Cosmology I

University of Trieste, master degree programme in Physics

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Detailed programme

1. **Introduction:** Observational facts in cosmology. A preface to curvature: inertial frames with gravity, weak equivalence principle. **Sources:** slides, Schutz: Chap. 5.1, pagg 111-118.

Part 1: General Relativity

2. **Special relativity:** Principles of SR; SR as a geometrical theory of space-time; invariance of the interval; geometrical demonstration of Lorentz tranformations. Vectors in SR, basis vectors, velocity vector for massive particles and for photons. **Source:** Schutz, Chap. 1 (pagg. 1-21) and 2 (pagg. 33-50).

3. **Tensors in SR:** Definition of tensors; the metric tensor; definition of one-forms; transformation of one-forms; basis one-forms; gradient as prototype one-form; metric tensor; dual vector spaces; general tensors. **Source:** Schutz, Chap. 3 (pagg. 56-77).

4. **Fluids in SR, tensor calculus in SR:** number density of dust, number flux 4-vector; definition of stress-energy tensor; divergence of stress-energy tensor; perfect fluid case. General coordinate transformations; one-forms as gradient of scalar fields; vectors as tangents to curves; scalar product as total derivative of a scalar field; the case of polar coordinates: metric tensor. **Source:** Schutz, Chap. 4 (pagg. 84-93 and 96-102), Chap 5 (pagg. 118-124).

5. **Tensor calculus in SR for polar coordinates:** derivative of basis vectors; Christoffel symbols; definition of covariant derivative; divergence and Laplacian; covariant derivative of one-forms; generalization to any tensor; Christoffel symbols from the metric. **Source:** Schutz, Chap. 5 (pagg. 124-139).

6. **Curved manifolds:** definition of a manifold; construction of tensors on a manifold; Riemannian manifolds; metric tensor; length of a curve; volume element; local flatness theorem; covariant differentiation; divergence and Gauss law; parallel transport of a vector; geodesics, affine parameters. **Source:** Schutz, Chap. 6 (pagg. 142-157).

7. **The curvature tensor:** definition of curvature; Riemann tensor from parallel transport on a loop; properties of the Riemann tensor; number of independent components; flat manifolds, case of polar coordinates; geodesic deviation; Bianchi identities, Ricci tensor, Ricci scalar, Einstein tensor; curvature of a sphere. **Sources:** Schutz, Chap 6 (pagg. 157-166). See also Carroll, section 3.7, pagg. 127-128.

8. **The Einstein's equations:** basic principles of general relativity; strong equivalence principle; Einstein's derivation of his equations; cosmological constant moved to the stress-energy tensor; Einstein's equation in vacuum; Hilbert's derivation form least action principle, Einstein-Hilbert action. **Sources:** Schutz, Chap. 7 (pagg. 171-175), Chap. 8 (pagg. 184-189); Carroll, section 4.2 and 4.3 (pagg. 155-165).

9. **Physics in a curved space-time:** weak field limit with nearly Lorentz coordinates; gauge transformations; field equations of linearised GR in the Lorentz gauge; Poisson equation in the weak field limit; geodesic equation; Newtonian equations of motion; conserved quantities; energy in the Newtonian limit. **Sources:** Schutz, Chap. 7 (pagg. 175-181), Chap. 8 (pagg. 189-195); Carroll, pagg. 201-204.

10. **Black holes:** metric in spherical symmetry; solving Einstein's equations in spherical symmetry; Schwartzschild metric; Schwartzschild radius and Laplace argument; motion of a particle around a black hole; the event horizon; gravitational redshift; meaning of the event horizon. **Source:** Schutz, Chap. 10 (pagg. 256-260), Chap. 11 (pagg. 281-287 and 298-301).

Part 2: Friedmann-Robertson-Walker models

11. **Observational cosmology:** distances in astronomy; the cosmological principle; the cosmic distance ladder; the Hubble diagram with nearby and distant SNe; galaxies as tracers of the large-scale structure of the Universe; galaxy weak and strong lensing; galaxy clusters; the Lyman-alpha forest. **Sources:** Slides; Introduzione all'astrofisica.

12. **Robertson-Walker metric:** metric for a homogeneous and isotropic Universe; space part of the metric; flat, open and closed universes; length of a circle, area of spherical surface, volume. **Sources:** Schutz, Chap. 12, pagg. 341-345; Vittorio, Chapter 1; Carroll, Chap. 8.

