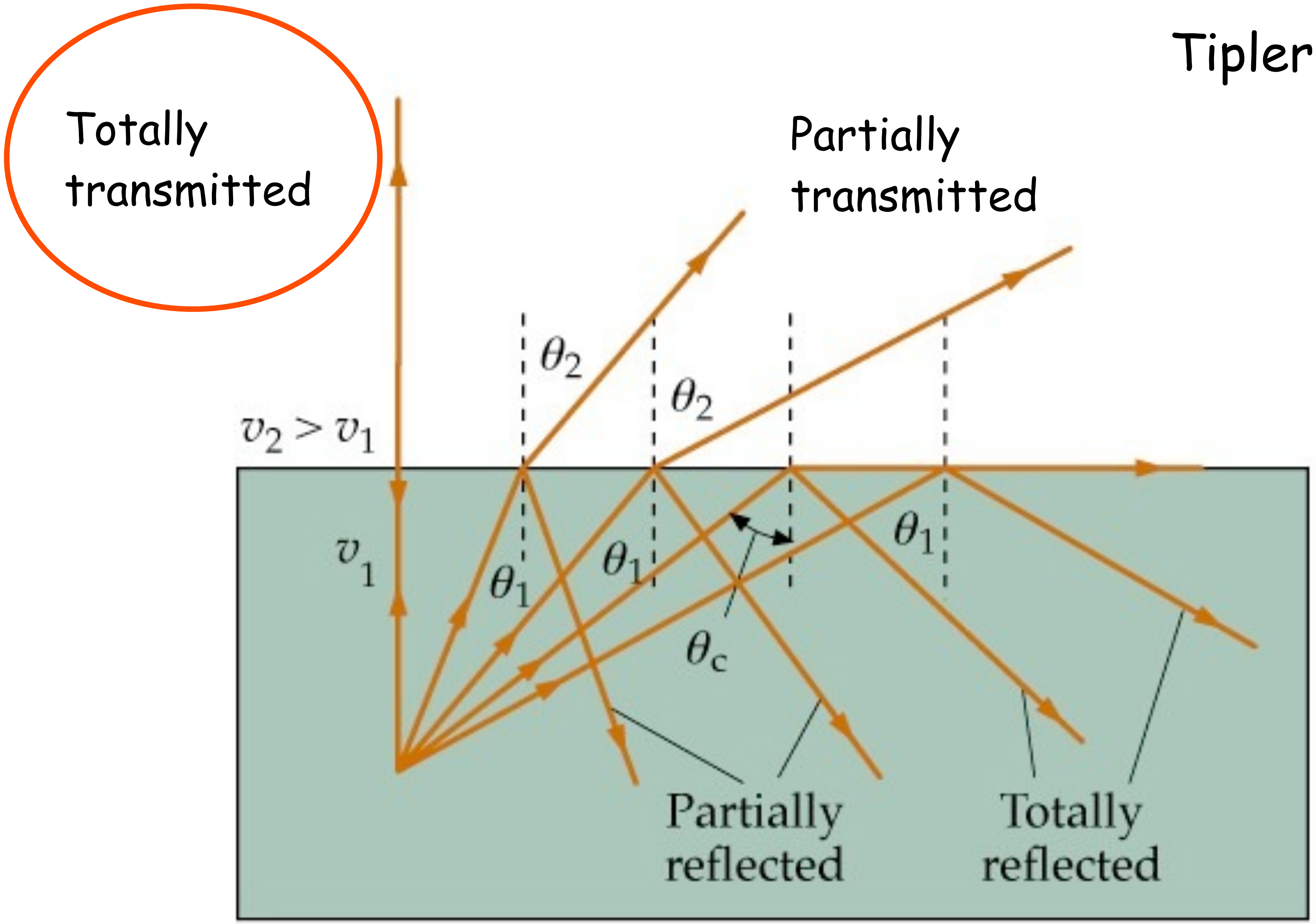
This is a general principle for determining the paths of light rays.

When a light ray travels between any two points P and Q, its actual path will be the one that takes the least time.

Fermat's principle is sometimes referred to as the "principle of least time".

An obvious consequence of Fermat's principle is that when light travels in a single homogeneous medium the paths are straight lines.

Tipler figure 33-22a



Some definitions:

Index of refraction :

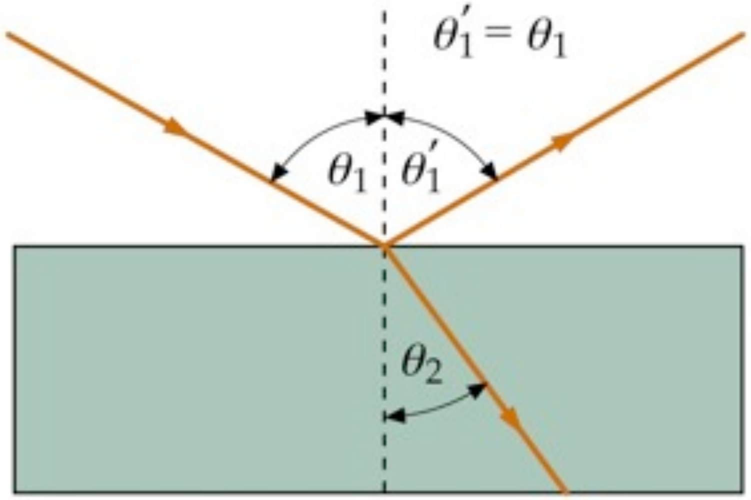
A transparent medium is characterised by the **index of refraction** n, where n is defined as the ratio of the speed of light in a vacuum c, to the speed of light in the medium v

$$n = \frac{c}{v}$$

air	n=1.0003
water	n=1.33
diamond	n=2.4

Law of reflection :

$$\theta_1 = \theta_1'$$



Intensity of transmitted and reflected light

For normal incidence at a boundary (ie $\theta_1 = \theta_1' = 0$)

the reflected intensity I is given by:

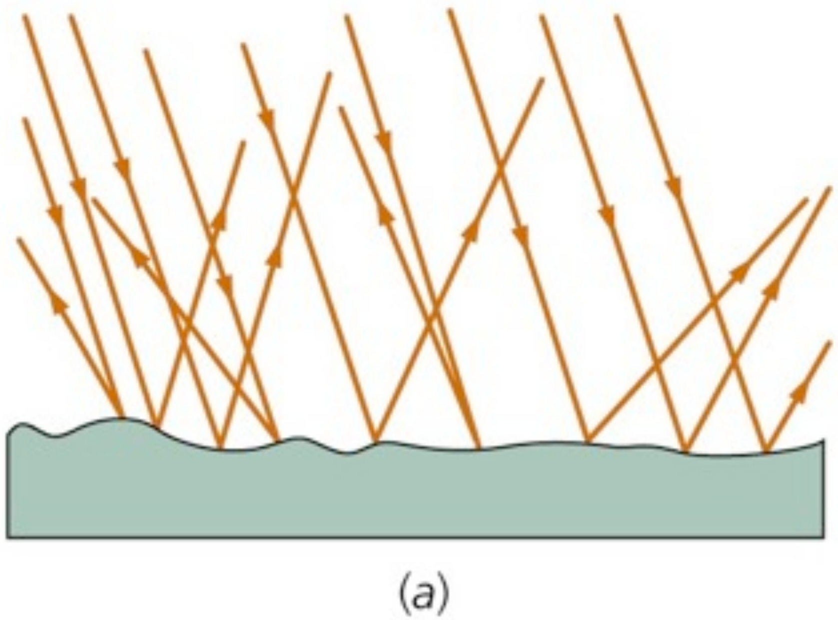
$$I = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 I_o$$

I_o = incident intensity, n_1 and n_2 are the refractive indexes of medium 1 and 2

eg: for an air-glass interface, $n_1 = 1$ and $n_2 = 1.5$

giving $I = I_o/25$

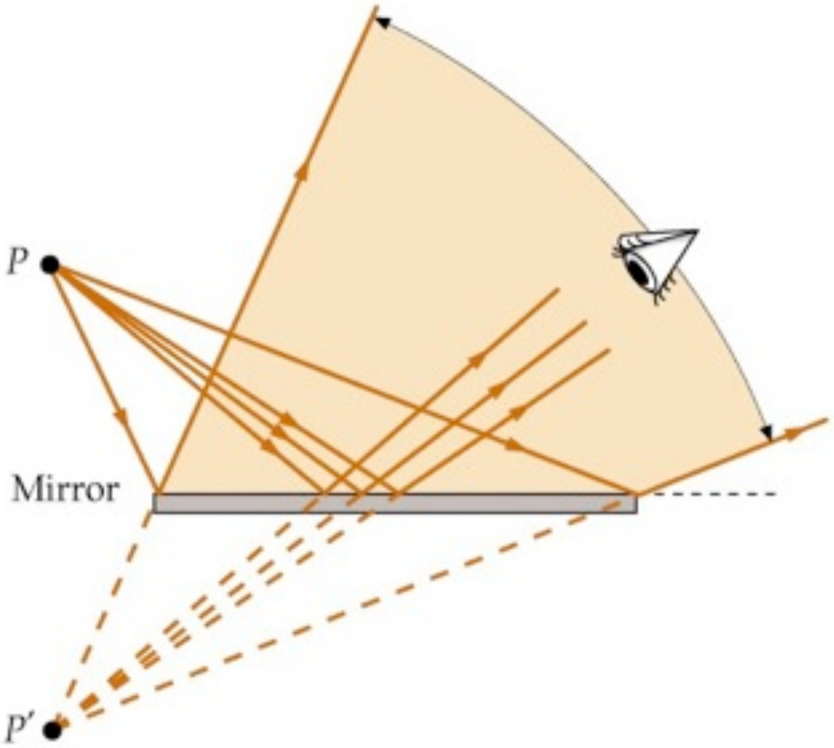
Reflection of light



Diffuse

reflection

(Tipler fig 33-20)

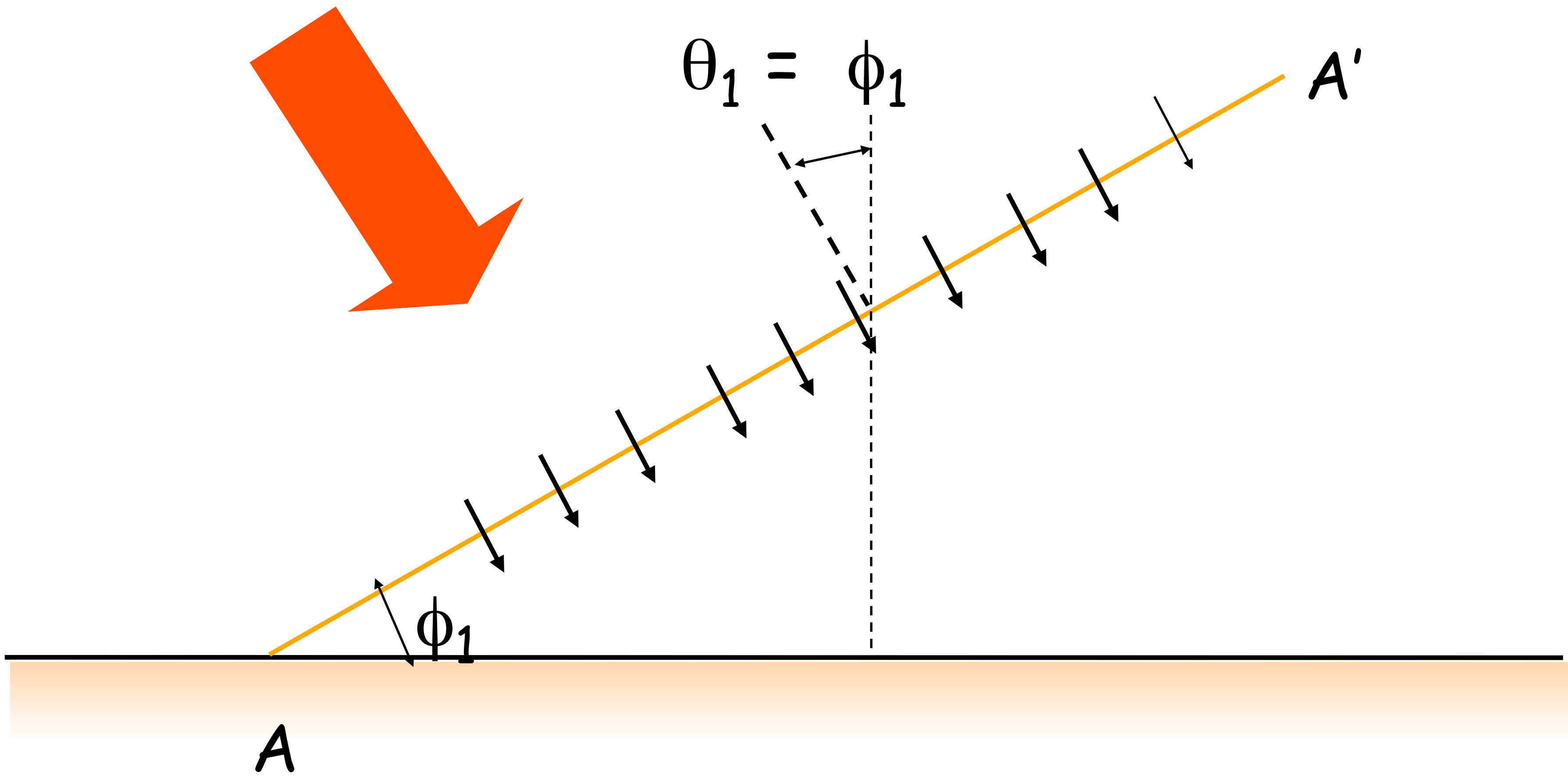


Specular

reflection

(Tipler fig 33-19)

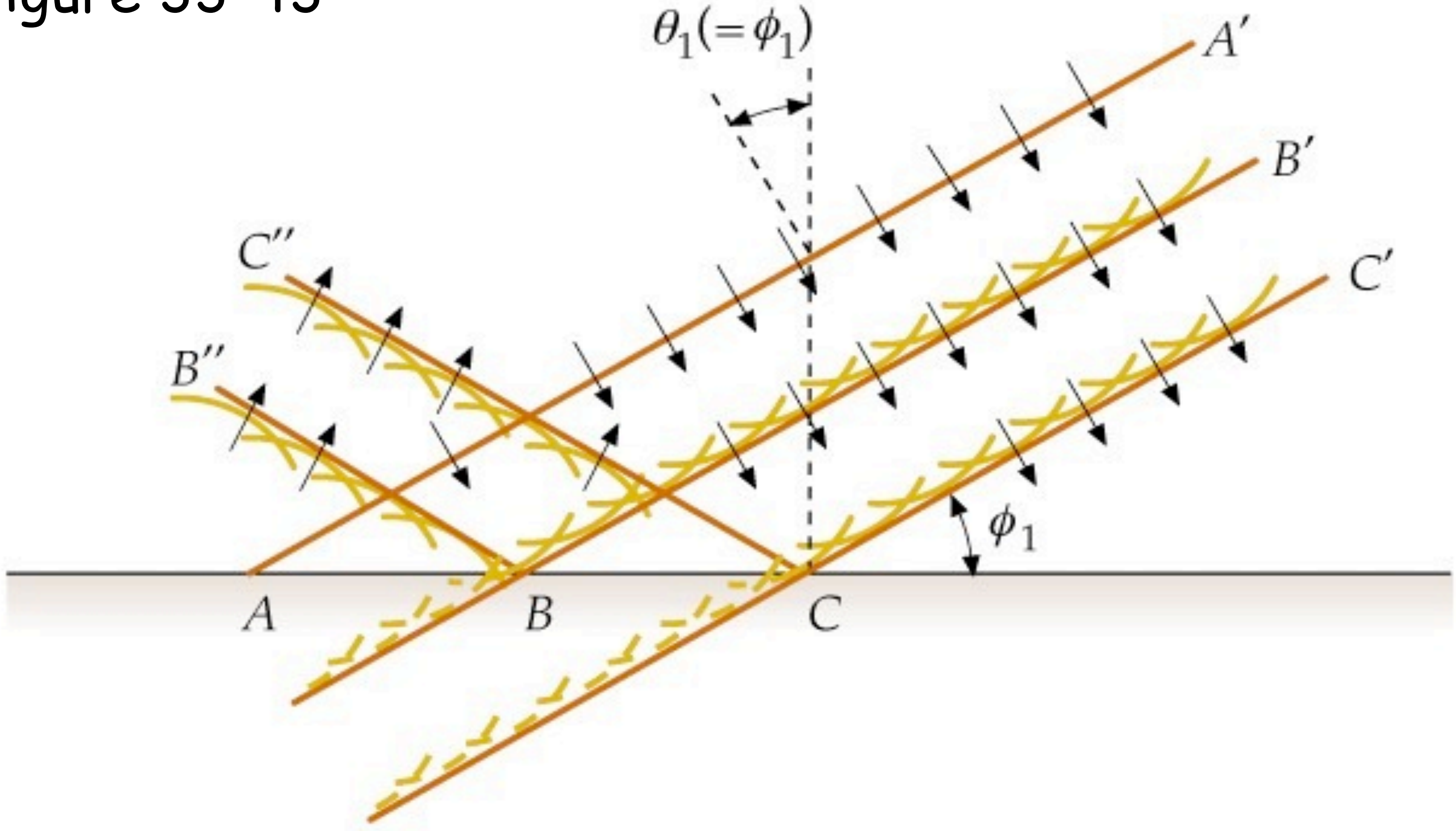
Derivation of law of reflection (Huygens)



