$$e^{iz} = \cos z + i \,\text{sen}\, z \qquad \cos z = \frac{e^{iz} + e^{-iz}}{2} \qquad \boxed{\text{sen}\, z = \frac{e^{iz} - e^{-iz}}{2i}} = \frac{1}{i}\,\text{senh}(z)$$

$$\text{sen}(z_1 + z_2) = \text{sen}(z_1)\cos(z_2) + \cos(z_1)\,\text{sen}(z_2)$$

$$\cos(z_1 + z_2) = \cos(z_1)\cos(z_2) - \text{sen}(z_1)\,\text{sen}(z_2)$$

$$\text{sen}(z) = i\,\text{senh}(iz) \quad F$$

$$\text{sen}(z) = \frac{1}{i}\,\text{senh}(iz)$$

$$\boxed{\text{senh}(z) = -i\,\text{sen}(iz)} \quad V$$

$$\text{sen}(iz) = \frac{1}{2i}\left(e^{-z} - e^{z}\right) = -\frac{1}{i}\,\text{senh}(z) = i\,\text{senh}(z)$$

$$\Rightarrow \text{senh}(z) = -i\,\text{sen}(iz)$$

$$\cos(iz) = i\cosh(z) \quad F$$

$$\cos(iz) = \frac{e^{-z} + e^{z}}{2} = \cosh(z)$$

$$\mathrm{sen}(\bar{z}) = \overline{\mathrm{sen}(z)}$$ VERA

$$\mathrm{sen}(x+iy) = \mathrm{sen}\,x \cos(iy) + \mathrm{sen}(iy)\cos(x)$$

$$= \mathrm{sen}\,x \cdot \cosh y + i \cos x \,\mathrm{senh}\, y$$

$$\mathrm{sen}(x-iy) = \mathrm{sen}\,x \cdot \cosh y - i \cos x \,\mathrm{senh}(y) = \overline{\mathrm{sen}(x+iy)}$$

$\mathrm{sen}^2 z + \cos^2 z = 1$ Vera

$$\left[\frac{1}{2i}\left(e^{iz} - e^{-iz}\right)\right]^2 + \left[\frac{1}{2}\left(e^{iz} + e^{-iz}\right)\right]^2 = -\frac{1}{4}\left(\cancel{e^{2iz}} - 2\underbrace{e^{iz}e^{-iz}}_{=1} + \cancel{e^{-2iz}}\right) +$$

$$+ \frac{1}{4}\left(\cancel{e^{2iz}} + 2\underbrace{e^{iz}e^{-iz}}_{=1} + \cancel{e^{-2iz}}\right)$$

$$= \frac{1}{4}(2+2) = 1 \;//$$

$|\mathrm{sen}\, z| \leq 1$ FALSA

$|\mathrm{sen}\, x| \leq 1 \quad \forall x \in \mathbb{R}$

$\mathrm{sen}(iz) = i\,\mathrm{senh}(z) \qquad z = iy$

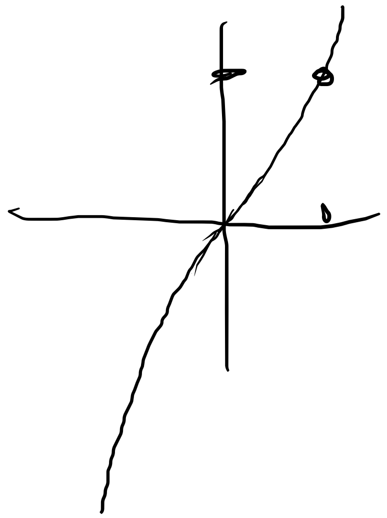
$|\mathrm{sen}(iy)| = |\mathrm{senh}\, y| \to +\infty$ se $y \to \pm\infty$!!!

seno e coseno non sono funzioni limitate in $\mathbb{C}$!!

$$\operatorname{senh}(iz + 2\pi) = \operatorname{senh}(iz)$$ **Falso!**

es: $z = 0$

$$\operatorname{senh}(2\pi) > 0$$
$$\operatorname{senh}(0) = 0$$



$$\operatorname{senh}(iz) = i \operatorname{sen} z$$

$$\operatorname{senh}(\underbrace{iz + i2\pi}_{i(z+2\pi)}) = i \operatorname{sen}(z+2\pi) = i \operatorname{sen}(z) = \operatorname{senh}(iz)$$

$$\operatorname{senh}(z + 2\pi) = \operatorname{senh}(z) \quad \forall z \in \mathbb{C}$$

Logaritmo

$e^z = w$ soluzioni

sia $w \in \mathbb{C}$ $w \neq 0$ $z = x + iy$ $e^{x+iy} = w$?

