Cyber-Physical Systems

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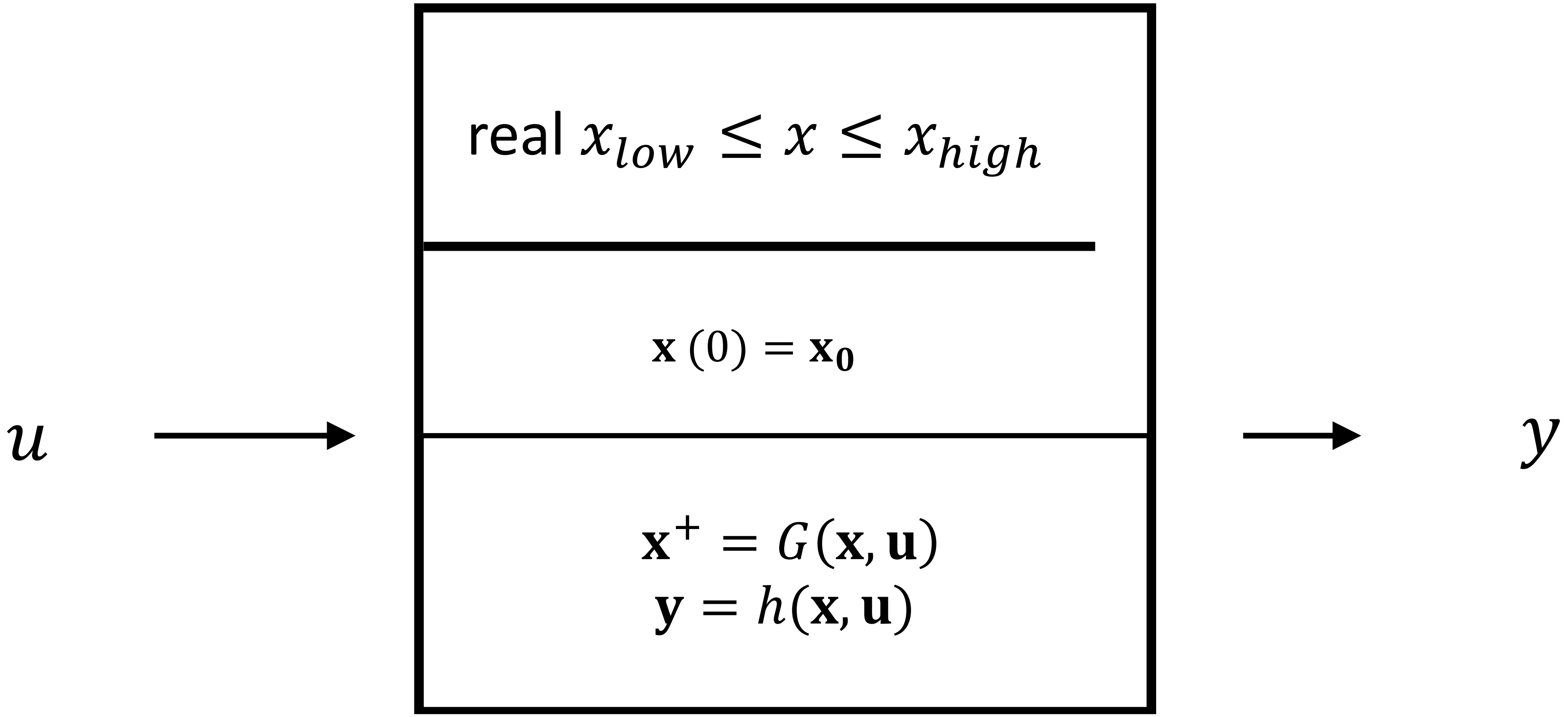
Lecture 3: Concurrent Modeling

[Many Slides due to J. Deshmukh, Toyota]