AA′ is a wavefront of incident light striking a mirror at *A*

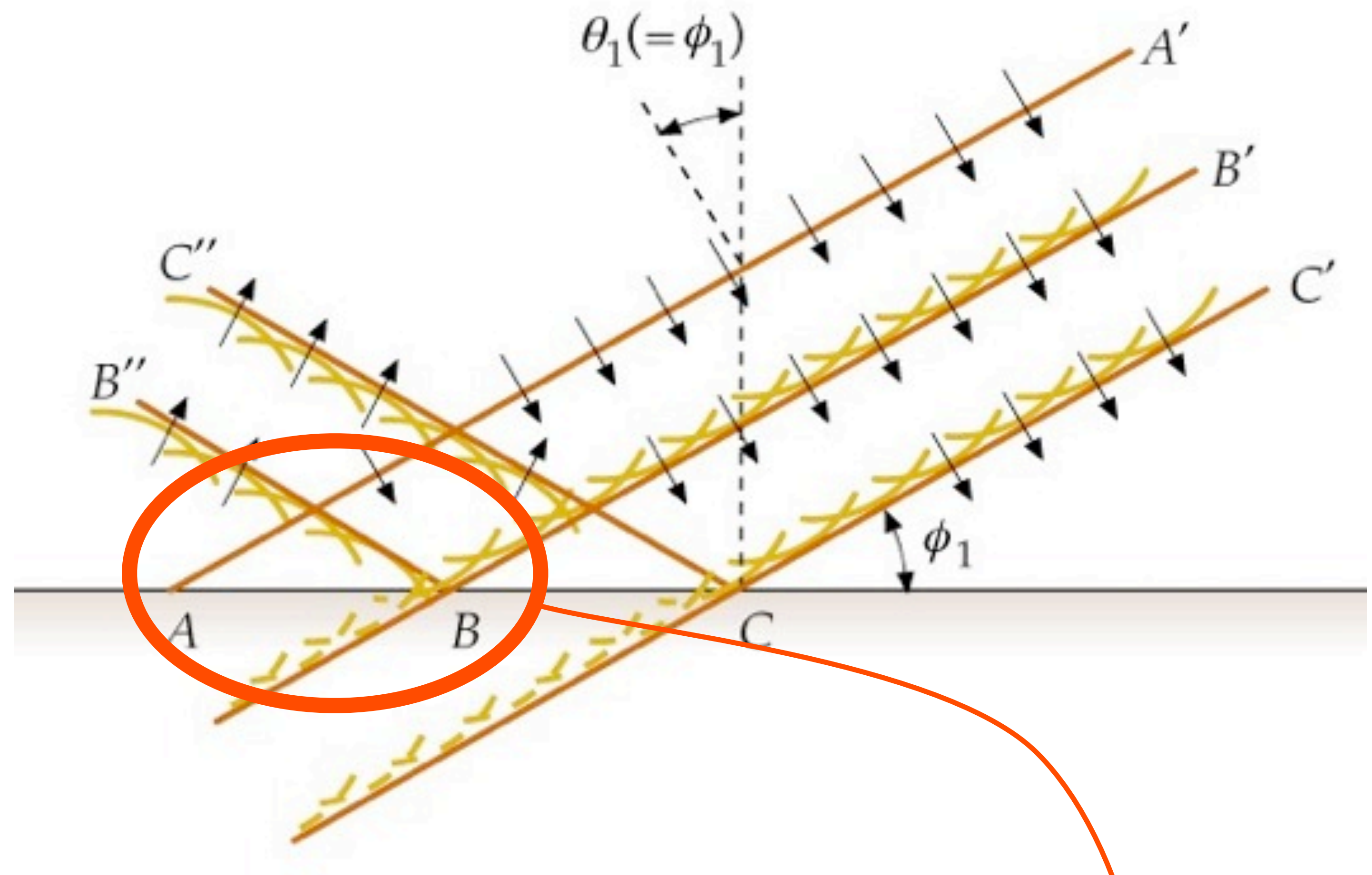
The angle between the wavefront and the mirror = the angle between the normal to the wavefront and the vertical direction (normal to the mirror)

Tipler figure 33-43



According to Huygens each point on the wavefront can be thought of as a point source of secondary wavelets

The position of the wavefront after a time t can be found by constructing wavelets of radius ct with centres on AA′



Waves that do not strike the mirror form the new wavefront BB′ (and similarly CC′)

Consider a small portion of these waves........

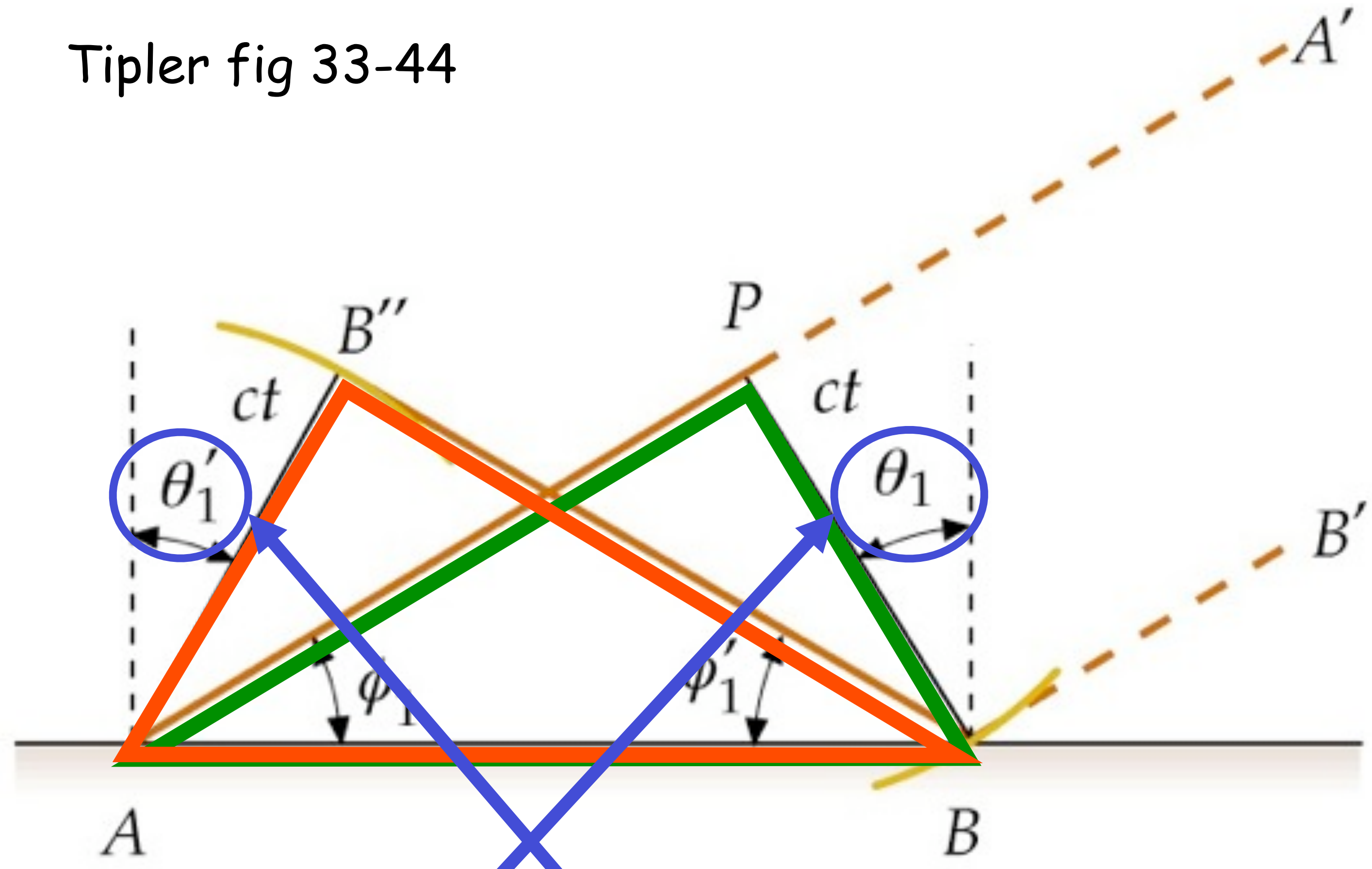
AP is part of AA′

in time t P moves to B

and the wavelet from A reaches B″

BB″ makes an angle ϕ_1' with the mirror, $\phi_1' = \theta_1'$

Tipler fig 33-44



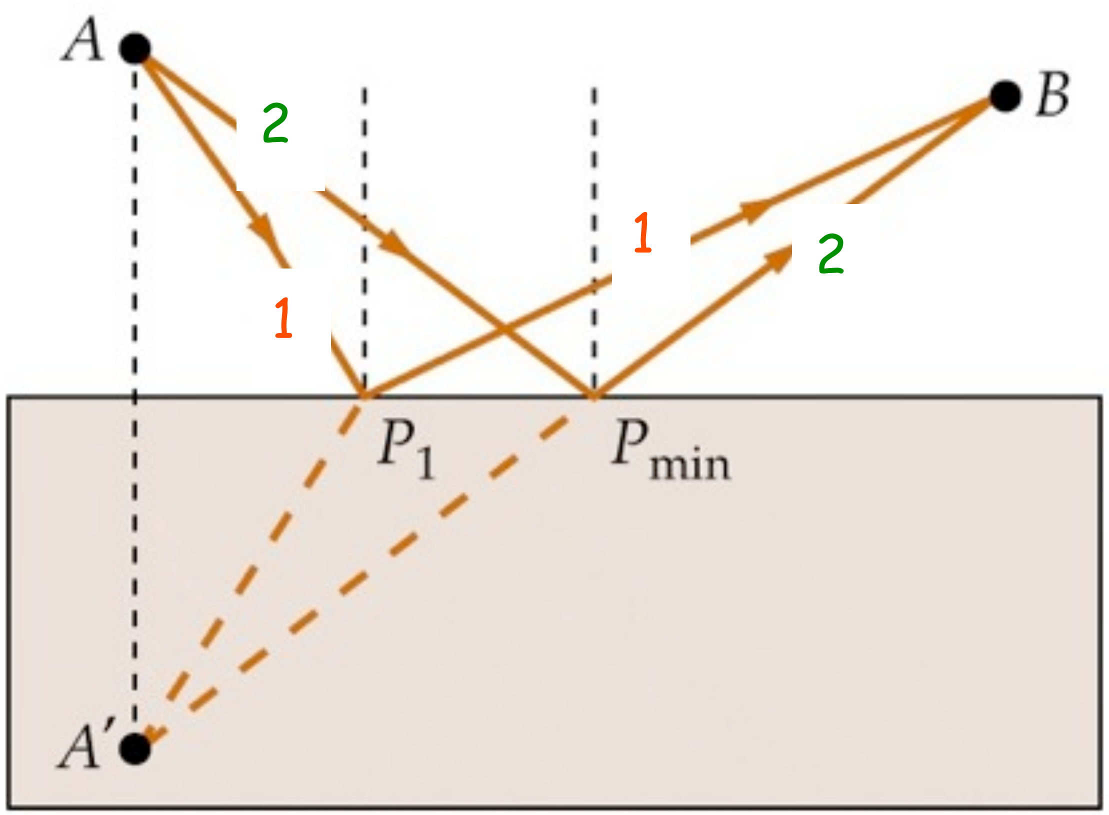
Now have 2 right angle triangles ABP and BAB″

AB is common AB″ = BP = ct triangles are congruent

$\phi_1 = \phi_1'$ ie angle of incidence θ_1 = angle of reflection θ_1'

Derivation of law of reflection (Fermat)

Light can travel from A to B (via mirror) on path 1 or path 2.



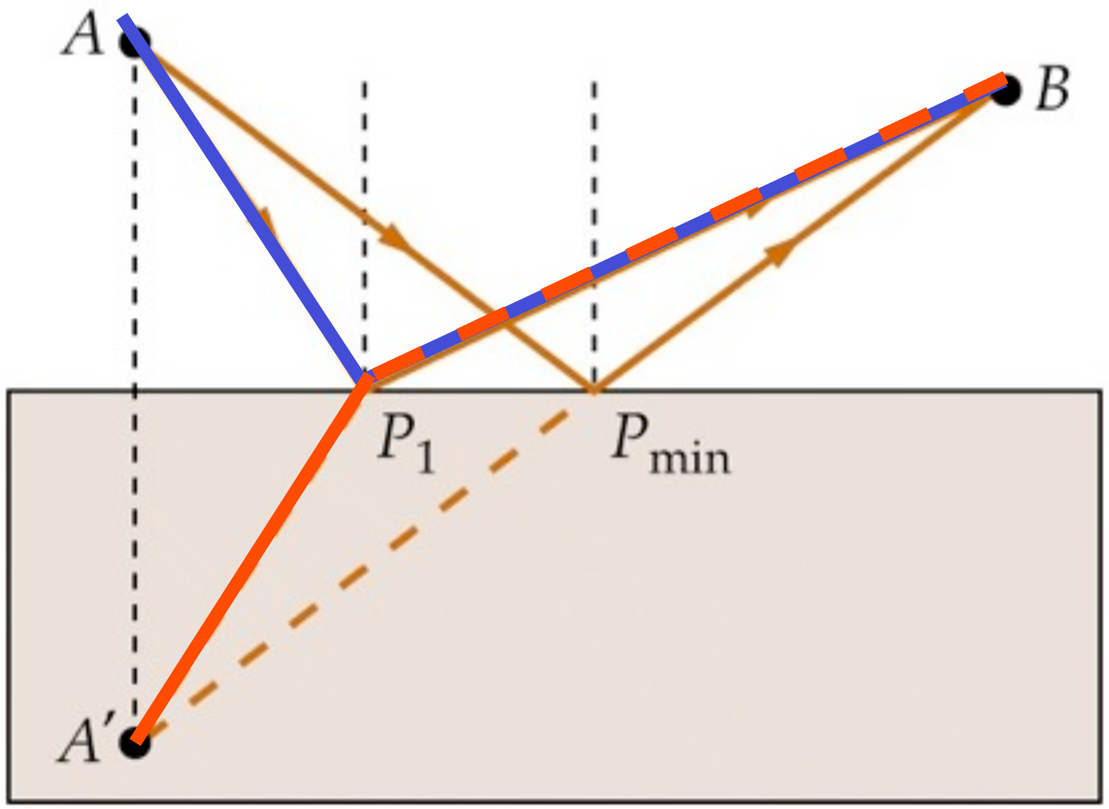
Tipler figure 33-46

If we want to apply Fermat's principle we need to know at which point P the wave must strike the mirror so that APB takes the least time.

As light is travelling in the same medium at all times the shortest time will also be the shortest distance.

Where ever P is located
the distance APB = A'PB
where A' is the position of
the image of the light
source.

