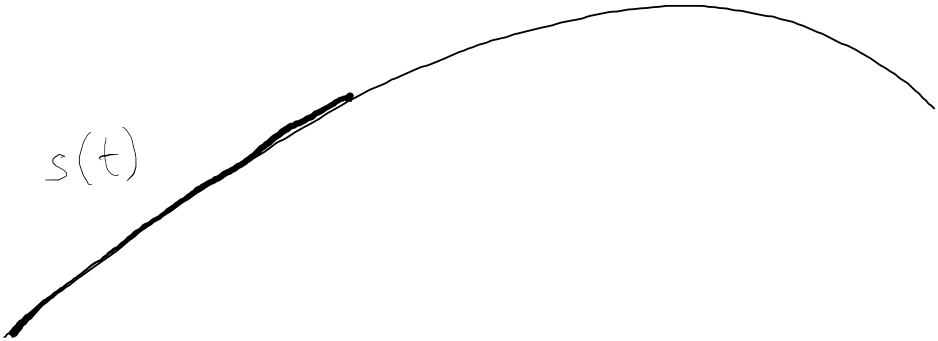
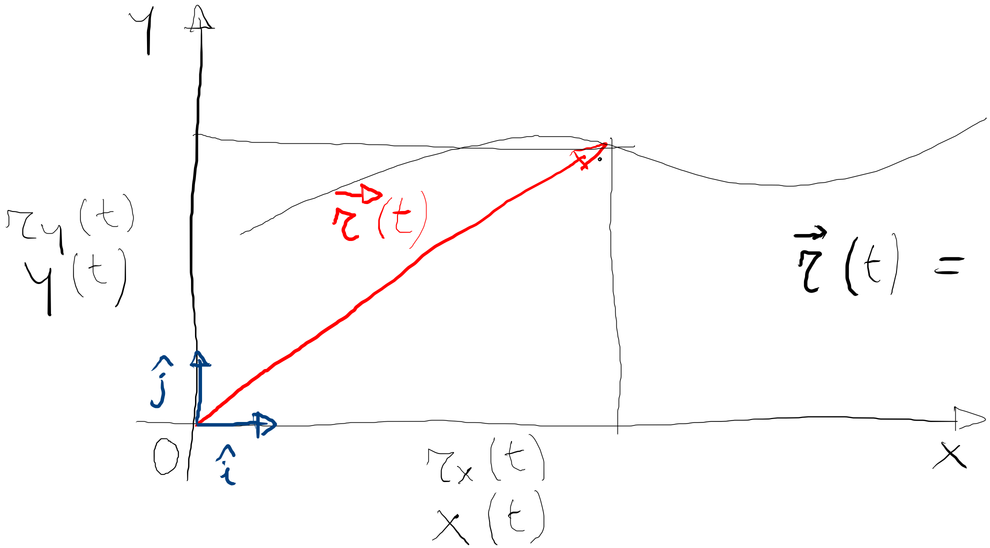
CINEMATICA