$w = \rho e^{i\vartheta}$ $\rho > 0$

$e^{x+iy} = \rho e^{i\vartheta}$

$e^x e^{iy} = \rho e^{i\vartheta}$

$\Rightarrow \begin{cases} e^x = \rho & x = \log \rho \\ y = \vartheta + 2k\pi & k \in \mathbb{Z} \end{cases}$ infinite soluzioni

$\forall k \in \mathbb{Z}$ il numero $z_k = \log \rho + i(\vartheta + 2k\pi)$ è soluzione dell'equazione

Diremo determinazione principale del logaritmo il numero

$\mathrm{Log}(w) = \log|w| + i\,\mathrm{Arg}(w)$ $\mathrm{Log}\, w$ definito su $\mathbb{C} \setminus \{0\}$

$\mathrm{Log}\, w$ è l'inversa della funzione $f(z) = e^z$ ristretta all'insieme

$\{x + iy,\ -\pi < y \leq \pi\}$

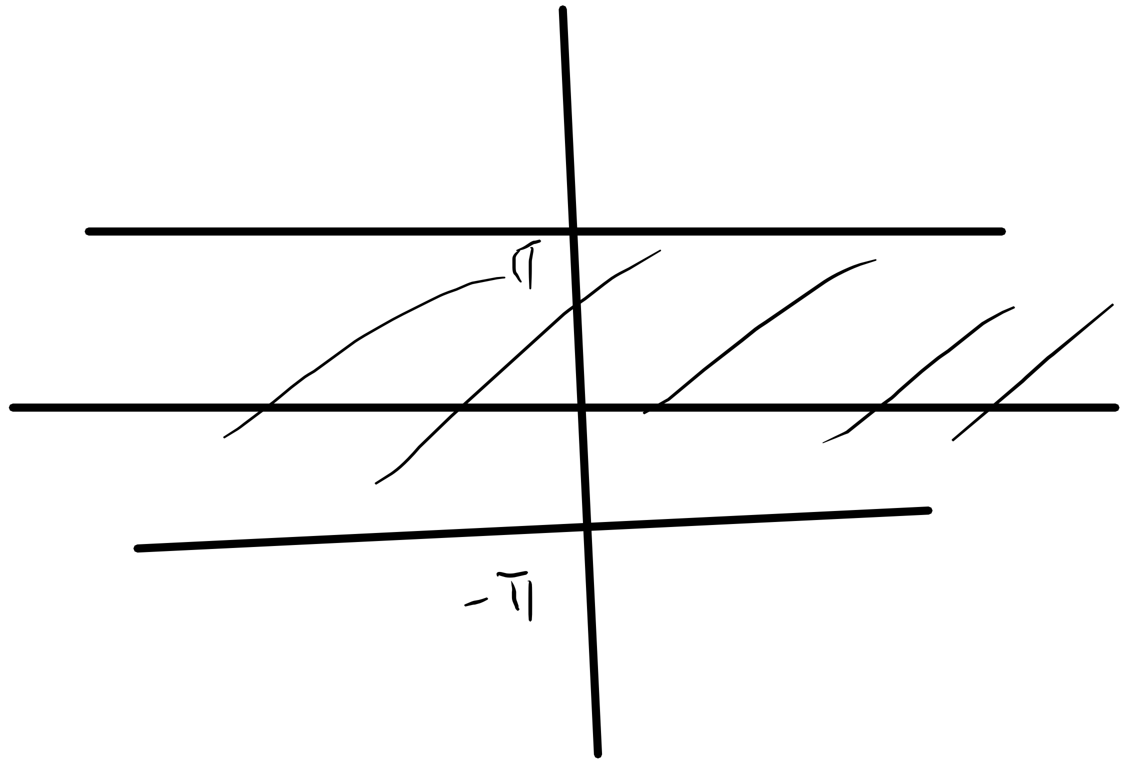
$$\operatorname{Log} w = \log|w| + i\,\operatorname{Arg}(w)$$

$$\operatorname{Log}(e^{z}) = z \quad \forall z \in \mathbb{C}$$

F

$$\operatorname{Log}(e^{2\pi i}) = \operatorname{Log}(1) = 0 \neq 2\pi i$$

$$e^{2\pi i} = 1$$



$$e^{\operatorname{Log} z} = z$$

Vera

$$e^{\operatorname{Log} z} = e^{\log|z| + i\,\operatorname{Arg}(z)} = |z|\, e^{i\,\operatorname{Arg}(z)} = z$$

$$\cancel{\mathrm{Log}(z\cdot w) = \mathrm{Log}(z)\cdot \mathrm{Log}(w)}$$

NO !!!