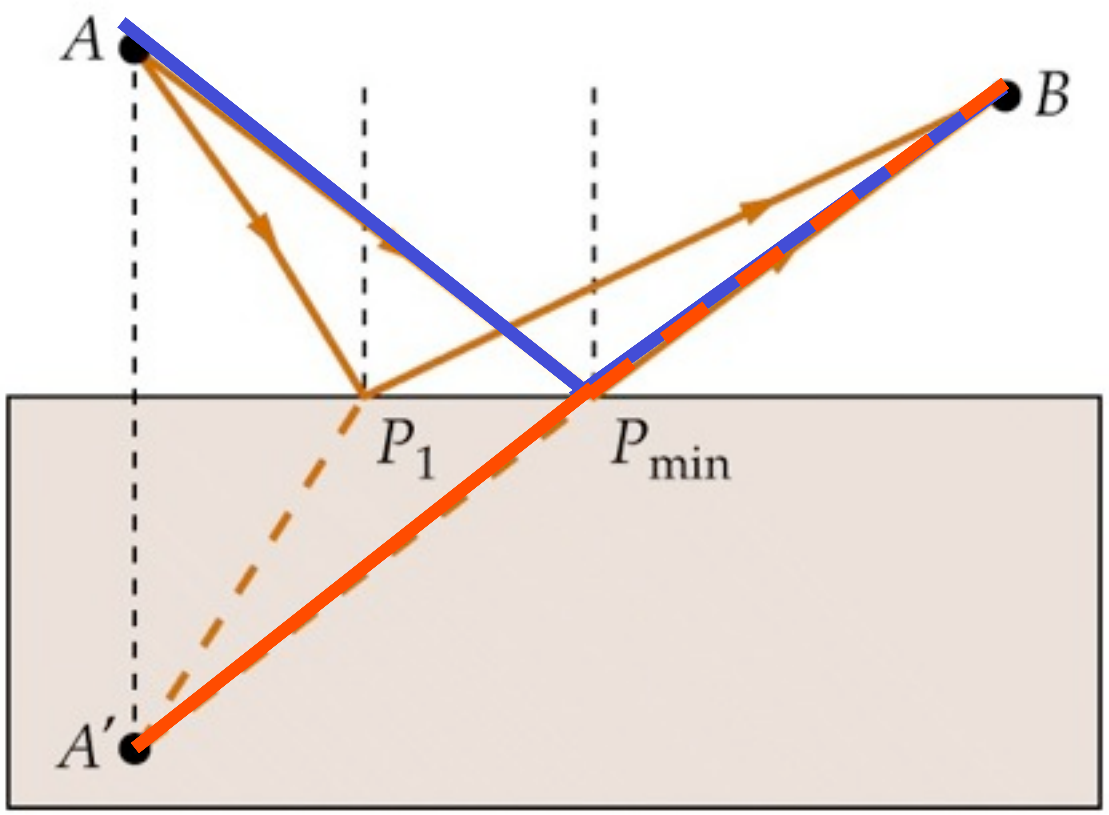
As the position of P varies the shortest distance will be when $A(P=P_{min})B$ lie in a straight line.



Tipler figure 33-46

Where ever P is located the distance APB = A'PB where A' is the position of the image of the light source.

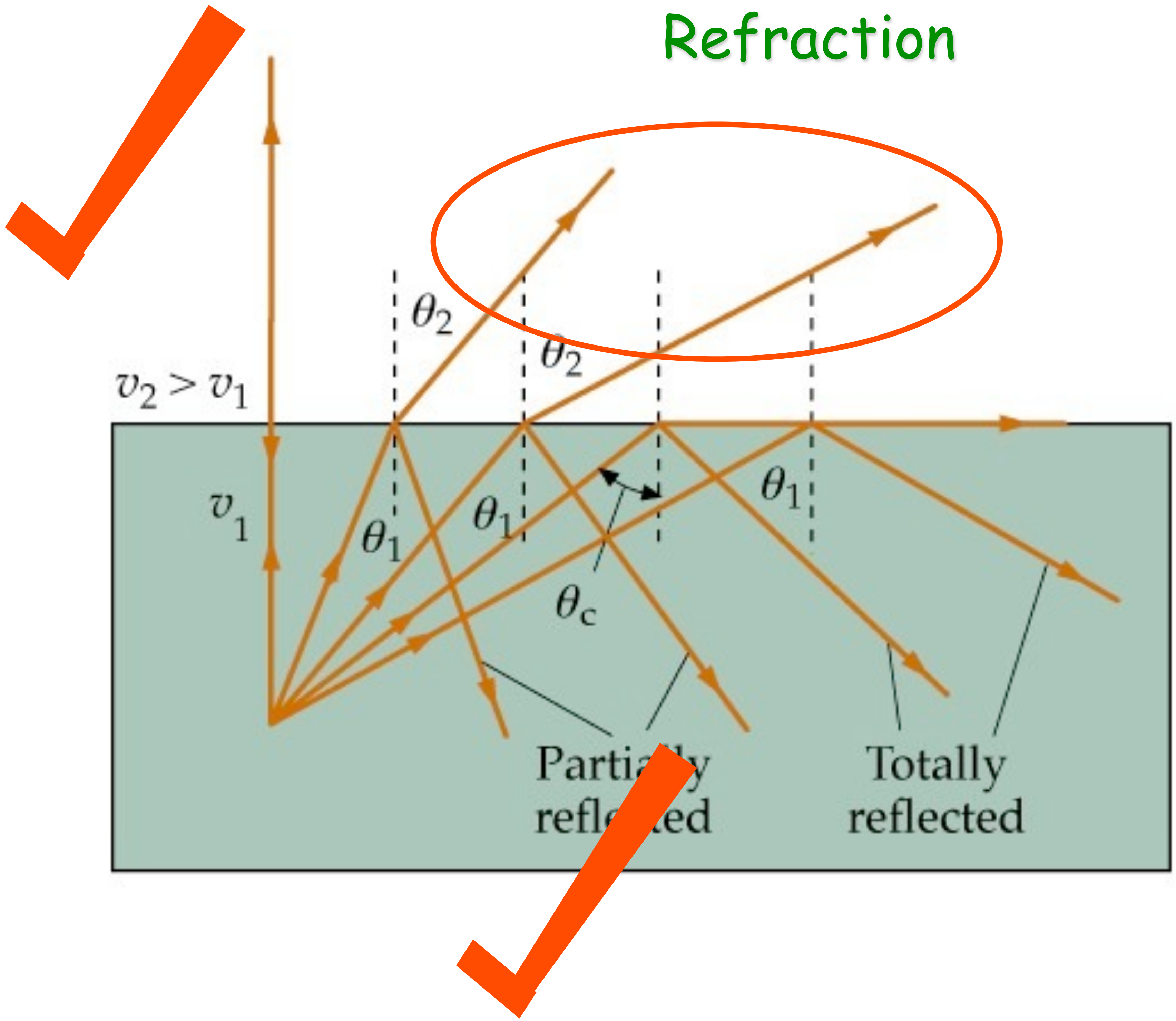
As the position of P varies the shortest distance will be when A'($P=P_{min}$)B lie in a straight line.



Tipler figure 33-46

This will be when the angle of incidence = angle of reflection

Refraction



θ_2
θ_2
$v_2 > v_1$
v_1
θ_1
θ_1
θ_1
θ_c
Partially reflected
Totally reflected

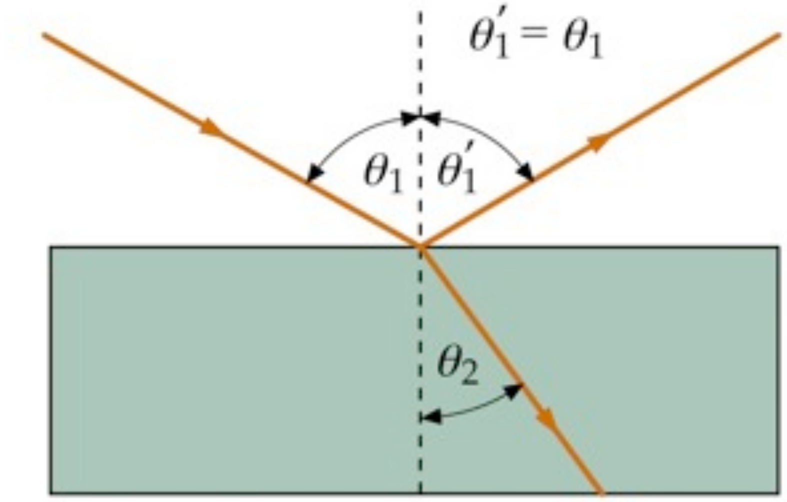
Refraction of light

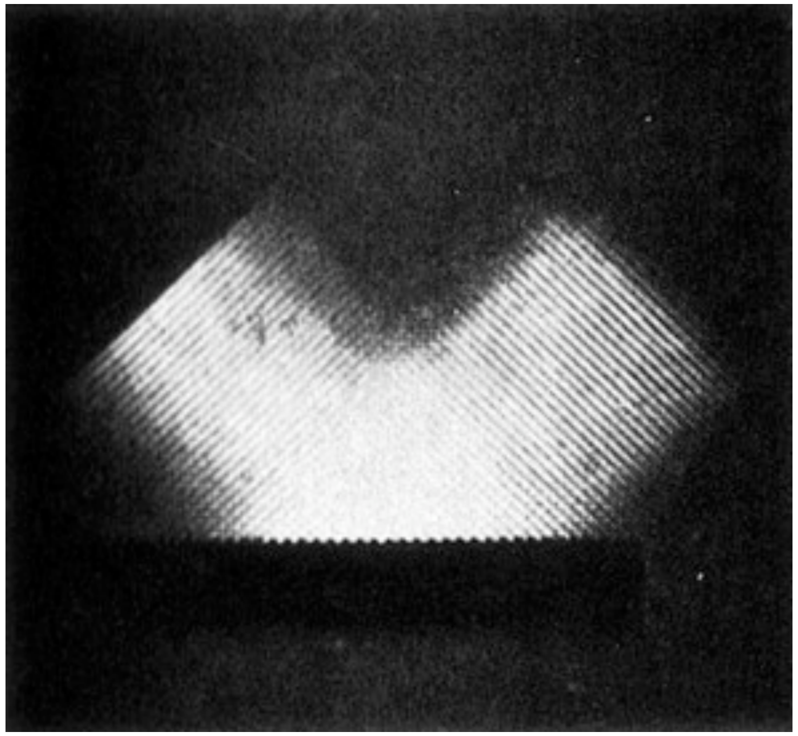
Snell's law of refraction :

if v is the speed of the light in the medium

$$\frac{1}{v_1}\sin\theta_1 = \frac{1}{v_2}\sin\theta_2$$

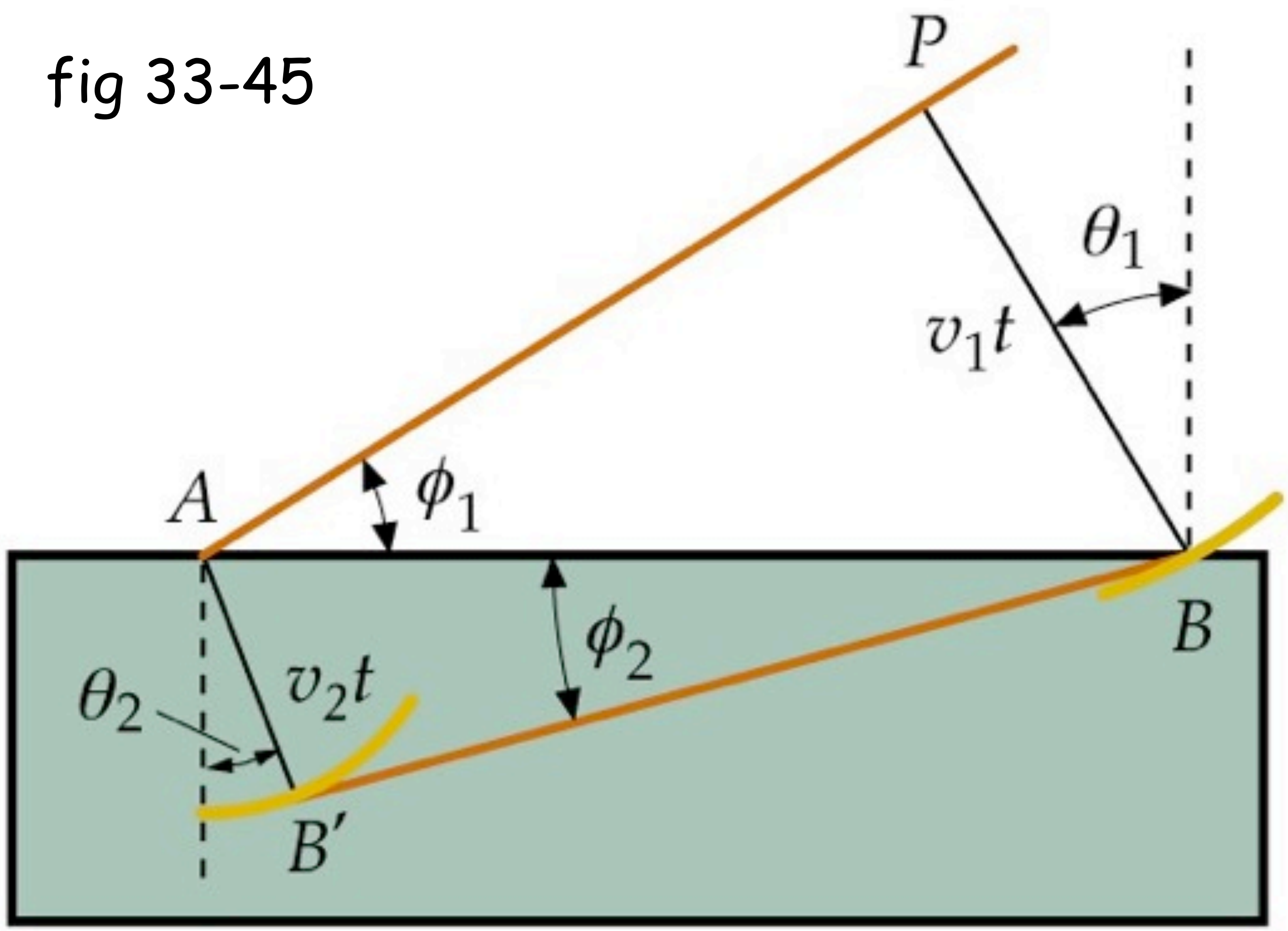
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$





Derivation of law of refraction (Huygens)

fig 33-45



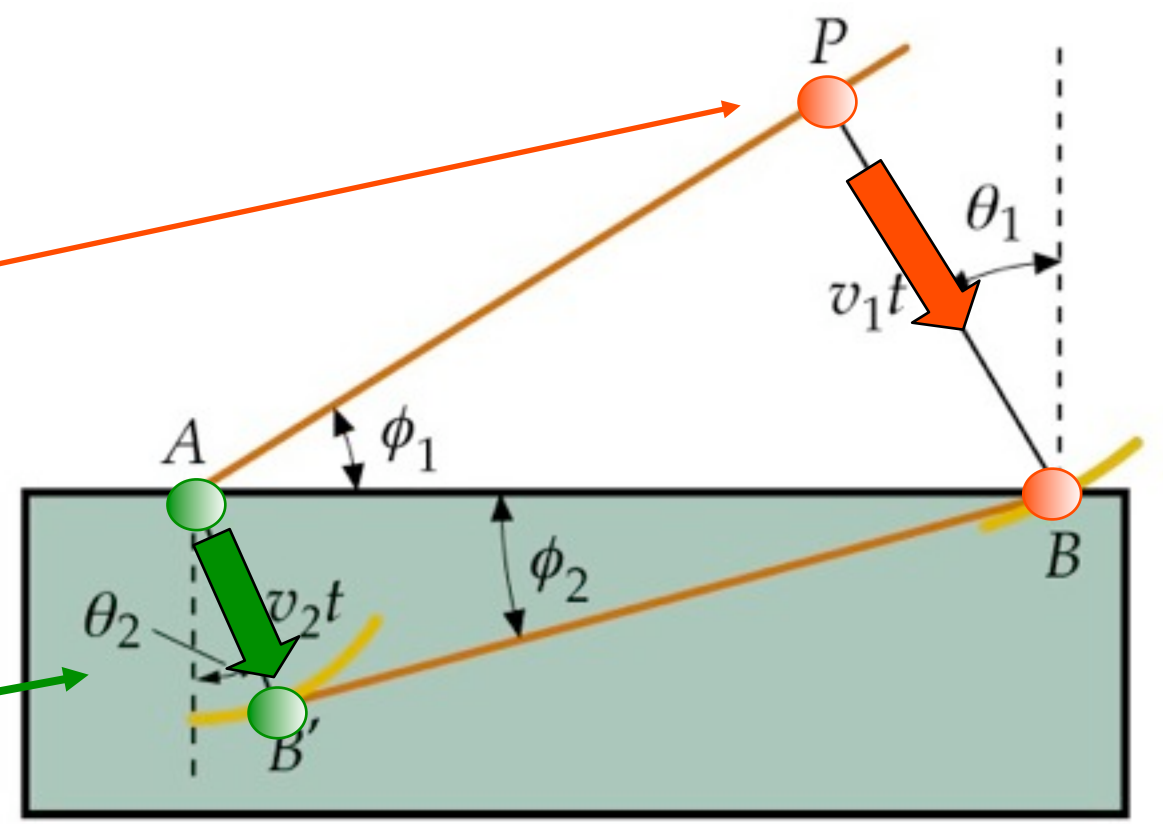
Consider a plane wave incident on a glass interface.