punto materiale m

traiettoria



POSIZIONE

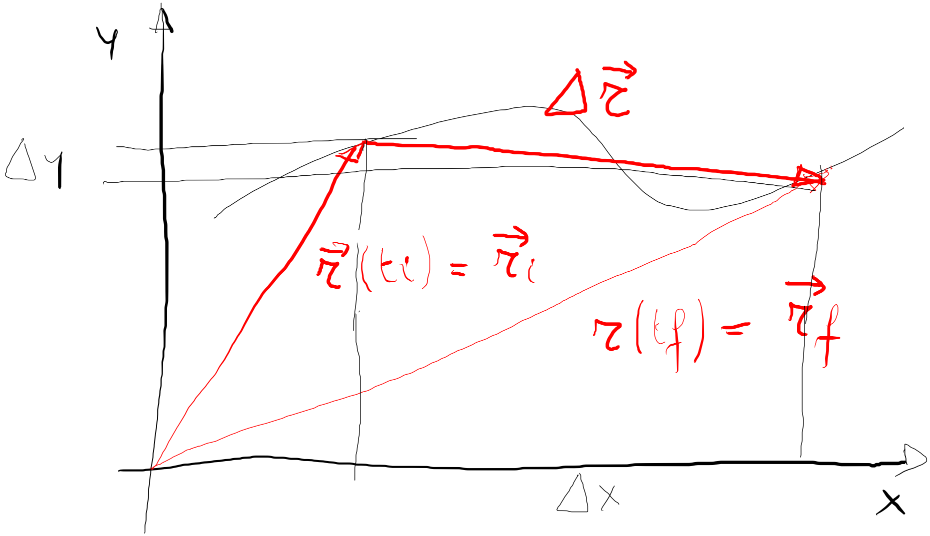


$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j}$$

SPOSTAMENTO

$\Delta t = t_f - t_i$

t_f
t_i



$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$

$\Delta \vec{r} = \Delta x \, \hat{i} + \Delta y \, \hat{j}$

$\Delta x = x_f - x_i$

$\Delta y = y_f - y_i$

VELOCITÀ MEDIA

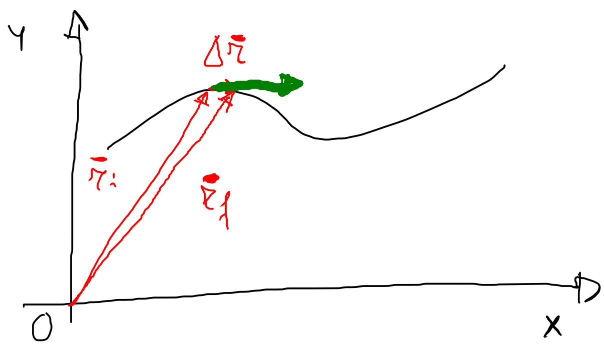
$$\vec{v}_m = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$

$$v_{mx} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

VELOCITÀ ISTANTANEA

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$



$$\vec{v} = \left| \frac{d\vec{r}}{dt} \right|$$

dir tangente

verso moto

$\frac{m}{s}$ **SI** MKS

$\frac{cm}{s}$ c.g.s.

$\frac{km}{h}$

$$v_m = \frac{100\,m}{10\,s} = 10\,\frac{m}{s} = 36\,\frac{km}{h}$$

mph

nodi

$$1\,\frac{km}{h} = 1\,\frac{\cancel{km}}{\cancel{h}} \cdot \left(\frac{10^3\,m}{1\,\cancel{km}}\right) \cdot \left(\frac{\cancel{1h}}{3600\,s}\right) = \frac{\cancel{10^3}}{3{,}6 \cdot \cancel{10^3}}\,\frac{m}{s}$$

$$1\,\frac{km}{h} = \frac{1}{3{,}6}\,\frac{m}{s}$$

$$1\,\frac{m}{s} = 3{,}6\,\frac{km}{h}$$

ACCELERAZIONE MEDIA e ISTANTANEA

$$\vec{a}_m = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{\Delta \vec{v}}{\Delta t}$$

$$a_{mx} = \frac{\Delta v_x}{\Delta t}$$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}$$