13. **The Hubble law:** cosmological redshift; proper distance; comoving distance; Hubble parameter; luminosity distance; diameter distance; Hubble law; superluminal motion; deceleration parameter. **Sources:** Schutz, Chap. 12, pagg. 345-351; Vittorio, Chapter 2.

14. **Friedmann equations from Einstein equations:** from the metric to the Ricci tensor; 00 component: first Friedmann equation; space components: second Friedmann equation; continuity equation: third Friedmann equation; critical density; density parameter. **Sources:** Vittorio, Chapter 1; Carroll, Chap. 8.

15. **Flat models, horizons:** solution of Friedmann equations for dust in a flat universe; Hubble parameter, deceleration parameter, density versus time, distances; age of the universe; particle horizon; speed of light sphere; past light cone; conformal diagrams. **Source:** Vittorio, Chap. 2; Ellis & Rothman.

16. **Flat and non-flat models:** generic equation of state; solutions for a flat model; the radiation case; curved models; solutions for matter-dominated models; evolution of the density parameter. **Source:** Vittorio, Chaps. 1 and 2.

17. **Models with Λ:** Friedmann equations with a cosmological constant; flat models with Λ; Einstein's static universe; models with Λ and matter; age of the universe; observational evidence for acceleration; models in the $\Omega_m - \Omega_\Lambda$ plane. **Source:** Vittorio, Chaps. 1 and 2.

Part 3: The early universe

18. **Introduction to the early Universe:** the Cosmic Microwave Background; evolution of temperature, recombination; the CMB dipole; temperature fluctuations; parameters of the ΛCDM model; matter-radiation equivalence; matter-dark energy equivalence; thermal history of the Universe; phase transitions; the horizon problem; big bang nucleosynthesis. **Sources:** slides; Introduzione all'astrofisica, Chap. 5 (Sections 5.4 to 5.6); Vittorio, Chap. 3; Bonometto, Chap. V.

19. **Thermodynamics of the early Universe:** statistics of ultra-relativistic species; evolution of entropy and entropy density; entropy per baryon and η; entropy production during phase transitions. **Source:** Bonometto, Chap. IV, appendices IV.A1 and IV.A2; Vittorio, Chap. 3.

20. **Quantum fields in a FRW universe:** the Planck era; natural units; dynamics of a quantum scalar field; effective pressure and vacuum energy; cosmological constant as a vacuum energy. **Source:** Vittorio, Chap. 4; Bonometto, Chap. III.9, Chap. XI.

21. **Inflation:** from the Planck era to the GUT breaking; phase transitions in the early Universe; the monopole problem; the horizon problem; the flatness problem; inflation; cosmological horizons; types of inflation; inflation with conformal diagrams, solution of the horizon problem; dilution, solution of the flatness problem. **Source:** Vittorio, Chap. 4.

22. **Big bang nucleosynthesis:** the end of phase transitions; hadron era; baryon-anti baryon annihilation; neutrino decoupling and freezing of neutron to proton ratio; lepton era; electron-positron annihilation and radiation era; the deuterium bottleneck; heavier elements. **Source:** Bonometto, Chap. VI; Vittorio, Chap. 3.

23. **Recombination:** coupling of baryons and radiation; evolution of temperature for the coupled plasma; recombination; evolution of the CMB spectrum; evolution of matter temperature. **Source:** Vittorio, Chap. 3; Bonometto, Chapter VIII.4, VIII.5 and VIII.A1.

24. **Precision cosmology:** a final discussion on hot topics in modern cosmology. **Sources:** slides; Bean's paper.

Web site: `https://adlibitum.oats.inaf.it/monaco/cosmology1.html`

Textbooks:

- A FIRST COURSE IN GENERAL RELATIVITY, 2nd edition, Bernard F. Schutz, Cambridge University Press
- COSMOLOGY, Nicola Vittorio, CRC Press
- COSMOLOGIA & COSMOLOGIE, S. Bonometto, Zanichelli (in italian)
- SPACETIME AND GEOMETRY: AN INTRODUCTION TO GENERAL RELATIVITY, Sean M. Carroll, Pearsons

More material (available on the web page):

- Ellis, G.F.R. & Rothman, T., *Lost horizons*, 1993, American Journal of Physics, 61, 883
- Bean, R., *TASI 2009 - Lectures on cosmic acceleration*, 2010, arXiv:1003.4468
- Lecture notes of *Introduzione all'astrofisica*, P. Monaco, in Italian.

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