AP represents a portion of the incident wave - we can use Huygens' construction to calculate the transmitted wave.

AP hits the glass surface at an angle ϕ_1.

In a time t a wavelet from P travels v_1t to point B

In the same time a wavelet from A travels v_2t to B'



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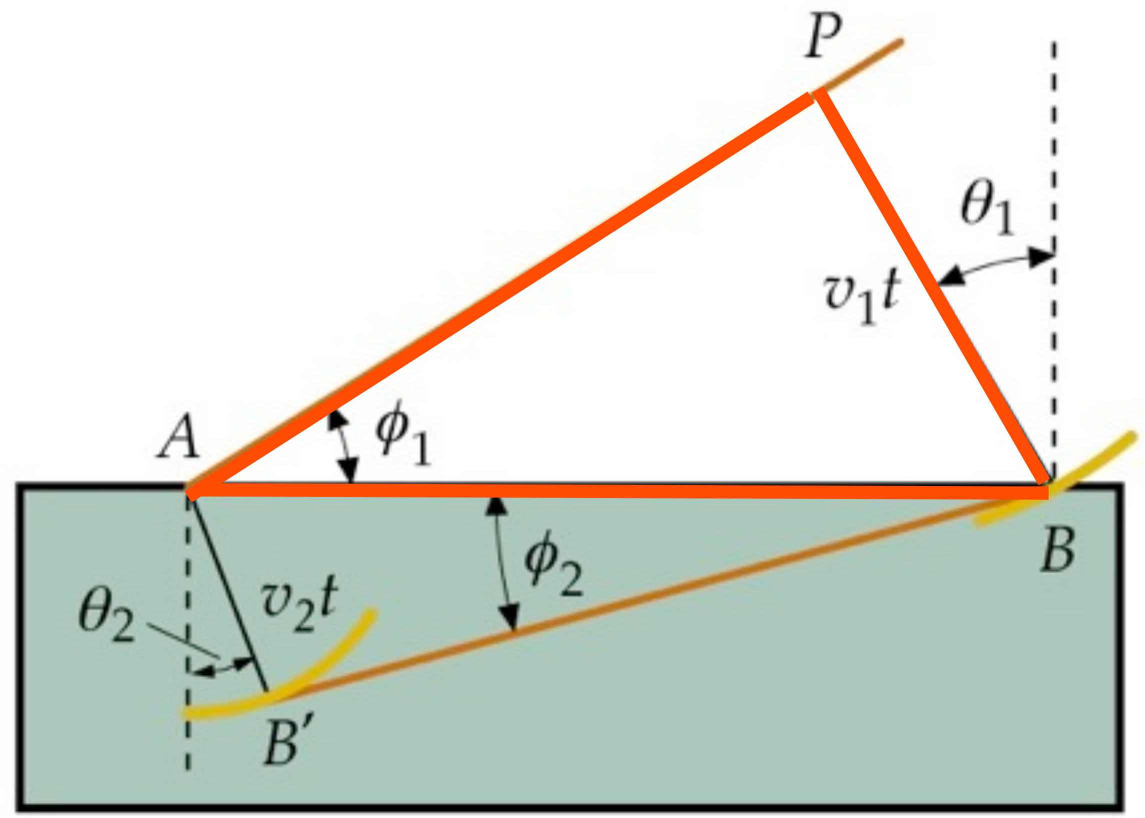


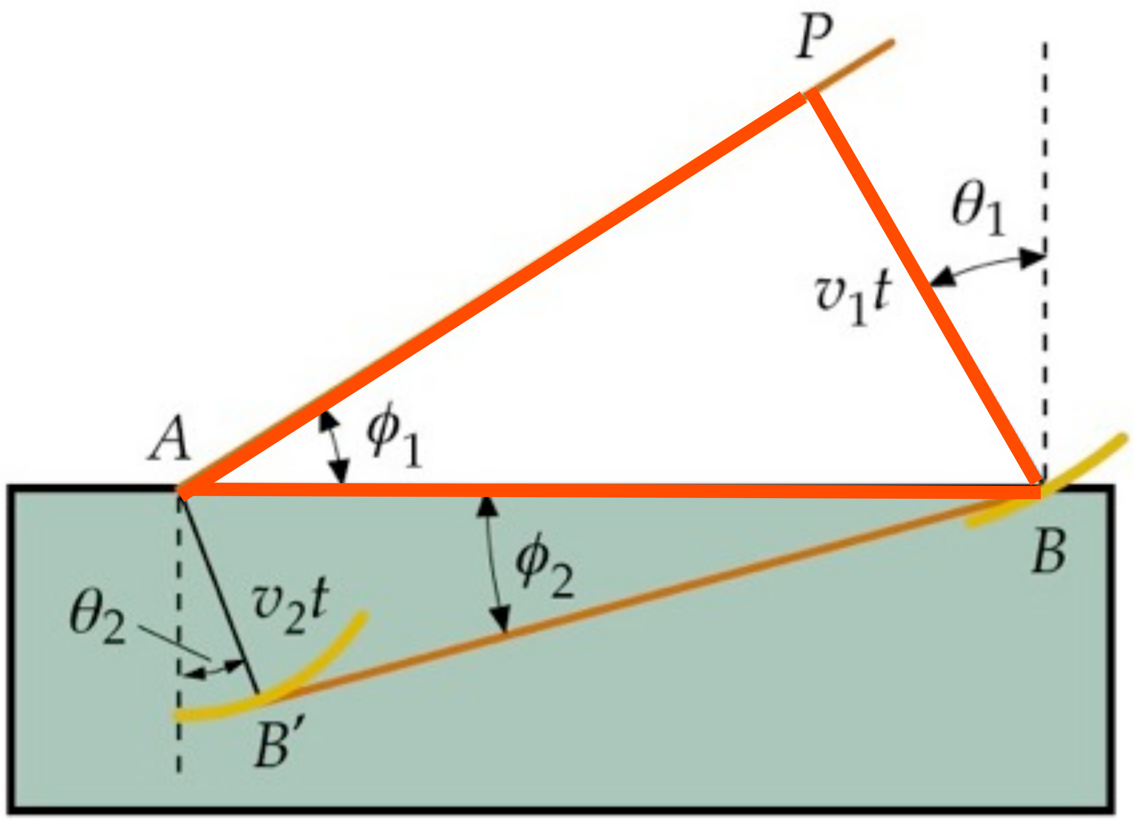
BB′ is not parallel to AP because $v_1 \neq v_2$

In ΔAPB $\quad \sin\phi_1 = \dfrac{v_1 t}{AB} \quad$ or $\quad AB = \dfrac{v_1 t}{\sin\phi_1}$

$$AB = \frac{v_1 t}{\sin \phi_1}$$

but $\phi_1 = \theta_1$

$$\therefore \quad AB = \frac{v_1 t}{\sin \theta_1}$$



$$AB = \frac{v_1 t}{\sin\phi_1}$$

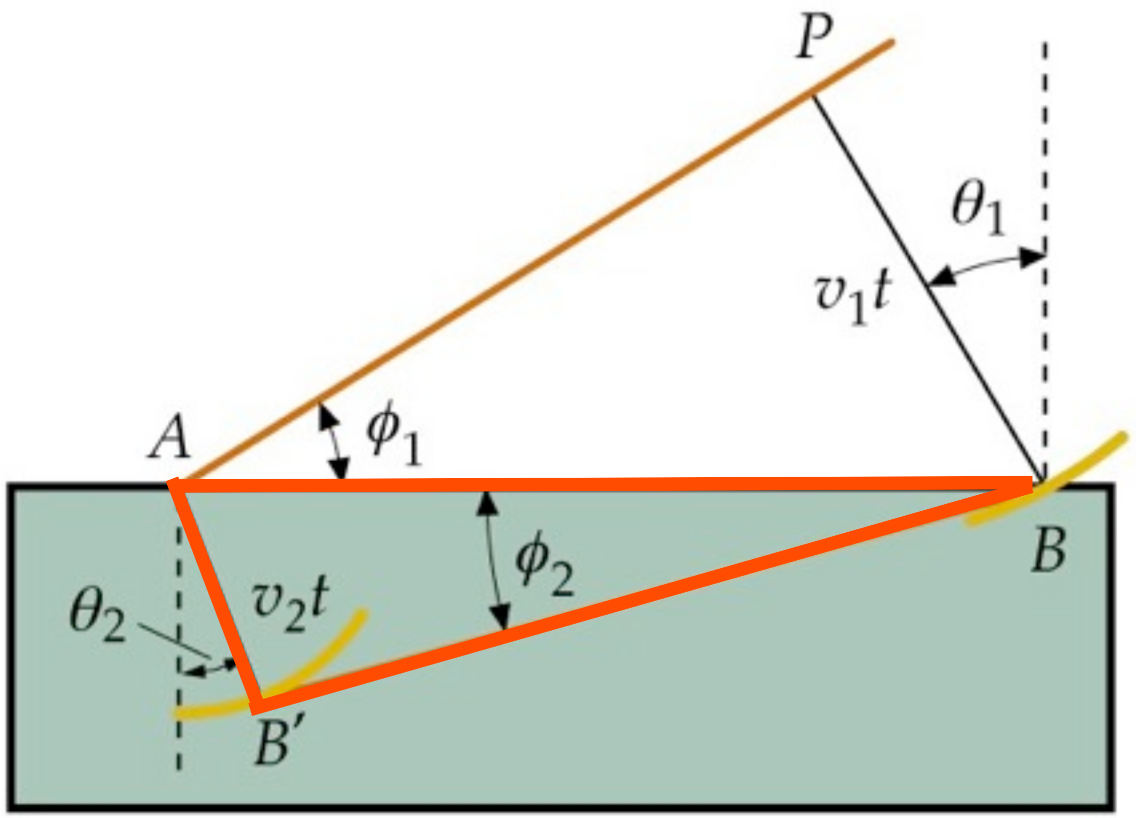
but $\phi_1 = \theta_1$

$$\therefore \quad AB = \frac{v_1 t}{\sin\theta_1}$$

Similarly in $\Delta ABB'$

$$\sin\phi_2 = \frac{v_2 t}{AB}$$

or $$AB = \frac{v_2 t}{\sin\theta_2}$$



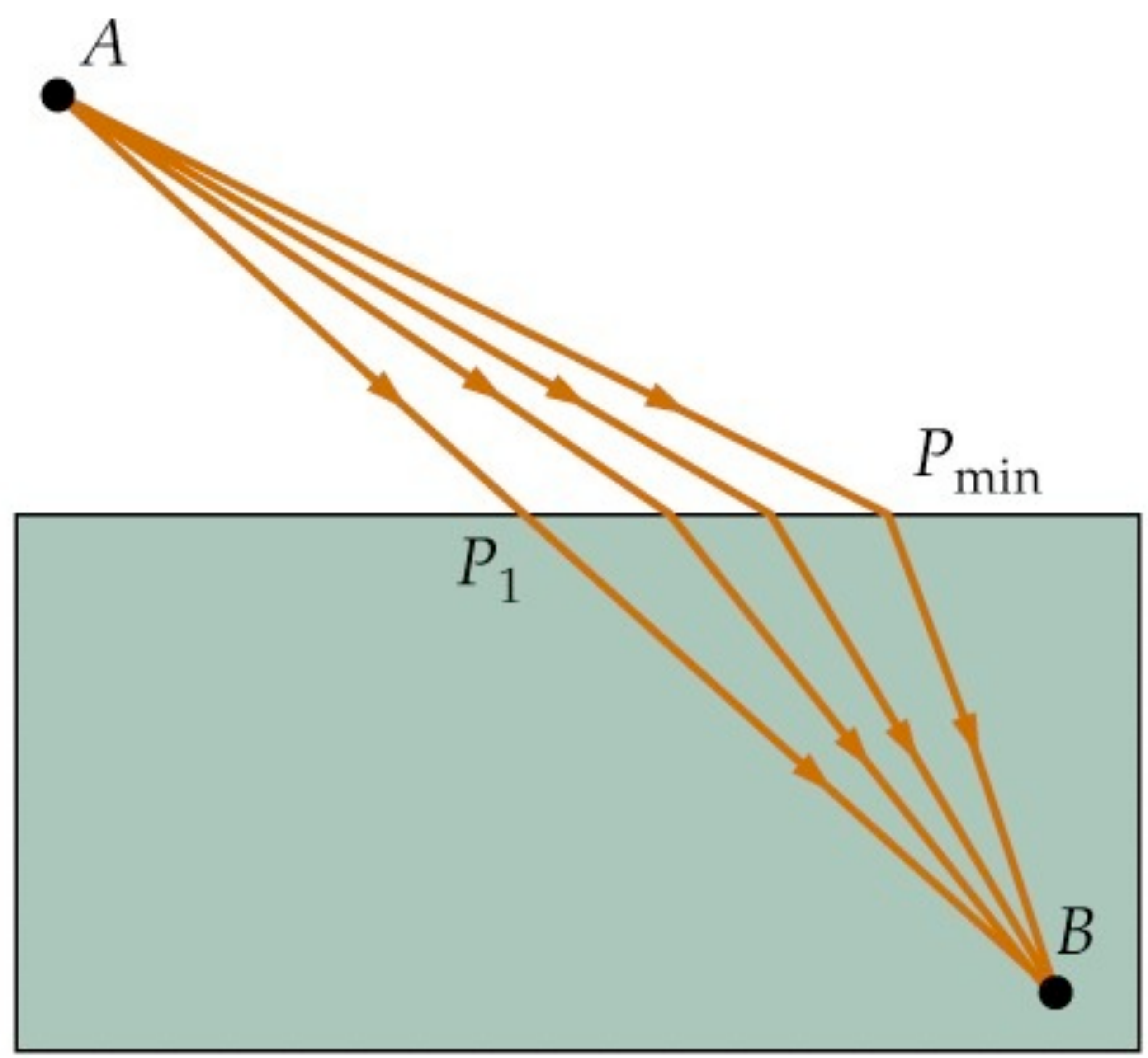
$$\frac{v_1 t}{\sin\theta_1} = \frac{v_2 t}{\sin\theta_2}$$

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$

But $v_1 = c / n_1$ and $v_2 = c / n_2$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