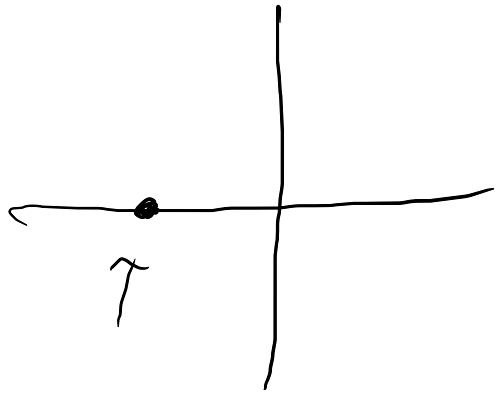
$$\left[\text{In } \mathbb{R} \qquad \log(x\cdot y) = \log x + \log y\right]$$

NO

$$\mathrm{Log}(z\cdot w) \cancel{\overset{?}{=}} \mathrm{Log}(z) + \mathrm{Log}(w) \qquad (\text{vere su } \mathbb{R})$$

$$\mathrm{Log}((-1)\cdot(-1)) = \mathrm{Log}\, 1 = 0$$

$$\underbrace{\mathrm{Log}(-1)}_{= i\pi} + \mathrm{Log}(-1) = i\pi + i\pi = 2i\pi \neq 0$$



$\rho = 1$

$\mathrm{Arg} = \pi$

$$\mathrm{Log}((-1)^2) = \mathrm{Log}(1) = 0$$

$$\neq$$

$$2\,\mathrm{Log}(-1) = 2i\pi \neq 0$$

$D \, \mathrm{Log}\, z = \frac{1}{z}$ $\quad \cancel{\forall z \neq 0}$ $\quad \mathrm{Log}\, z = \log|z| + i \,\underline{\mathrm{Arg}\, z}$

Falso

$\forall z \in \,]-\infty, 0[$ non è derivabile!

$\boxed{\forall z \in \mathbb{C} \setminus \,]-\infty, 0]}$

vero

$\boxed{12.15}$

$\boxed{\operatorname{sen} z = 2}$ risolvere in $\mathbb{C}$

$z = x + iy$ $\qquad \operatorname{sen}(x+iy) = 2$

$$\operatorname{sen} x \cosh y + i \cos x \operatorname{senh} y = 2 + i \cdot 0$$

$$\Rightarrow \begin{cases} \operatorname{sen} x \cosh y = 2 \\ \cos x \operatorname{senh} y = 0 \end{cases} \Longleftarrow \boxed{\cos x = 0} \vee \cancel{\operatorname{senh} y = 0}$$

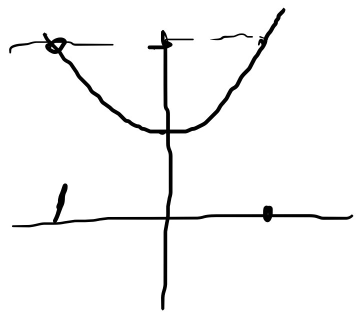
$\cos x = 0 \Downarrow$ $\qquad \operatorname{sen} x = -1 \vee \boxed{\operatorname{sen} x = 1}$

$\operatorname{senh} y = 0 \Downarrow$ $\qquad y = 0 \rightarrow \operatorname{sen} x \cdot \underbrace{\cosh(0)}_{1} = 2 \Rightarrow \operatorname{sen} x = 2$ IMPOSSIBILE

$-\cosh y = 2 \qquad \cosh y = -2$ IMPOSSIBILE

$\boxed{\cosh y = 2}$

y_1, y_2



$x = \frac{\pi}{2} + 2k\pi \qquad k \in \mathbb{Z}$

$z = \frac{\pi}{2} + 2k\pi + i y_1 \qquad y_1 = \log(2+\sqrt{3})$

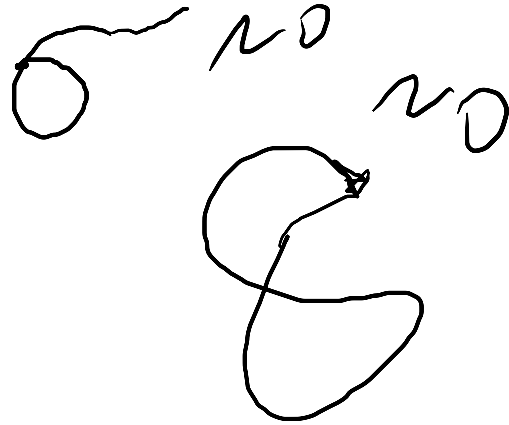
$\frac{\pi}{2} + 2k\pi + i y_2 = \log(2-\sqrt{3})$