Difference Equation



u

real $x_{low} \le x \le x_{high}$

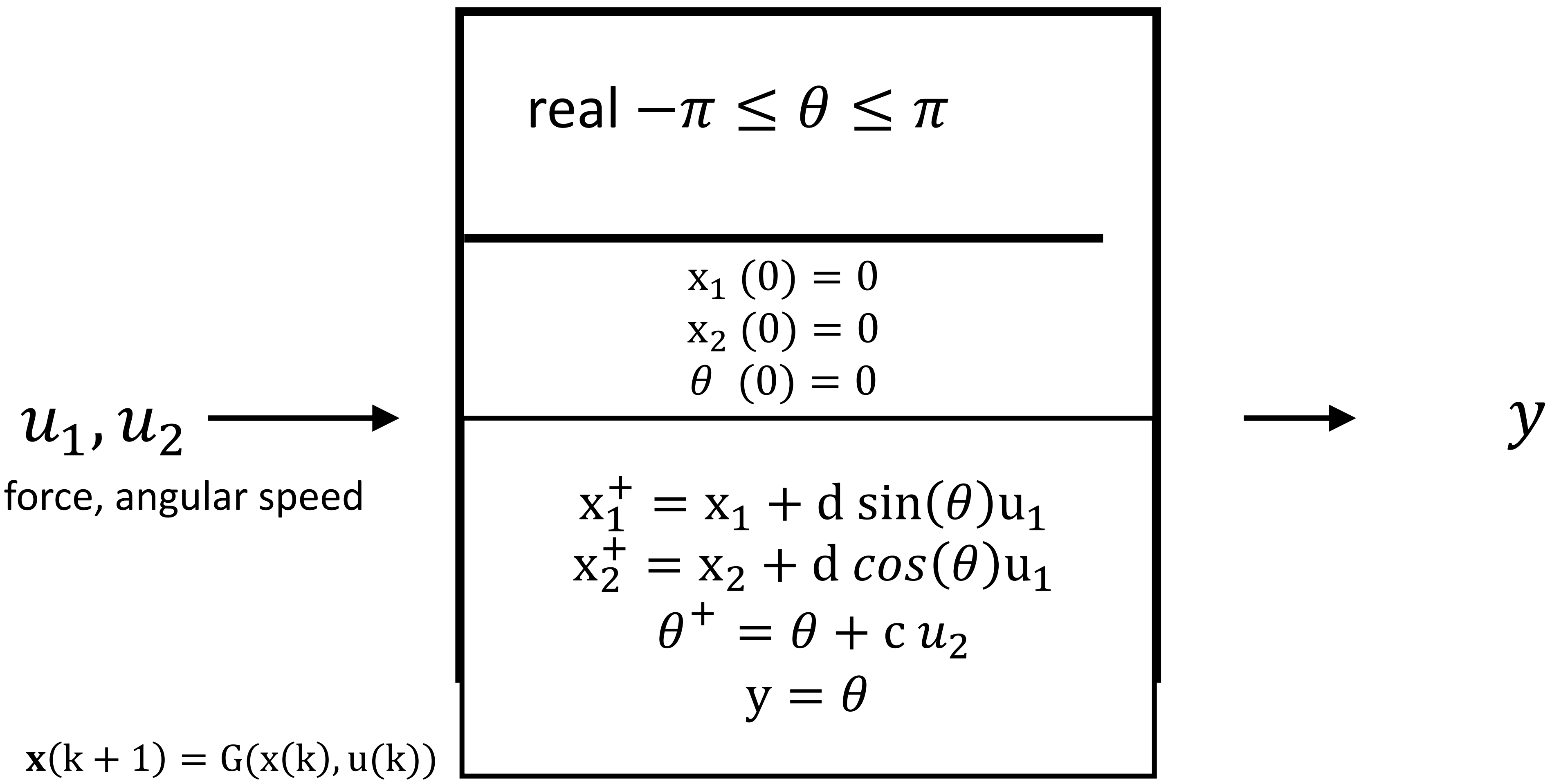
$\mathbf{x}\,(0) = \mathbf{x_0}$

$$\mathbf{x}^+ = G(\mathbf{x},\mathbf{u})$$
$$\mathbf{y} = h(\mathbf{x},\mathbf{u})$$

y

$\mathbf{x}(\mathrm{k}+1) = \mathrm{G}(\mathrm{x}(\mathrm{k}),\mathrm{u}(\mathrm{k}))$

Difference Equation



u_1, u_2

force, angular speed

real $-\pi \leq \theta \leq \pi$

$$\mathrm{x}_1\,(0) = 0$$
$$\mathrm{x}_2\,(0) = 0$$
$$\theta\;\;(0) = 0$$

$$\mathrm{x}_1^+ = \mathrm{x}_1 + \mathrm{d}\,\sin(\theta)\mathrm{u}_1$$
$$\mathrm{x}_2^+ = \mathrm{x}_2 + \mathrm{d}\,cos(\theta)\mathrm{u}_1$$
$$\theta^+ = \theta + \mathrm{c}\,u_2$$
$$\mathrm{y} = \theta$$

y

$\mathbf{x}(\mathrm{k}+1) = \mathrm{G}(\mathrm{x}(\mathrm{k}), \mathrm{u}(\mathrm{k}))$