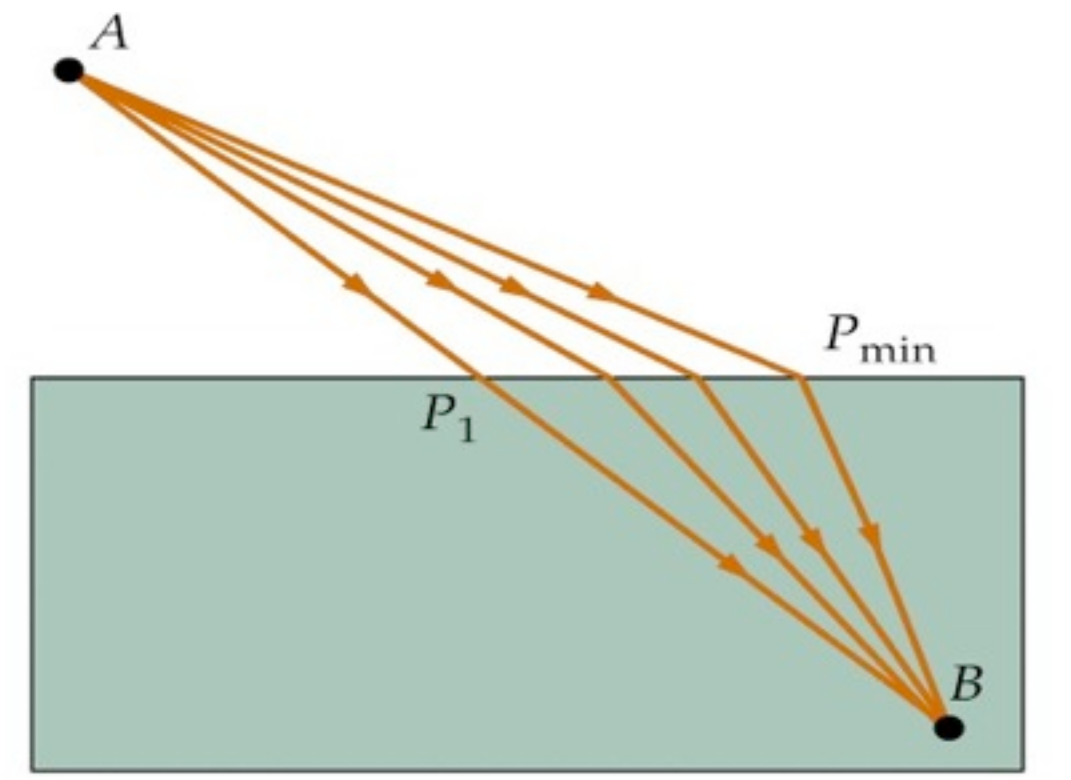
Derivation of law of refraction (Fermat)



Can also use Fermat's principle to derive Snell's Law (more complicated but important)

Derivation of law of refraction (Fermat)

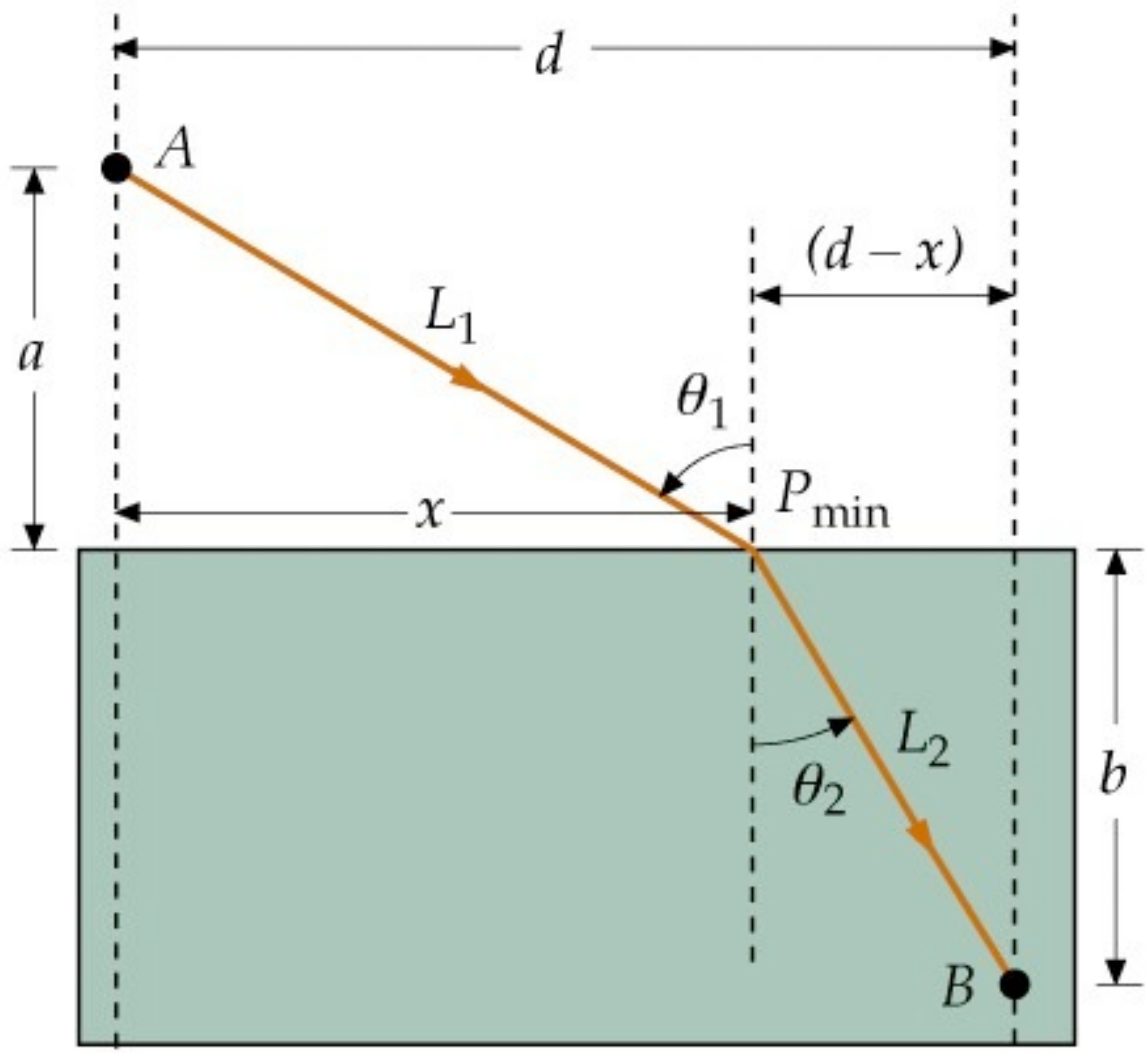
Several possible paths from A (in air) to B (in glass)



Remember that light travels more slowly in glass than in air so A-P_1-B (straight line) will not have the shortest travel time.

If we move to the right the path in the glass is shorter, but the overall path is longer - how do we choose the shortest route ?

Geometry for finding the path of least time



Tipler fig.33-48

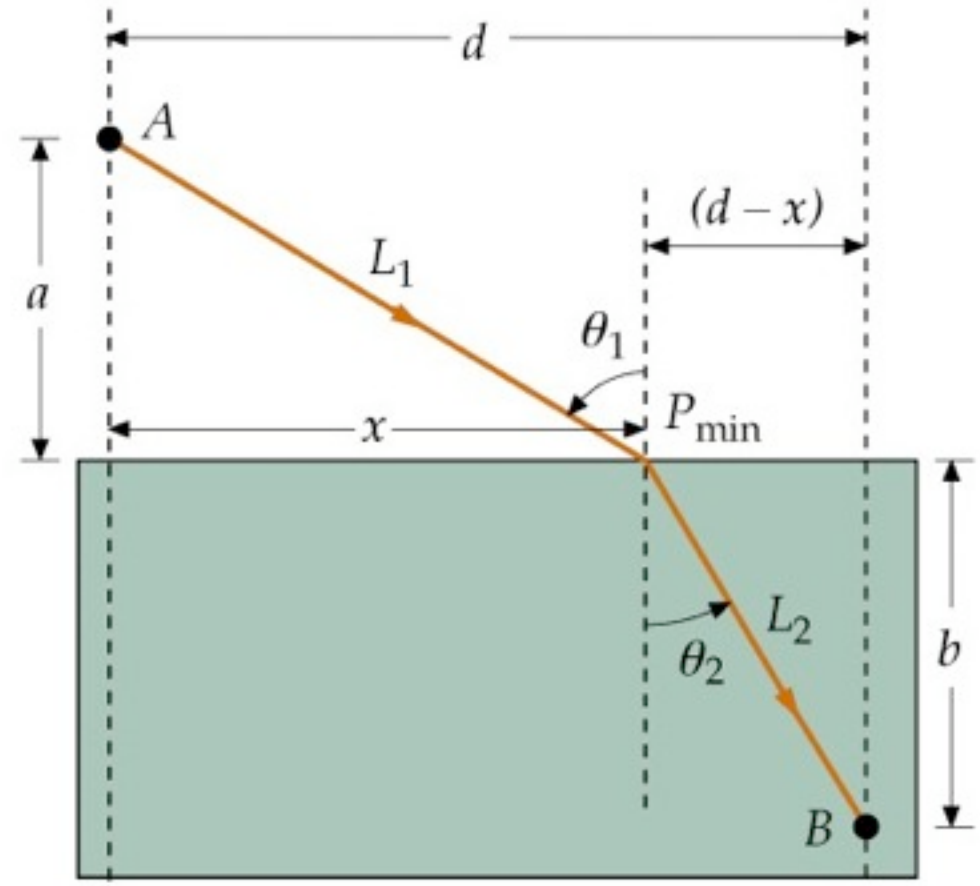
Geometry for finding the path of least time

t = time taken to travel A-B

$$t = \frac{L_1}{v_1} + \frac{L_2}{v_2}$$

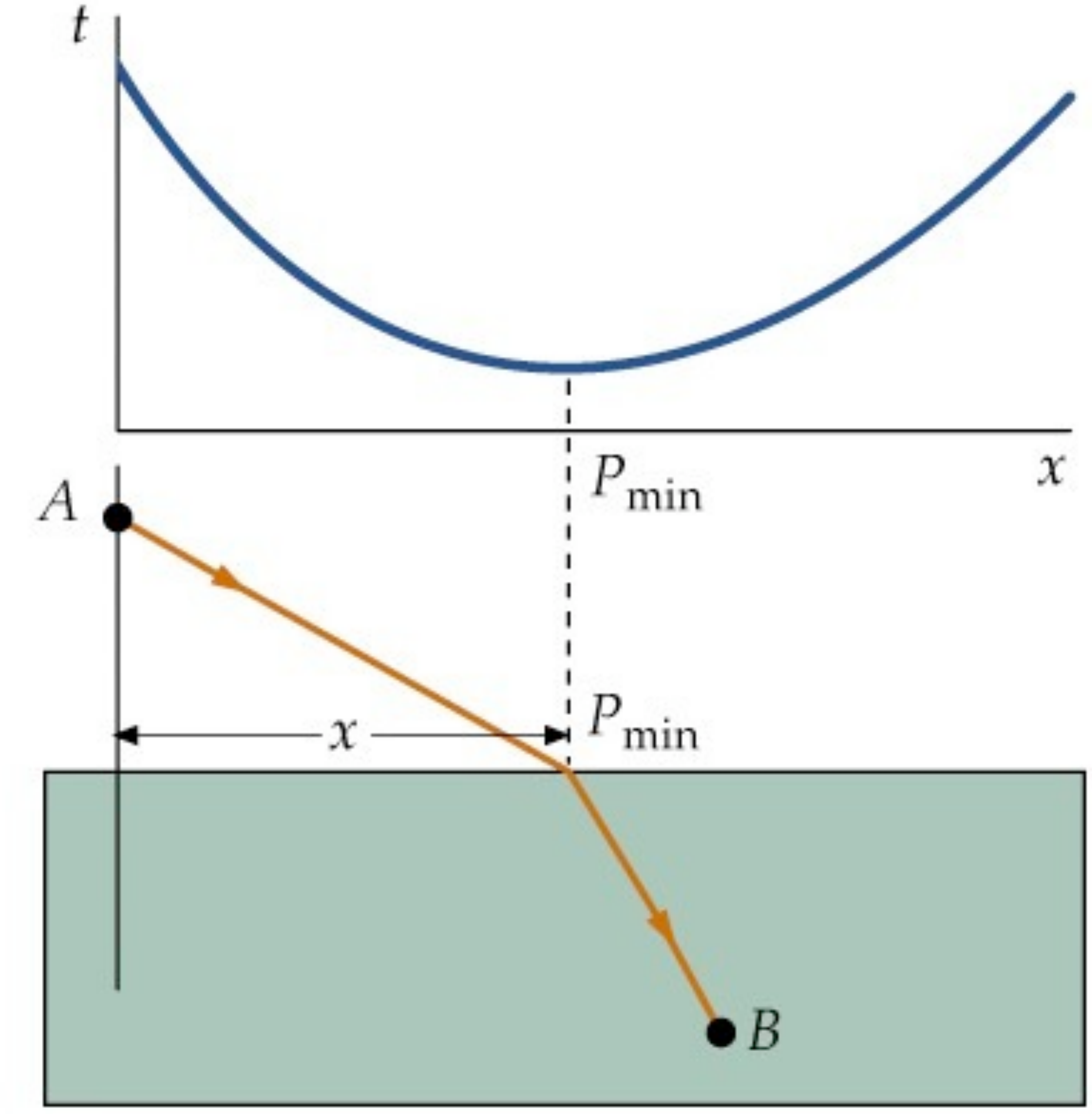
$$= \frac{L_1}{c/n_1} + \frac{L_2}{c/n_2}$$

$$= \frac{n_1 L_1}{c} + \frac{n_2 L_2}{c}$$



$$L_1^2 = a^2 + x^2 \quad \text{and} \quad L_2^2 = b^2 + (d-x)^2$$

We can combine these three equations and plot the time as a function of x

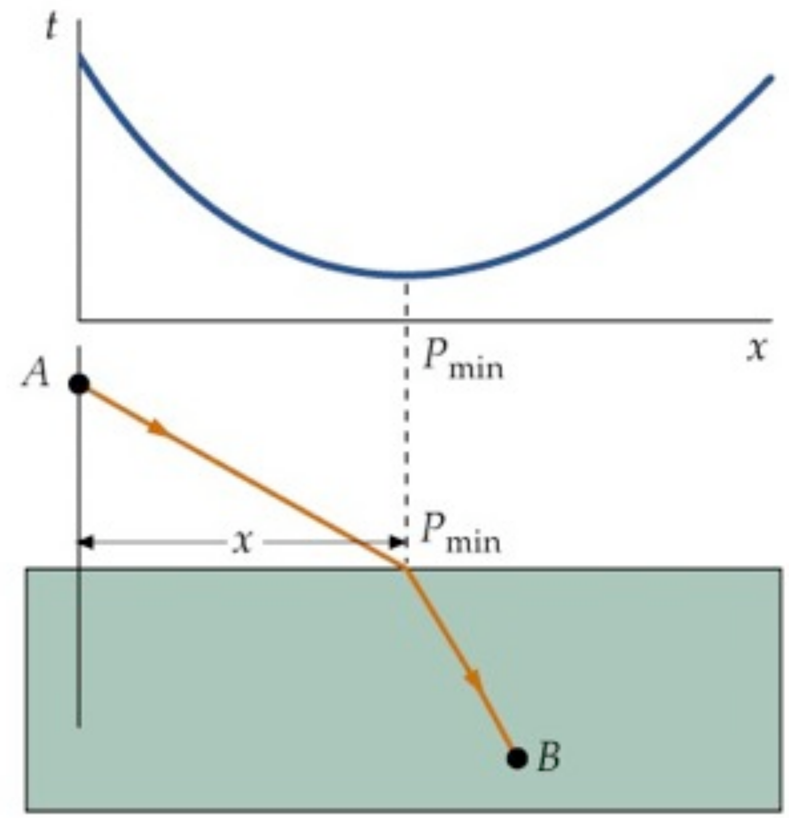


Tipler fig.33-49

Find the minimum time taken by finding when

$$\frac{dt}{dx} = 0$$

$$\frac{dt}{dx} = \frac{1}{c}\left(\frac{n_1\, dL_1}{dx} + \frac{n_2\, dL_2}{dx}\right) = 0$$