$\operatorname{sen} z = i$

$\parallel$

$\operatorname{sen} x \cosh y + i \cos x \operatorname{senh} y = 0 + 1 i$

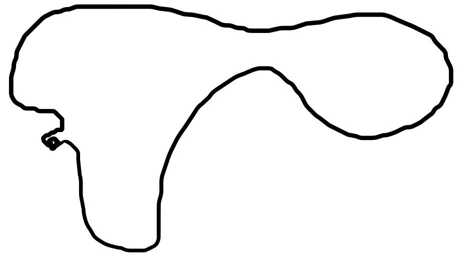
$$\boxed{\int_\gamma f(z)\, dz}$$



Integrazione su curve in $\mathbb{C}$

$\gamma : I \subseteq \mathbb{R} \to \mathbb{C}$ $\qquad \gamma(t) = x(t) + i\, y(t)$ curva parametrica [continua]

γ si dice chiusa se $\gamma : [a,b] \to \mathbb{C}$ $\qquad \gamma(a) = \gamma(b)$



γ si dice semplice se non ha intersezioni

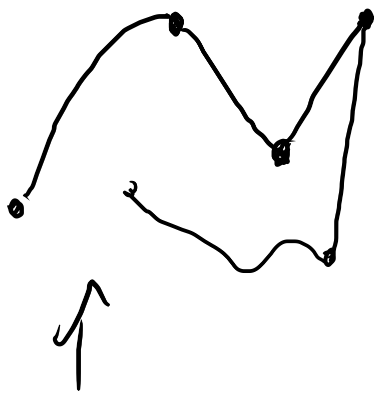
(γ iniettiva tranne eventualmente negli estremi dove $\gamma(a) = \gamma(b)$)

$\gamma : I \to \mathbb{C}$ è regolare se esiste $\forall t$ $\gamma'(t) \neq 0$

($\gamma' : I \to \mathbb{C}$ è una curva ed è la tangente)

$\tilde{\gamma} : I \to \mathbb{R}^2$

$\gamma : I \to \mathbb{C}$



concatenazione (somma) di curve

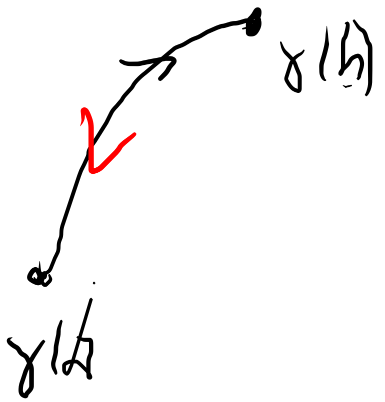
$\gamma = \gamma_1 + \gamma_2$

$\gamma : I_1 \cup I_2 \to \mathbb{C}$

$$\gamma(t) = \begin{cases} \gamma_1(t) & t \in I_1 \\ \gamma_2(t) & t \in I_2 \end{cases}$$