$$a = \frac{d^2 x}{dt^2}$$

$$\frac{\frac{m}{s}}{s} = \frac{m}{s^2}$$

$$g = 9{,}8\ \frac{m}{s^2} = 980\ \frac{cm}{s^2}$$

$$\frac{cm}{s^2}$$

$$5g = 5 \cdot 9{,}8\ \frac{m}{s^2} = 49\ \frac{m}{s^2}$$

$$\cancel{\frac{km}{h^2}}$$

$s(t)$

$x(t)$

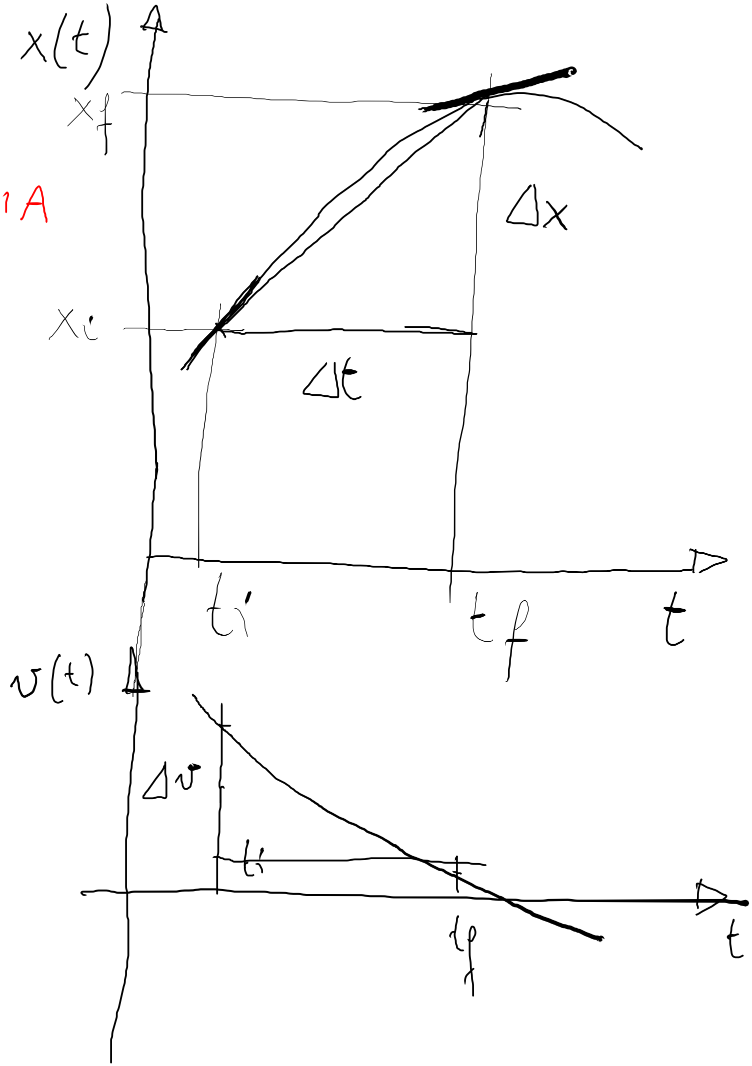
LEGGE ORARIA 1D

$$v_m = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

$$v = \frac{dx}{dt}$$

$$a_m = \frac{\Delta v}{\Delta t}$$

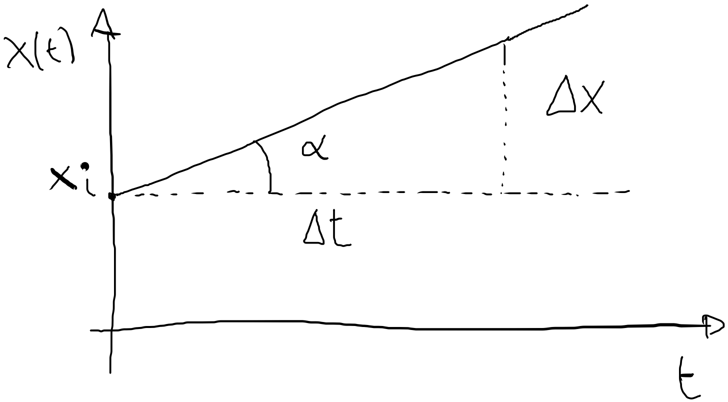
$$a = \frac{dv}{dt}$$



MOTO RETTILINEO UNIFORME

RETTILINEO → 1D

UNIFORME → v cost.