$$\frac{dL_1}{dx} = \frac{1}{2}(a^2 + x^2)^{-1/2} \times 2x$$

$$= \frac{x}{L_1}$$

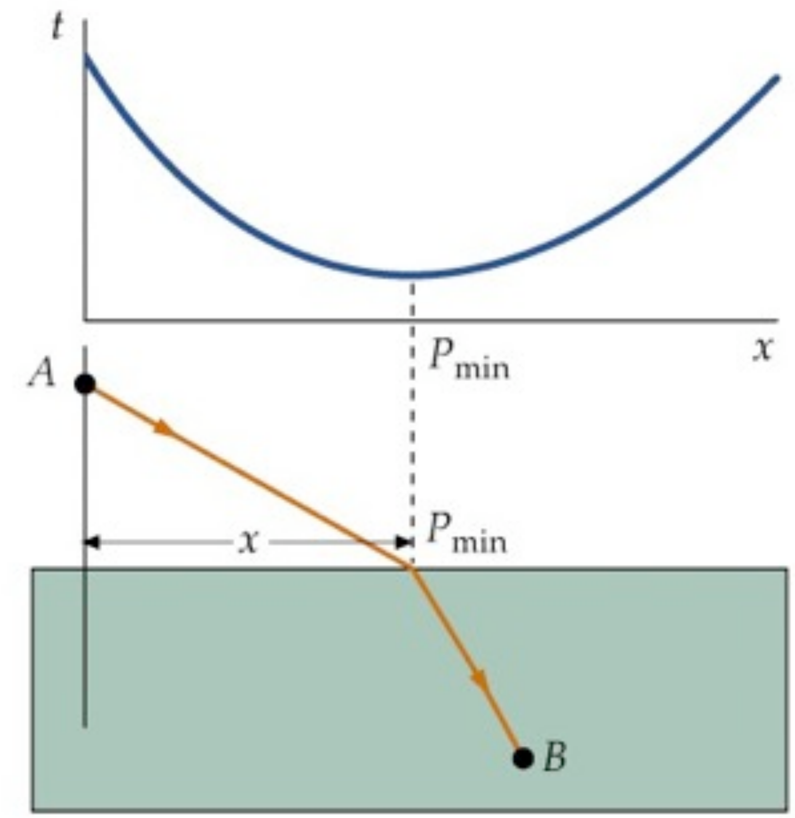
but $\frac{x}{L_1} = \sin\theta_1$

Similarly for L_2

$$\frac{dL_2}{dx} = \frac{1}{2}(b^2 + (d-x)^2)^{-1/2} \times -2(d-x)$$

$$= -\frac{(d-x)}{L_2}$$

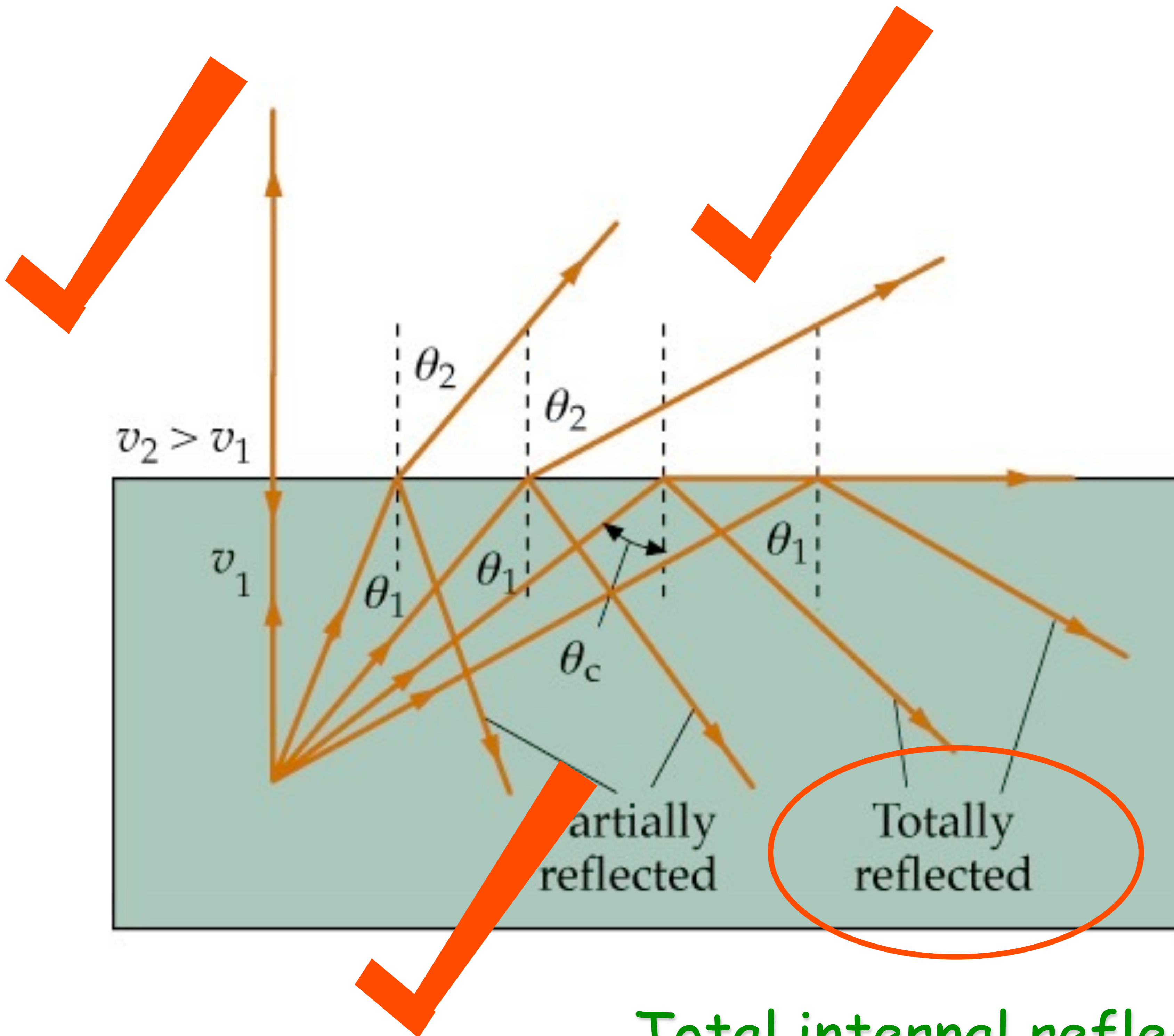
but $-\dfrac{d-x}{L_2} = -\sin\theta_2$



we want $\dfrac{dt}{dx} = 0$

$$n_1 \sin\theta_1 + n_2(-\sin\theta_2) = 0$$

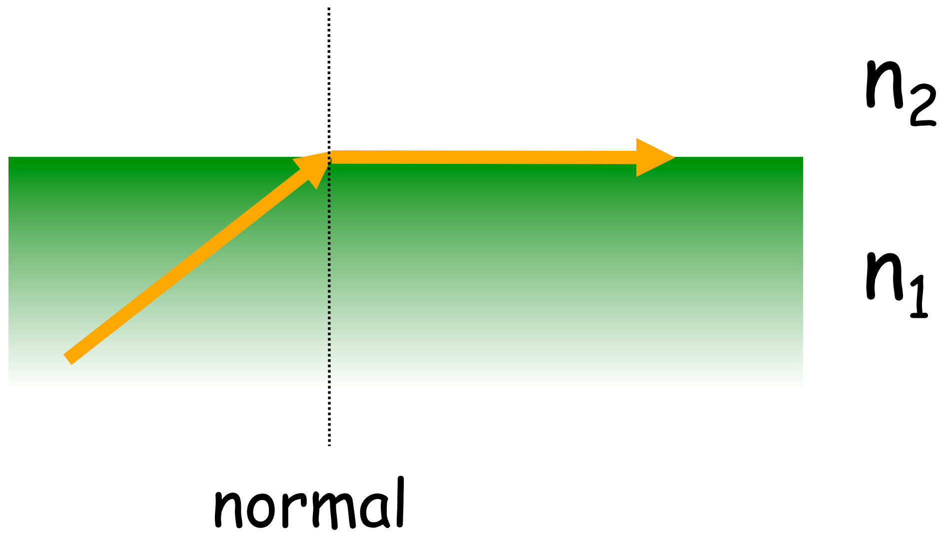
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



Total internal reflection

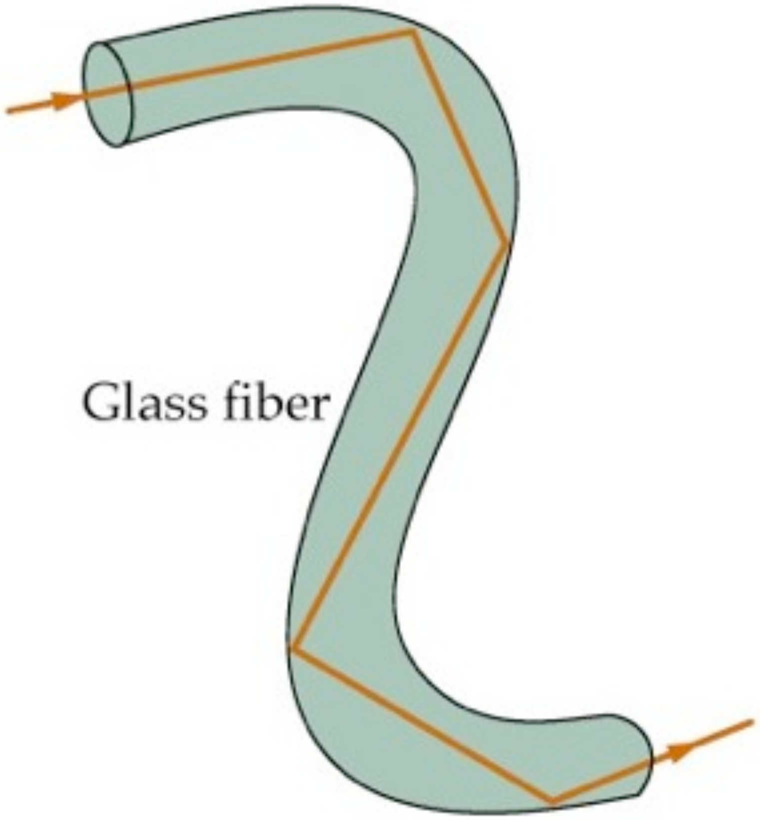
Total internal reflection

Total internal reflection occurs when light attempts to move from a medium of refractive index n_1 to a medium of refractive index n_2 and $n_1 > n_2$

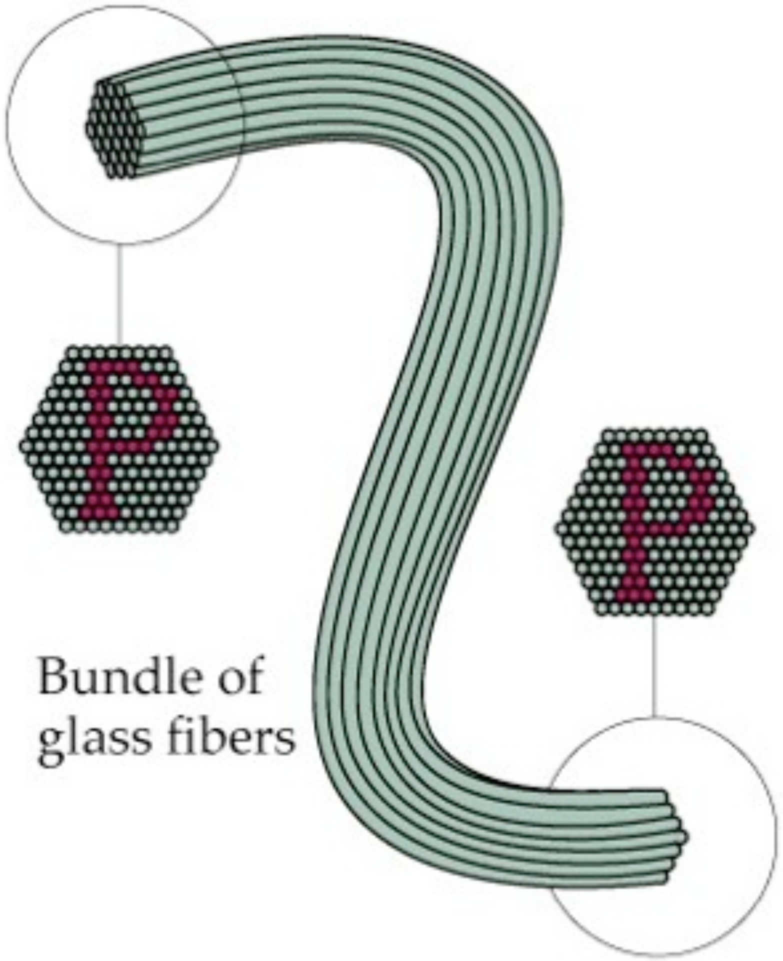


At a particular angle of incidence (the critical angle θ_c) the refracted ray will travel along the boundary ie $\theta_2 = 90°$

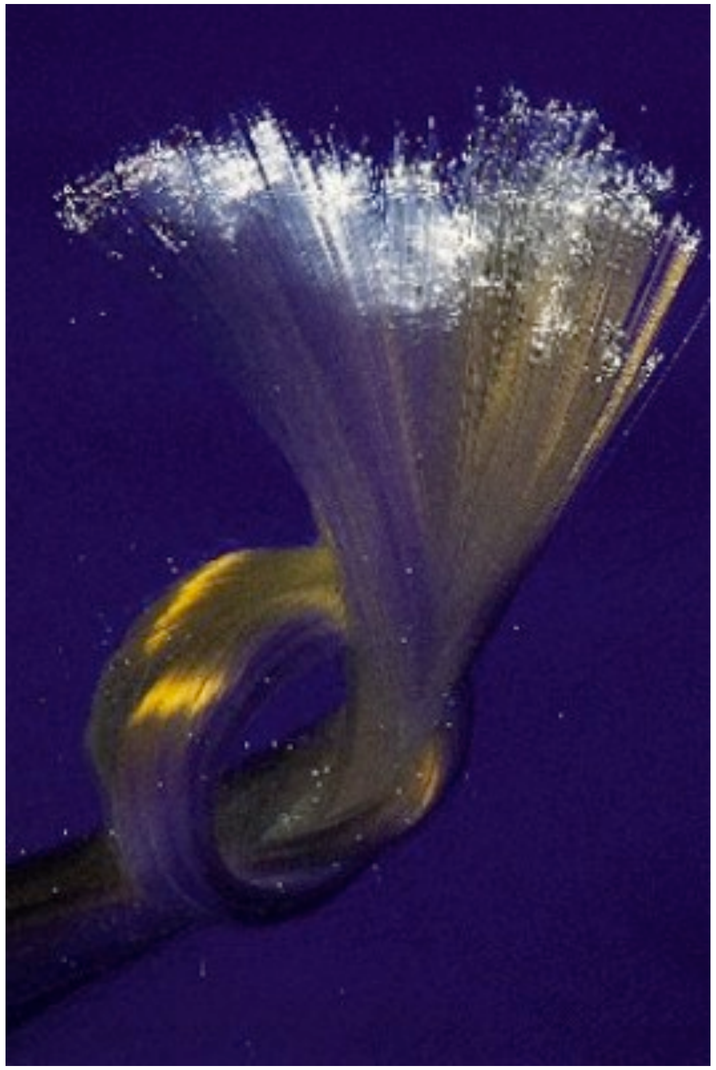
If the angle of incidence > θ_c the light is entirely reflected