Functional Components

1. *Classical* model of computation: Functional or Transformational Programs

- Start from a given input,
- Produce a certain output and then **terminate**
- Desired functionality can be described by a mathematical function
- Emphasis is on data computation

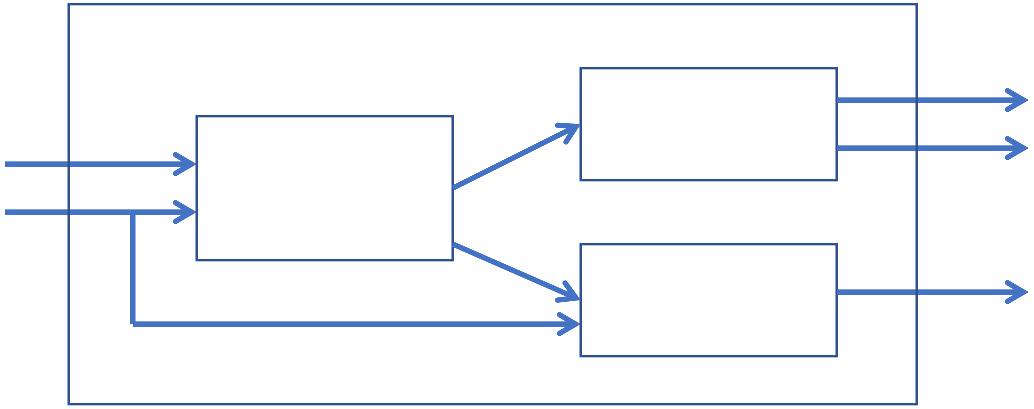
Reactive Components

2. Reactive Programs:

    - It maintains and internal state
    - Continuously interact with the environment at a rate decided by the environment
    - Emphasis is on system-environment interaction; e.g. airline autopilot, mail-servers, etc.

Synchronous Models

- All components execute in a sequence of rounds in lock-step



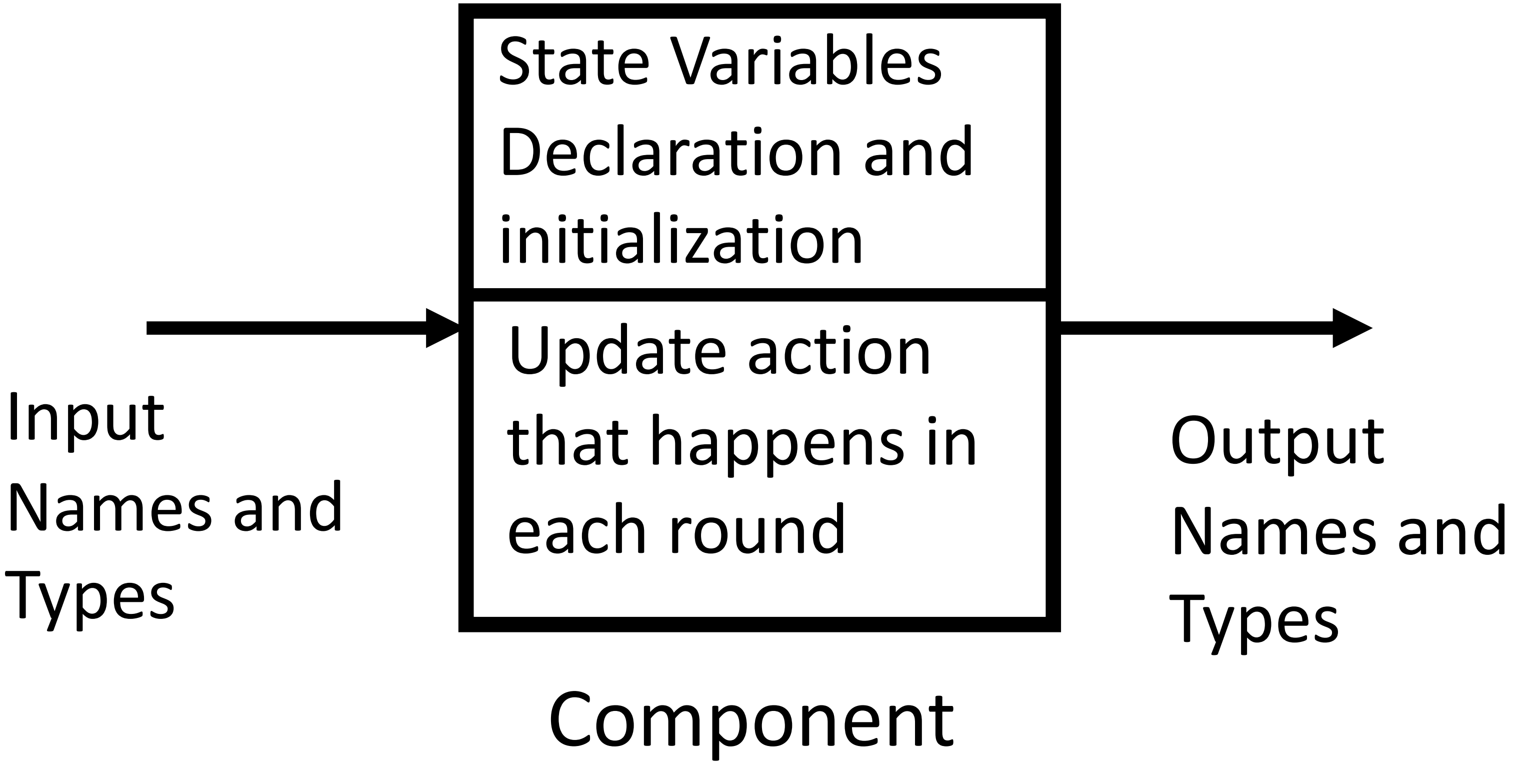
- Example:
  - Components in a digital hardware circuit with a central global clock
  - Fixed-step Simulation Models of Discrete Components in Simulink