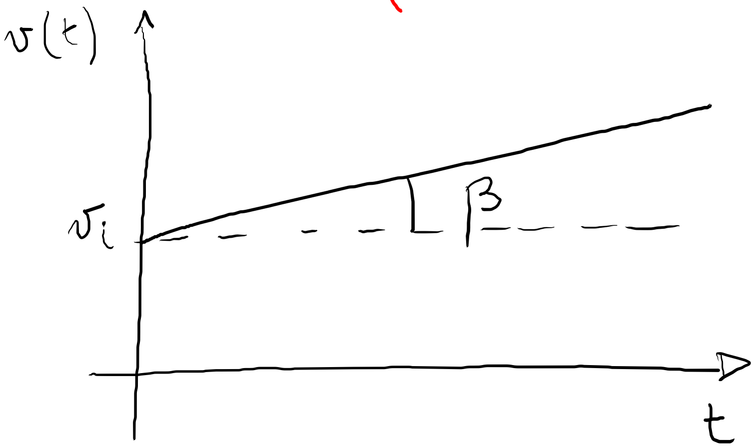
$$v_m = v = \frac{\Delta x}{\Delta t} = tg\,\alpha$$

$$x(t) = x_i + v t$$

$$\vec{r}(t) = \vec{x}_i + \vec{v}\, t$$

MOTO UNIFORMEMENTE ACCELERATO

(a costante nel tempo)



$$\operatorname{tg} \beta = \frac{\Delta v}{\Delta t} = a$$

1 $$\boxed{v(t) = v_i + a t}$$

2 $$\boxed{x(t) = x_i + v_i t + \frac{1}{2} a t^2}$$

$$v(t) = \frac{dx(t)}{dt} = v_i + a t$$

$$x(t=0) = x_i$$

CONTINUA

$$\begin{cases} v = v_i + at \\ x = x_i + v_i t + \frac{1}{2} a t^2 \end{cases}$$

$$\begin{cases} t = \frac{v - v_i}{a} \\ x = x_i + v_i \left(\frac{v - v_i}{a} \right) + \frac{1}{2} a \left(\frac{v - v_i}{a} \right)^2 \end{cases}$$

$$x - x_i = \frac{v_i v}{a} - \frac{v_i^2}{a} + \frac{1}{2} \cancel{a} \left(\frac{v^2 - 2 v v_i + v_i^2}{a^{\cancel{2}}} \right)$$

$$= \cancel{\frac{v_i v}{a}} - \frac{v_i^2}{a} + \frac{v^2}{2a} - \cancel{\frac{v v_i}{a}} + \frac{v_i^2}{2a}$$

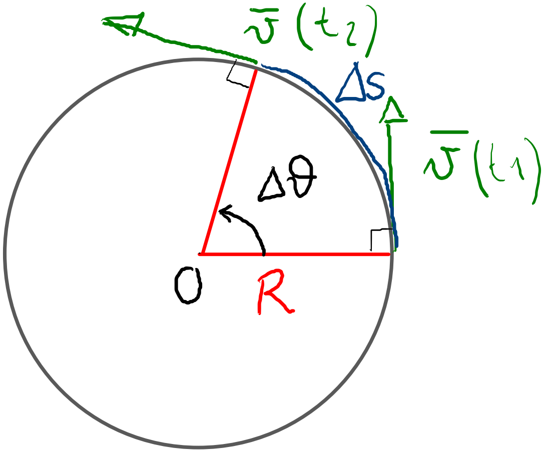
$$= -\frac{v_i^2}{2a} + \frac{v^2}{2a}$$

$$= \frac{1}{2a} \left(v^2 - v_i^2 \right)$$

$$2a (x - x_i) = v^2 - v_i^2$$

3 $$\boxed{v^2 = v_i^2 + 2a (x - x_i)}$$

MOTO CIRCOLARE UNIFORME



$$|\bar{v}(t_1)| = |\vec{v}(t_2)| = v$$

v costante nel tempo

$$\Delta t = t_2 - t_1$$

$$\Delta\vartheta$$

$$\omega_m = \frac{\Delta\vartheta}{\Delta t}$$ velocità angolare media $\frac{rad}{s}$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\vartheta}{\Delta t} = \frac{d\vartheta}{dt}$$

$$\Delta\vartheta = \frac{\Delta s}{R}$$

$$\omega_m = \frac{\Delta\vartheta}{\Delta t} = \frac{\Delta s}{R \cdot \Delta t} = \frac{1}{R}\frac{\Delta s}{\Delta t} = \frac{1}{R} v_m = \frac{v}{R}$$