γ regolare a tratti

$-\gamma$



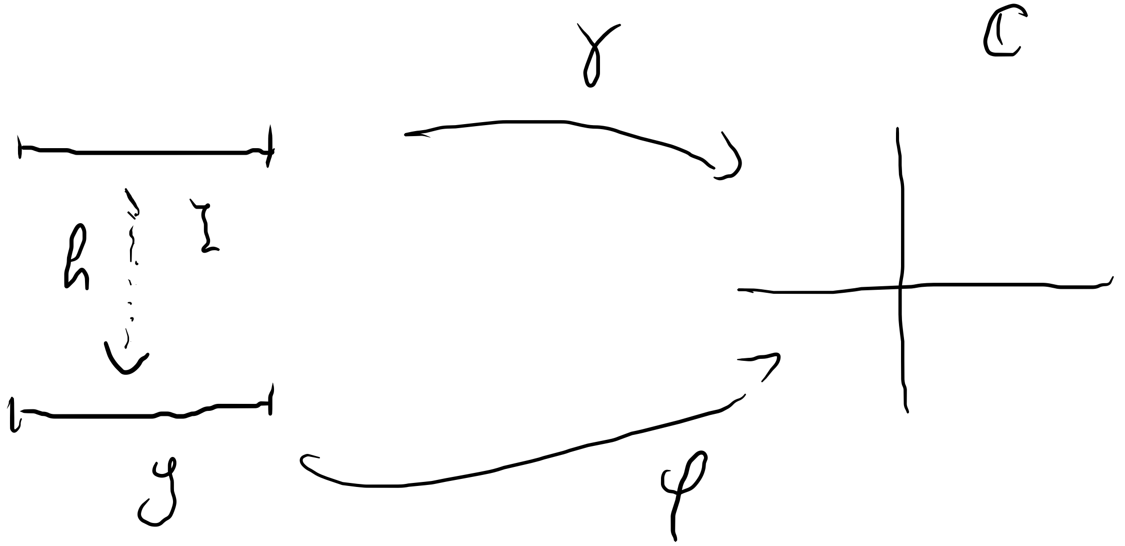
$\gamma : [a,b] \to \mathbb{C}$

$-\gamma : [a,b] \to \mathbb{C}$

$-\gamma(t) = \gamma(a+b-t)$

Curve equivalenti $\gamma : I \to \mathbb{C}$ $\quad \varphi : J \to \mathbb{C}$

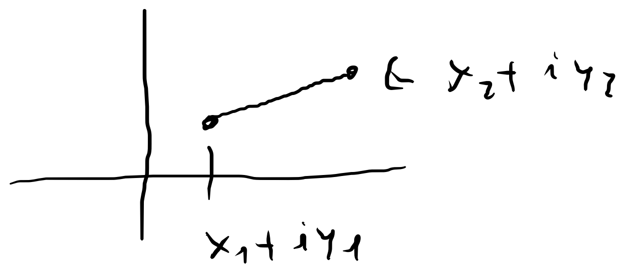
γ e φ sono equivalenti se esiste $h \in C^1(I, J)$ invertibile con inverso C^1 tale che $\varphi \circ h = \gamma$

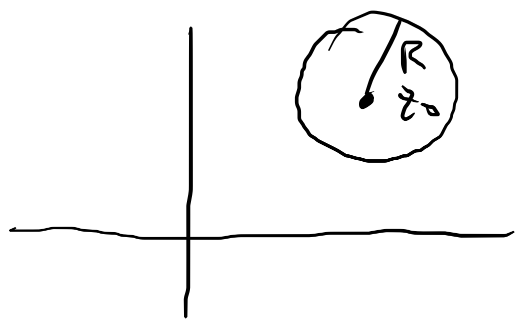


se $h' > 0$ φ e γ sono equiverse

se $h' < 0$ controverse

Segmento $\gamma : [0,1] \to \mathbb{C}$ $\quad \gamma(t) = x_1 + iy_1 + t(x_2 - x_1 + i(y_2 - y_1))$