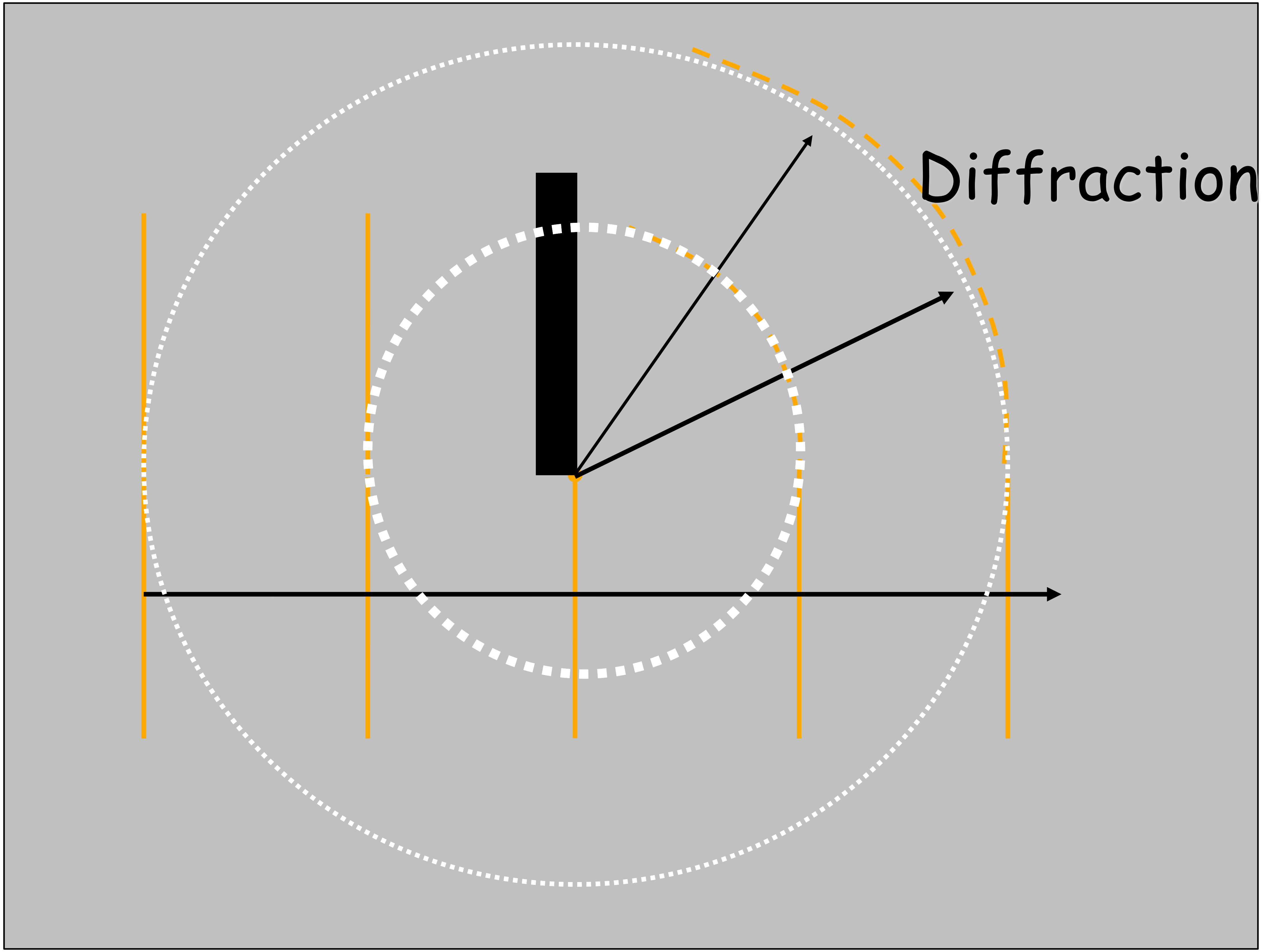
Glass fiber



Bundle of
glass fibers



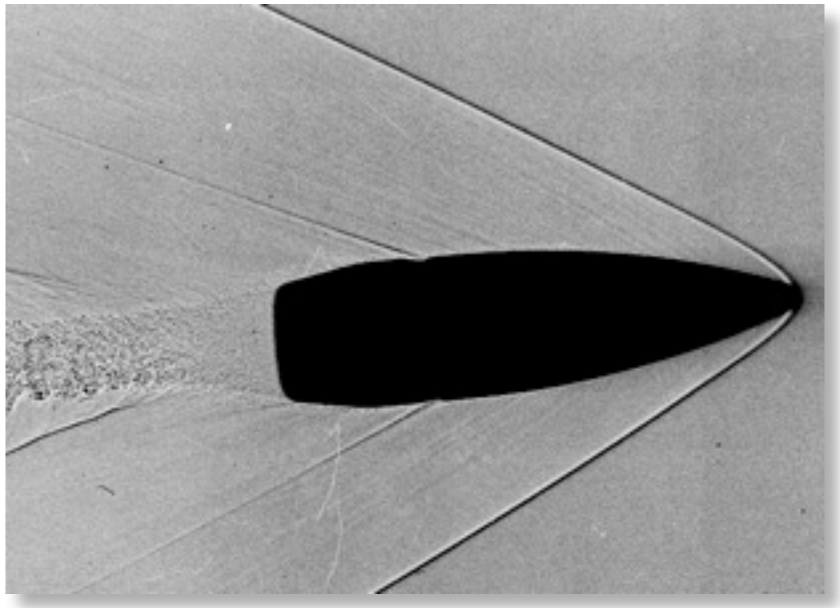
Visualization



Doppler effect

Shock waves and

Sonic booms



The Doppler effect

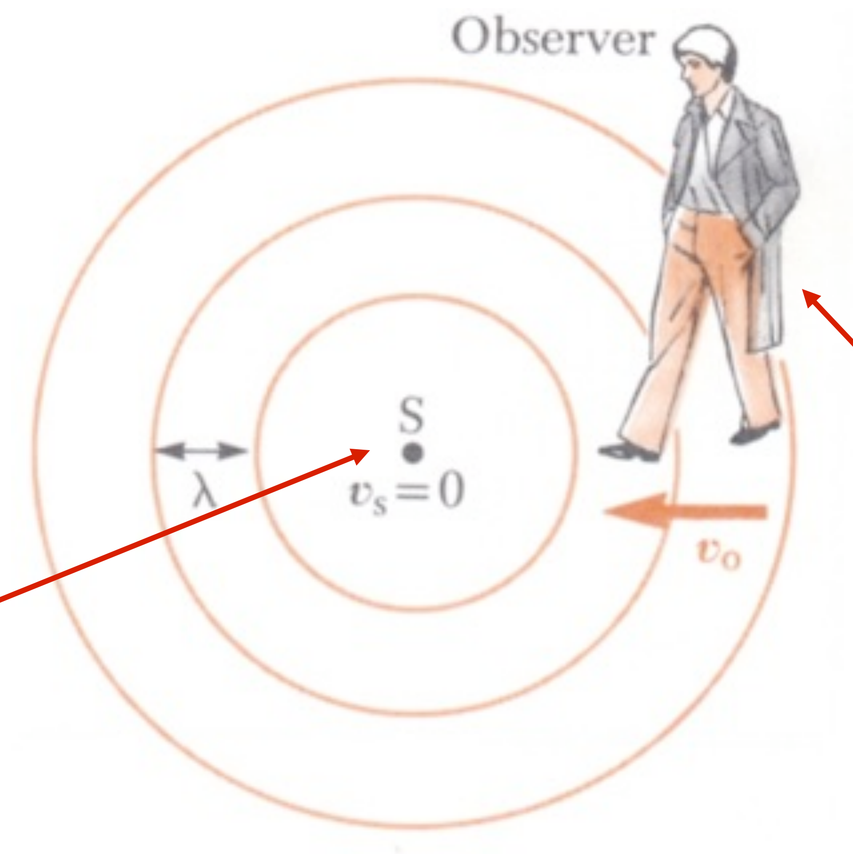
The Doppler effect is experienced whenever there is a relative motion between source and observer.

When the source and observer are moving towards each other the frequency heard by the observer is higher than the frequency of the source.

When the source and observer move away from each other the observer hears a frequency which is lower than the source frequency.

Although the Doppler effect is most commonly experienced with sound waves it is a phenomenon common to all harmonic waves.

Stationary source & observer moving to source



Point source S at rest ie $v_s = 0$

Observer moving **TOWARDS** S at speed v_o

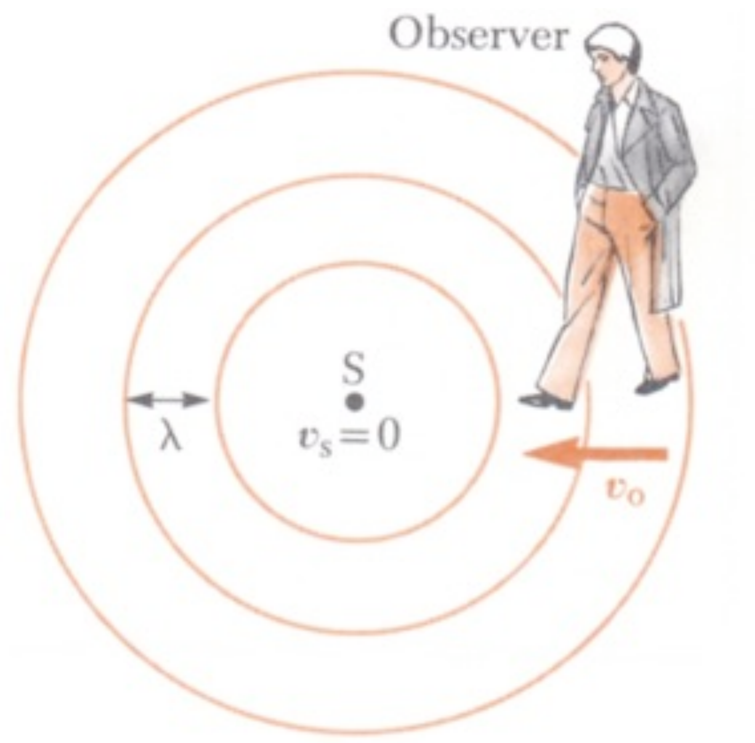
We are dealing with relative speeds ∴ "at rest" = at rest with respect to the air. We assume the air is stationary.

frequency of source = f

velocity of sound = v

frequency observed = f'

wavelength of sound = λ



If observer O was stationary he would detect f wavefronts per second

ie: if $v_o = 0$ and $v_s = 0$ $f' = f$

When O ⟶ S O moves a distance $v_o t$ in t seconds

During this time O detects an additional $\frac{v_o t}{\lambda}$ wavefronts

ie: an additional $\frac{v_o}{\lambda}$ wavefronts / second

As more wavefronts are heard per second the frequency heard by the observer is increased.

$$f' = f + \Delta f = f + \frac{v_o}{\lambda}$$

$$\text{but } v = f\lambda \quad \text{or} \quad \frac{1}{\lambda} = \frac{f}{v}$$

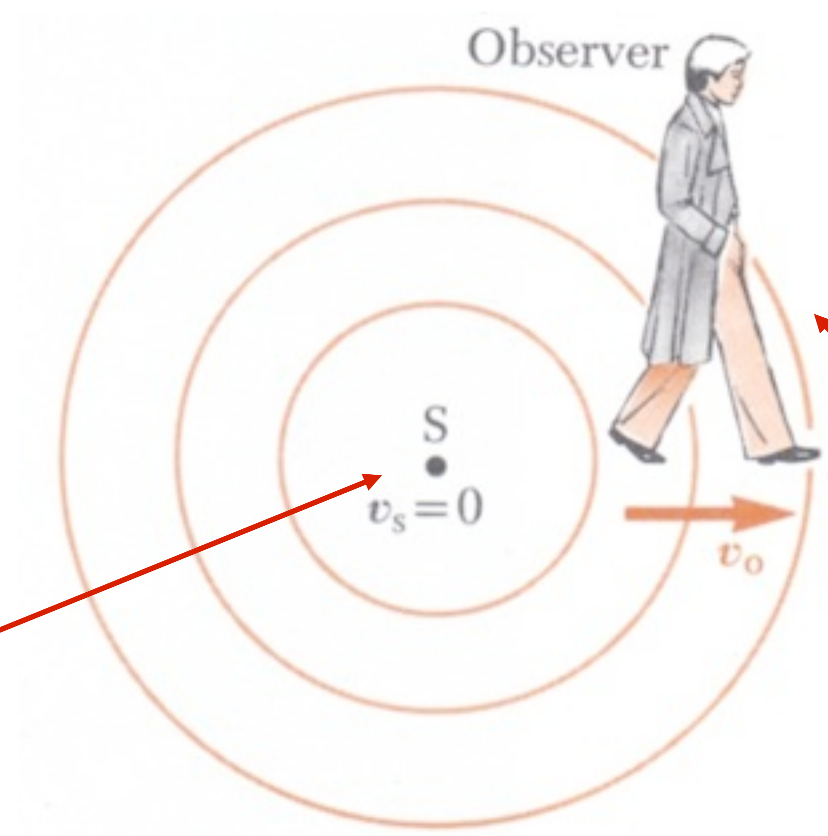
$$\therefore \frac{v_o}{\lambda} = \frac{v_o}{v} f$$

$$\therefore f' = f + \frac{v_o}{v} f$$

$$f' = f\left(\frac{v + v_o}{v}\right)$$

$(v + v_o)$ = speed of waves relative to O

Stationary source, observer moving from source



Observer moving **AWAY FROM** S at speed v_o

Point source S at rest ie $v_s = 0$

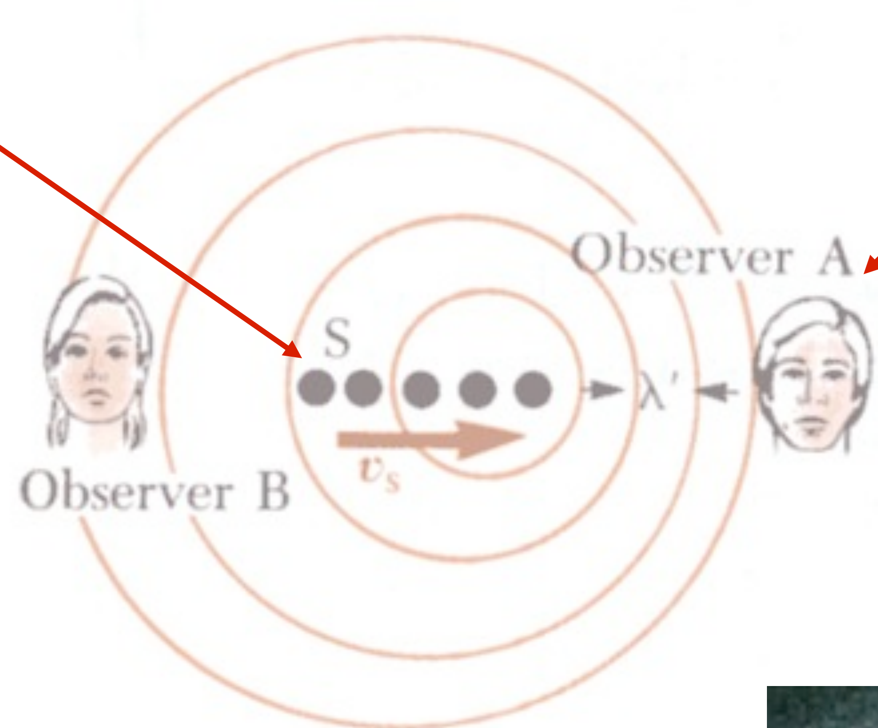
O now detects fewer wavefronts /second and therefore the frequency is lowered.

The speed of the wave relative to O is $(v - v_o)$

$$\therefore\ f' = f\left(\frac{v - v_o}{v}\right)$$

Stationary observer, source moving

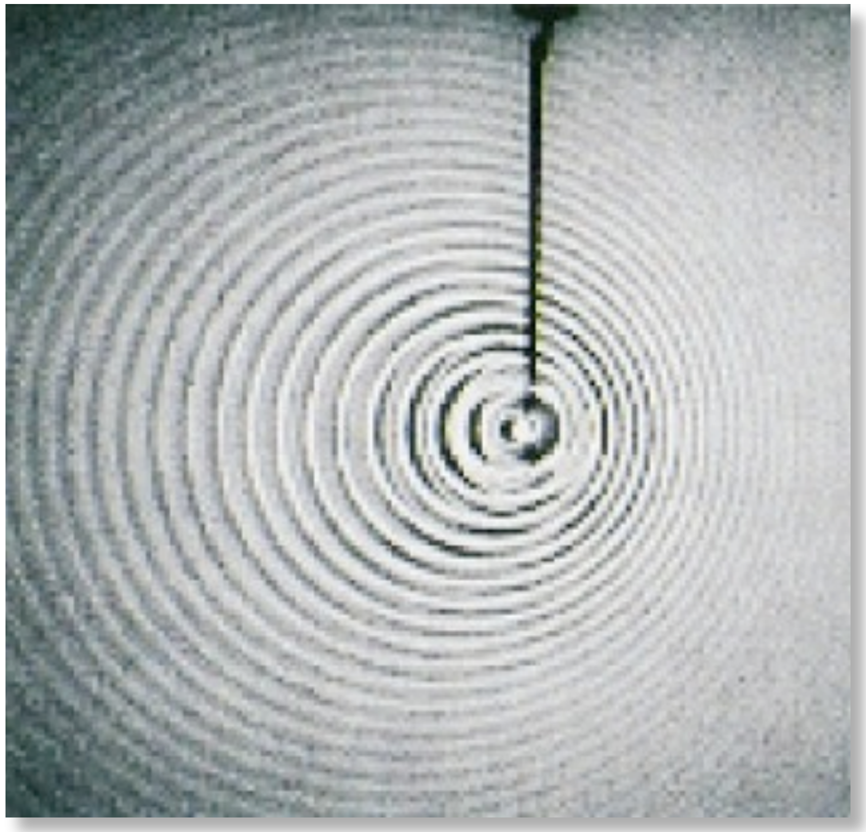
Point source S moving with speed v_s



S moving **TOWARDS** O_A at speed v_s

Source is moving towards O_A at speed v_s.