CONTINUA

$$\boxed{\omega = \frac{v}{R}} \qquad v = \omega R$$

ACCELERAZIONE ANGOLARE

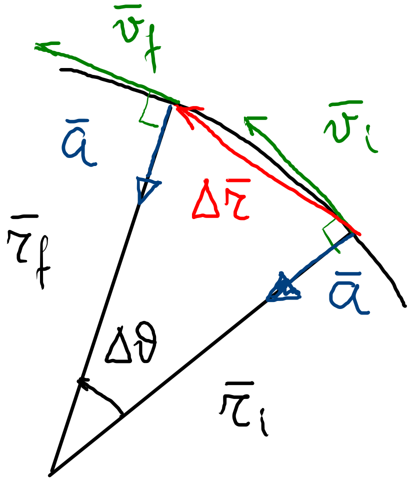
$$\alpha_m = \frac{\Delta\omega}{\Delta t}$$

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

NB:

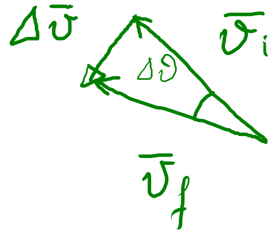
le parti tra parentesi quadre (come queste) NON sono fondamentali

ACCELERAZIONE NEL MOTO CIRCOLARE UNIFORME



$$|\bar{v}_i| = |\bar{v}_f| = v$$

$$|\bar{r}_i| = |\bar{r}_f| = R$$



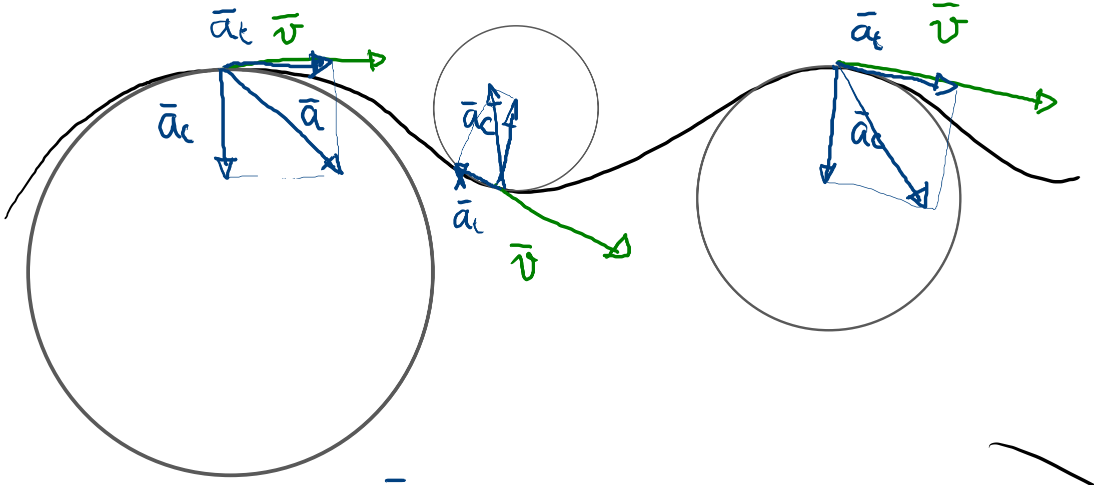
$$\frac{|\Delta \vec{r}|}{R} = \frac{|\Delta \vec{v}|}{v}$$