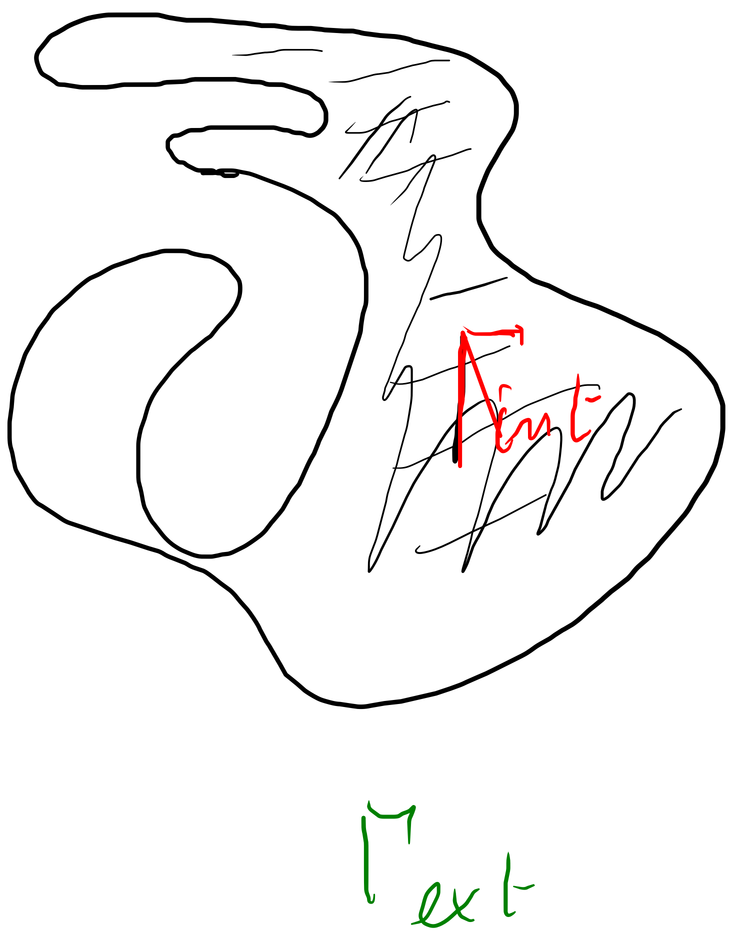


Circonferenza di centro z_0 e raggio R

$$\gamma(t) = z_0 + R e^{it}$$

$$z_0 + R(\cos t + i \sin t) \qquad t \in [0, 2\pi]$$

Teorema della curva chiusa di Jordan



Sia γ una curva chiusa semplice con sostegno $\Gamma = \gamma(I)$, allora il complementare di Γ è unione di due aperti connessi, uno limitato che si dice interno della curva Γ_{int} e uno illimitato che si dice esterno della curva Γ_{ext}

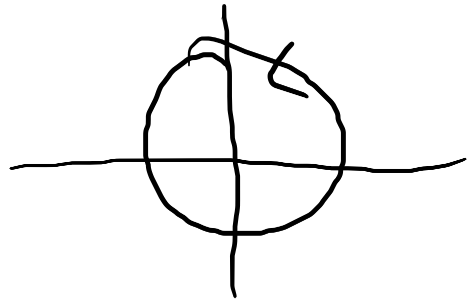
Diremo circuito una curva semplice chiusa.

La curva γ si dice orientata positivamente se considerata la coppia di vettori $(\nu(t), \tau(t))$

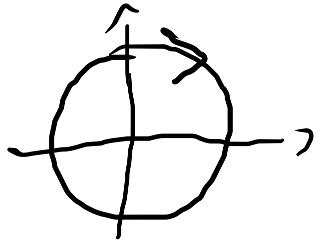
$\tau(t)$ vettore tangente
$\nu(t)$ vettore normale

questa è sovrapponibile con una rotazione alla coppia dei vettori di base (e_1, e_2)

$\gamma(t) = e^{it}$ $t \in [0, 2\pi]$ è orientata positivamente



$\varphi(t) = \sin t + i \cos t$ $t \in [0, 2\pi]$



Sia $A \subseteq \mathbb{C}$ aperto, $f : A \to \mathbb{C}$ continua, $\gamma : I \to A$ curva regolare con sostegno $\Gamma \subset A$. Diremo integrale di f sulla curva γ il numero

$$\int_\gamma f(z)\,dz := \int_I f(\gamma(t)) \cdot \gamma'(t)\,dt = *$$

$\uparrow$ prodotto di numeri complessi

$$f \rightsquigarrow u(x,y) + i\, v(x,y) \qquad \gamma(t) = x(t) + i\,y(t)$$

$$* \int_I \left[u(x(t), y(t)) + i\, v(x(t), y(t)) \right] \cdot \left[x'(t) + i\, y'(t) \right] dt =$$

$$= \int_I \left[u(x(t), y(t)) \cdot x'(t) - v(x(t), y(t)) \cdot y'(t) \right] dt + i \int_I \left[u(x(t), y(t)) \cdot y'(t) - v(x(t), y(t)) \cdot x'(t) \right] dt$$

⚠ se identifichiamo $\mathbb{C} \simeq \mathbb{R}^2$

l'integrale di linea di un campo su $\mathbb{R}^2$ $\quad f : \mathbb{R}^2 \to \mathbb{R}$

$$\int_\gamma f \, ds = \int_I f(\gamma(t)) \cdot |\gamma'(t)| \, dt$$

es: $f(z) = 1$

$\gamma : [a,b] \to \mathbb{C}$

$\gamma(t) = x(t) + i y(t)$

$$\int_\gamma 1 \, ds = \int_a^b |\gamma'(t)| \, dt$$ <u>lunghezza della curva</u>

$$\int_\gamma 1 \, dz = \int_a^b \gamma'(t) \, dt = \int_a^b x'(t) \, dt + i \int_a^b y'(t) \, dt =$$

$$= \big(x(b) - x(a)\big) + i \big(y(b) - y(a)\big) =$$

$$= \gamma(b) - \gamma(a)$$