Wavefronts are closer together as a result of the motion of the source.



The observed wavelength λ' is shorter than the original wavelength λ.

During one cycle (which lasts for period T) the source moves a distance $v_s T$ ($= v_s/f$)

In one cycle the wavelength is shortened by v_s/f

$$\lambda' = \lambda - \Delta\lambda = \lambda - \frac{v_s}{f}$$

$$\text{but } \lambda = \frac{v}{f} \quad \text{and} \quad \lambda' = \frac{v}{f'}$$

$$\therefore \; f' = \left(\frac{v}{\lambda - v_s/f} \right)$$

$$f' = \left(\frac{v}{\lambda - v_s/f}\right)$$

$$= \left(\frac{v}{v/f - v_s/f}\right)$$

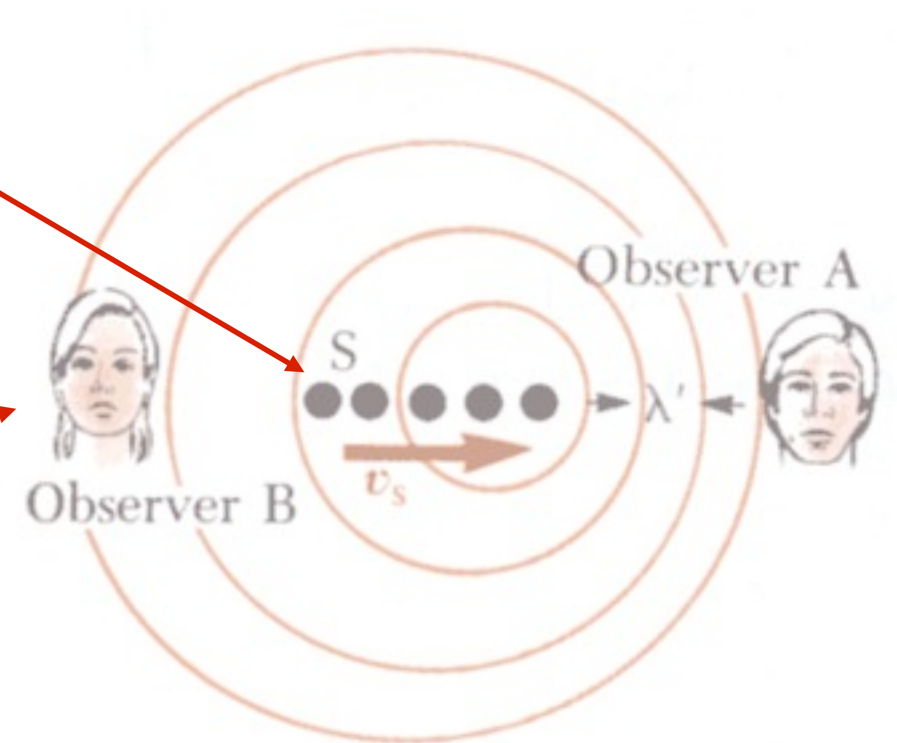
$$f' = f\left(\frac{v}{v - v_s}\right)$$

ie: the observed frequency is increased when the source moves towards the observer.

Note - the equation breaks down when $v_s \sim v$. We will discuss this situation later.

Stationary observer, source moving

Point source S
moving with speed v_s



S moving **AWAY FROM**
O_B at speed v_s

Source is moving away from O_B at speed v_s.

Wavefronts are further apart, λ is greater and O_B hears a decreased frequency given by

$$f' = f\left(\frac{v}{v + v_s}\right)$$

General equations

Frequency heard when observer is in motion

$$f' = f\left(\frac{v \pm v_o}{v}\right)$$

+ O towards S
- O away from S

Frequency heard when source is in motion

$$f' = f\left(\frac{v}{v \mp v_s}\right)$$

- S towards O
+ S away from O

Frequency heard when observer is in motion

$$f' = f\left(\frac{v \pm v_o}{v \mp v_s}\right)$$

upper signs = towards
lower signs = away from

Shock Waves

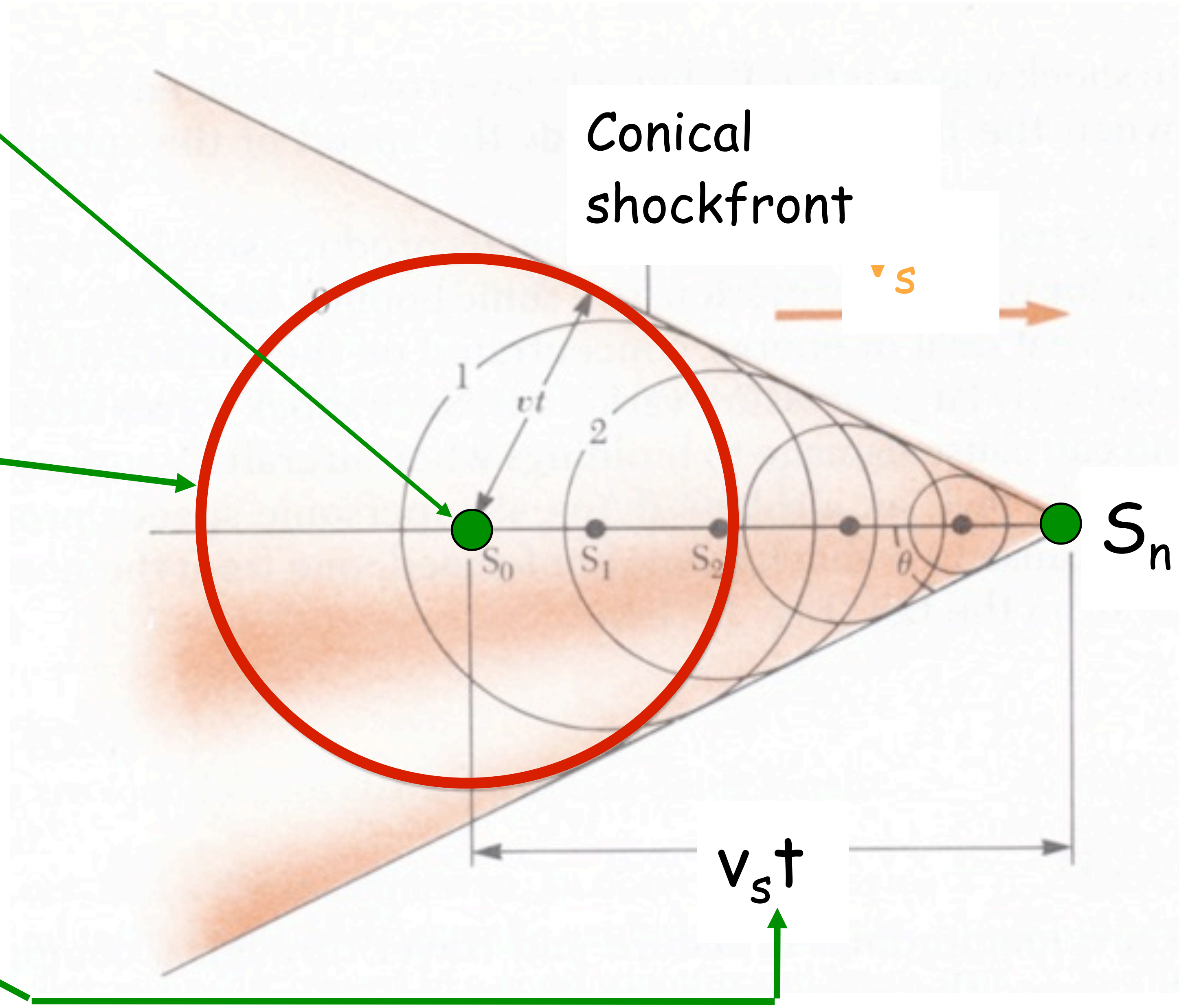
Go back to situation where source is moving with velocity v_s which exceeds wave velocity

At t=0 source is at S_o

At t = t S is at S_n

During this time the wavefront centred on S_o reaches a radius vt

Source has travelled from S_o to S_n = $v_s t$

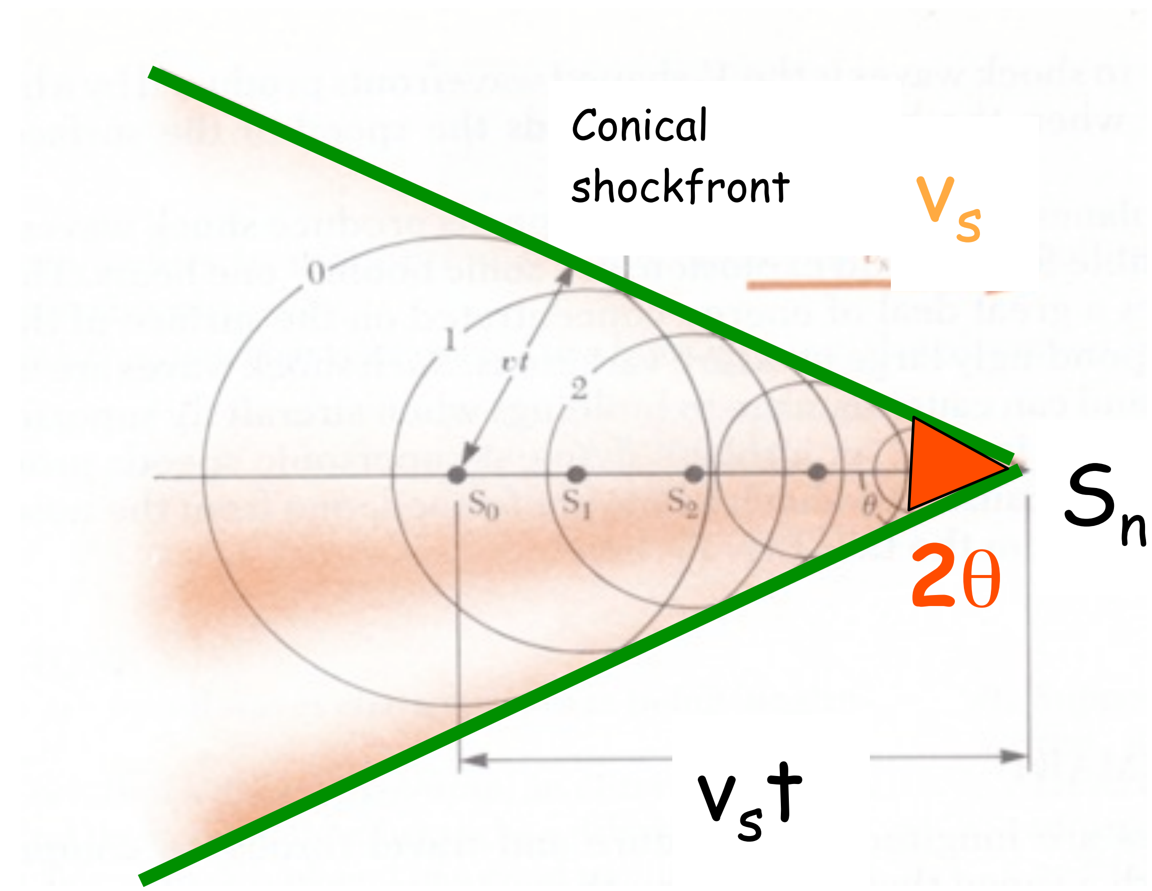


At t=t S is at S_n and waves are just about to be produced here

The line drawn from S_n to the wavefront centred on S_o is tangential to all wavefronts generated at intermediate times

The envelope of these waves is a cone whose apex half angle θ is given by

$$\sin\theta = \frac{vt}{v_s t} = \frac{v}{v_s}$$



The ratio $\frac{v}{v_s}$ is known as the **MACH** number.

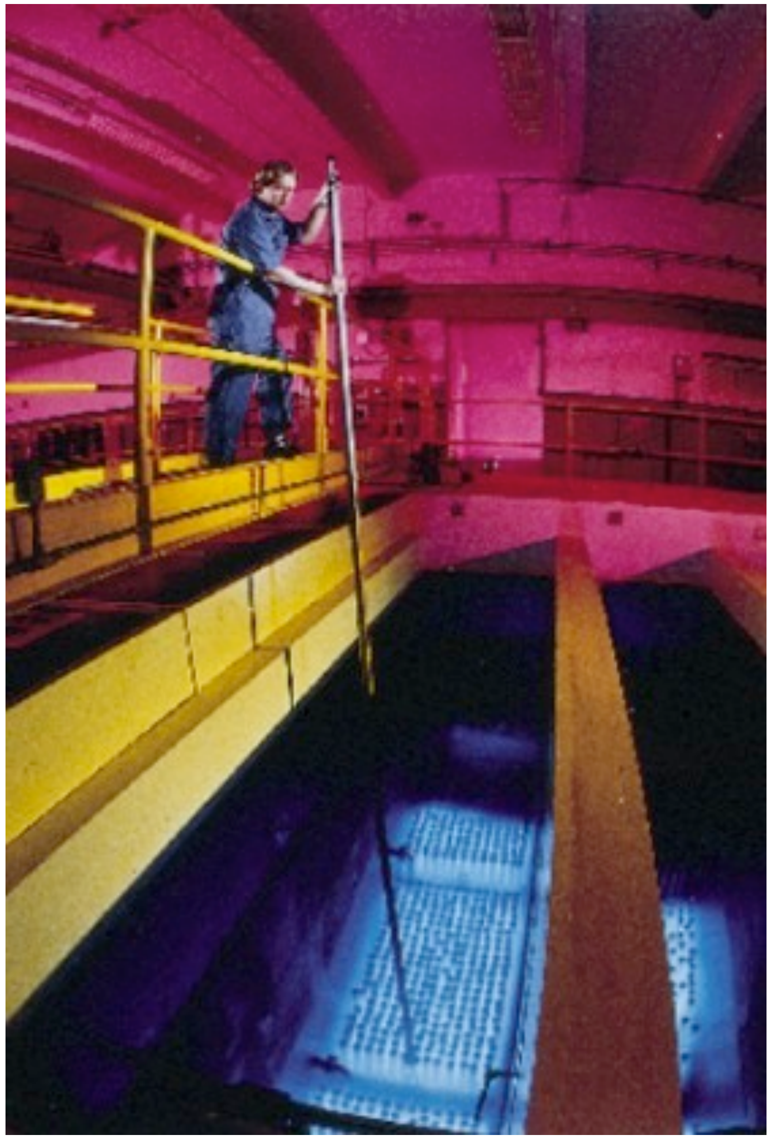
The conical wavefront produced when $v_s > v$ (supersonic speeds) is known as a shock wave.

An aeroplane travelling at supersonic speeds will produce shockwaves.

In this photo the cloud is formed by the adiabatic cooling of the shock wave to the dew point.

Examples of shockwaves: Cerenkov radiation

$$\sin\theta = \frac{vt}{v_s t} = \frac{v}{v_s}$$



This equation can also be applied to a form of electromagnetic radiation called Cerenkov radiation.

A charged particle moves in a medium with speed v that is greater than the speed of light in that medium

The blue glow surrounding the fuel elements in a nuclear reactor is an example of Cerenkov radiation