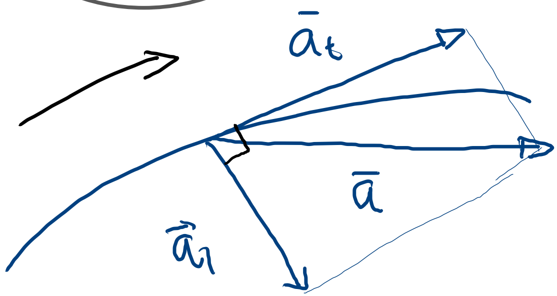
$$\bar{a}_m = \frac{\bar{v}_f - \bar{v}_i}{t_f - t_i} = \frac{\Delta \bar{v}}{\Delta t}$$

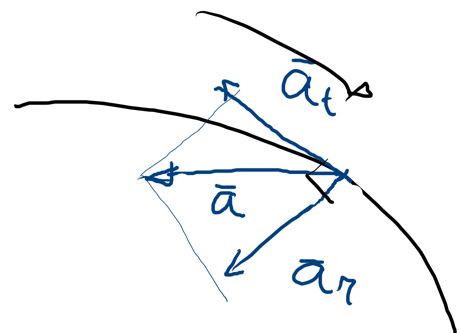
$$|\bar{a}_m| = \frac{|\Delta \bar{v}|}{\Delta t} = \frac{v}{R}\frac{|\Delta \bar{r}|}{\Delta t}$$

$$|\bar{a}| = \lim_{\Delta t \to 0} \frac{v}{R}\frac{|\Delta \bar{r}|}{\Delta t} = \frac{v}{R} \overbrace{\lim_{\Delta t \to 0} \frac{|\Delta \bar{r}|}{\Delta t}}^{v} = \frac{v^2}{R} = \omega^2 R$$

MOTO GENERICO CURVO







$$|\bar{a}_r| = \frac{v^2}{R} \qquad |\bar{a}_t| = R\alpha$$

moto dal moto circ. unif.

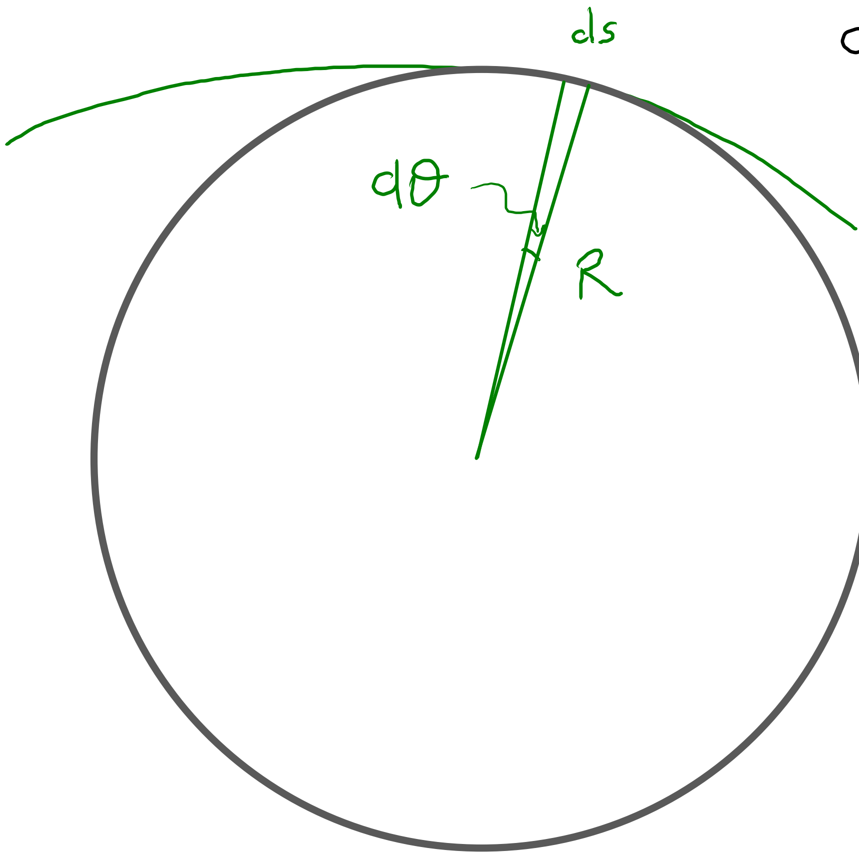
calcolo

$$|\bar{a}_t| = \frac{d|\bar{v}|}{dt} = \frac{d}{dt}\frac{ds}{dt} = \frac{d}{dt}R\frac{d\vartheta}{dt} = R\frac{d^2\vartheta}{dt} = R\frac{d\omega}{dt}$$

$$= R\alpha$$

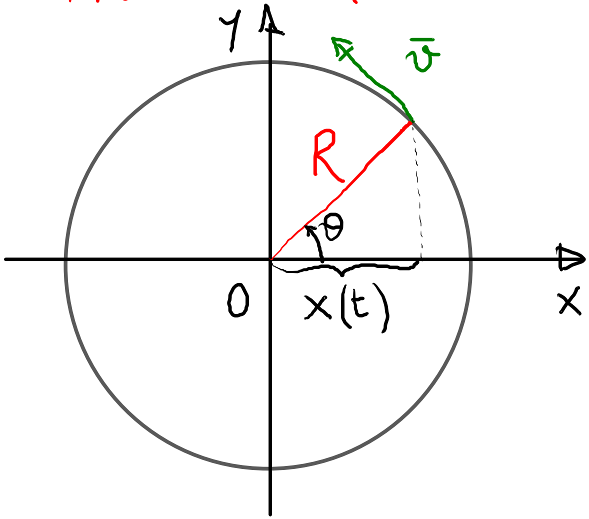
$$ds = R\,d\vartheta$$

$$d\vartheta = \frac{ds}{R}$$



calcolo di $|\bar{a}_t|$

MOTO ARMONICO



$$T = \frac{2\pi R}{v} = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\omega} \qquad \omega = \frac{2\pi}{T} = 2\pi\nu$$

pulsazione

$$\nu = \frac{1}{T} = \frac{\omega}{2\pi} \quad (Hz)$$

$$x(t) = R\cos\vartheta(t) = R\cos(\omega t)$$

$$\vartheta(t) = \omega \cdot t$$

$$x(t) = A\,\text{sen}\,(\omega t + \phi)$$

ampiezza — pulsazione

fase: tiene conto delle condizioni iniziali $(t=0)$

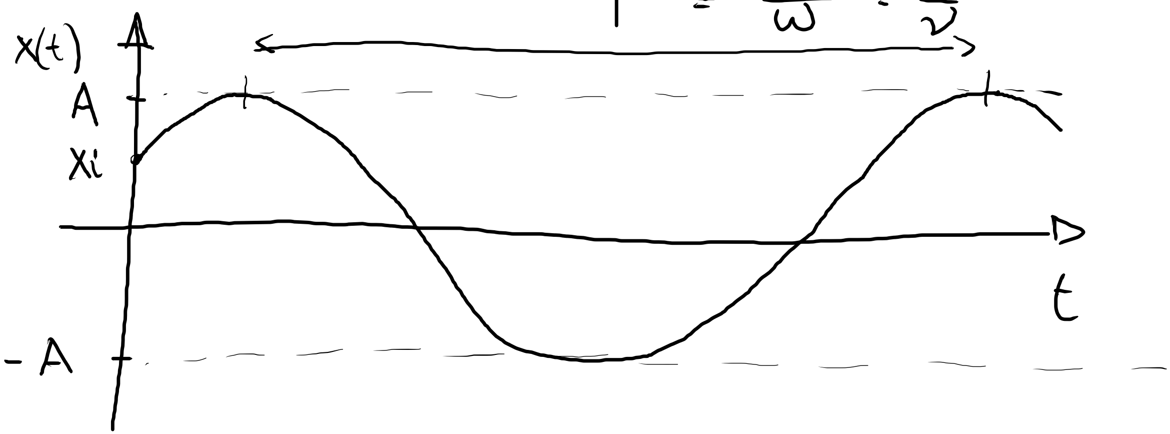
$$x(t=0) = A\,\text{sen}\,\phi$$

$$\mathrm{sen}\left(\omega t + \frac{\pi}{2}\right) = \cancel{\mathrm{sen}\,\omega t \cos\frac{\pi}{2}} + \cos\omega t \underbrace{\mathrm{sen}\frac{\pi}{2}}_{1}$$

$$= \cos\omega t$$

$$x(t) = A\,\mathrm{sen}(\omega t + \phi)$$

$$T = \frac{2\pi}{\omega} = \frac{1}{\nu}$$



$$x(t) = A\,\text{sen}(\omega t + \phi)$$

$$v(t) = \frac{dx(t)}{dt} = A\cos(\omega t + \phi) \cdot \frac{d}{dt}(\omega t + \phi) \quad \omega$$

$$= A\omega\cos(\omega t + \phi)$$

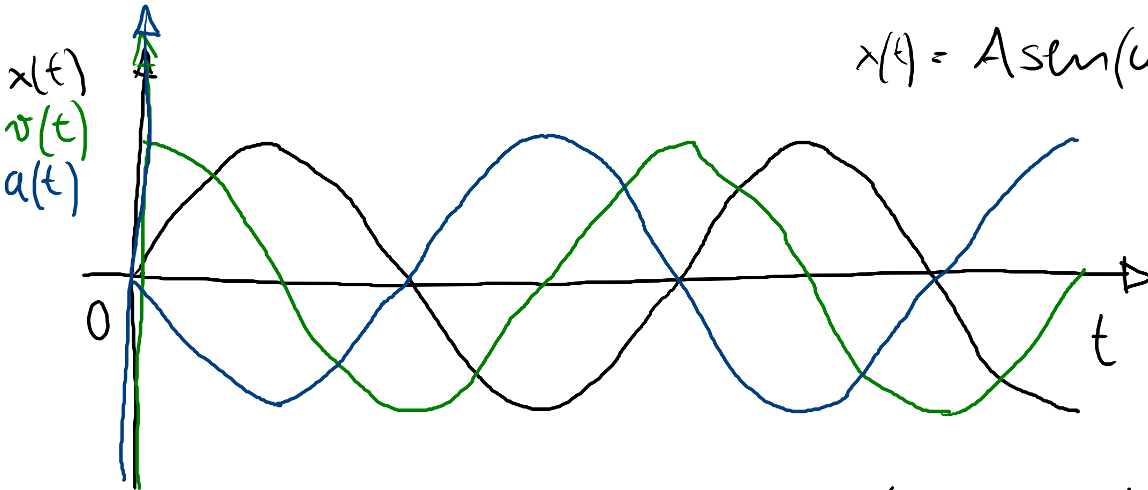
$$a(t) = \frac{dv(t)}{dt} = \frac{d^2 x(t)}{dt}$$

$$\omega$$

$$= A\omega\left[-\text{sen}(\omega t + \phi) \cdot \frac{d}{dt}(\omega t + \phi)\right]$$

$$= -A\omega^2\,\text{sen}(\omega t + \phi)$$

$$a(t) = -\omega^2 x(t)$$



$x(t) = A \operatorname{sen}(\omega t)$

$v(t) = +\omega A \operatorname{sen}(\omega t)$

$a(t) = -\omega^2 A \operatorname{sen}(\omega t)$

Domanda: $\omega = 2\pi \frac{rad}{s}$

$t = \frac{1}{4} s$

$\omega t = \cancel{2}\pi \frac{rad}{\cancel{s}} \cdot \frac{1}{\cancel{4}_2} \cancel{s} = \frac{\pi}{2} rad$