Synchronous languages

- Benefit: system design is simpler if we use a simple round-based computation

- Challenge: How do we ensure synchronous execution when components may execute on different hardware?

Simple Representation of a Synchronous Component



Input Names and Types

State Variables Declaration and initialization

Update action that happens in each round

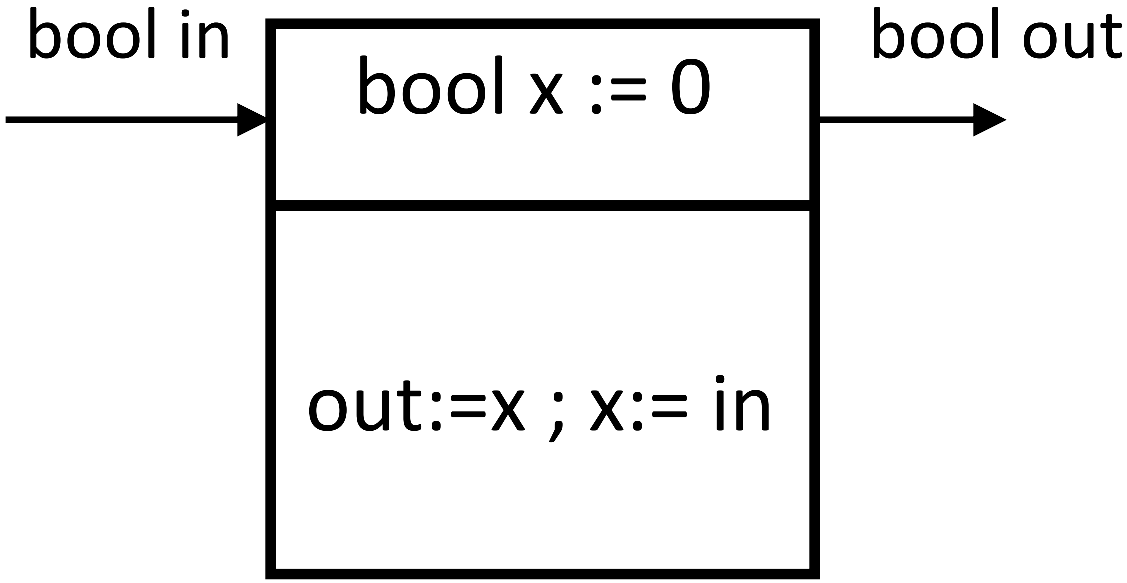
Output Names and Types

Component

Simplest synchronous component: delay

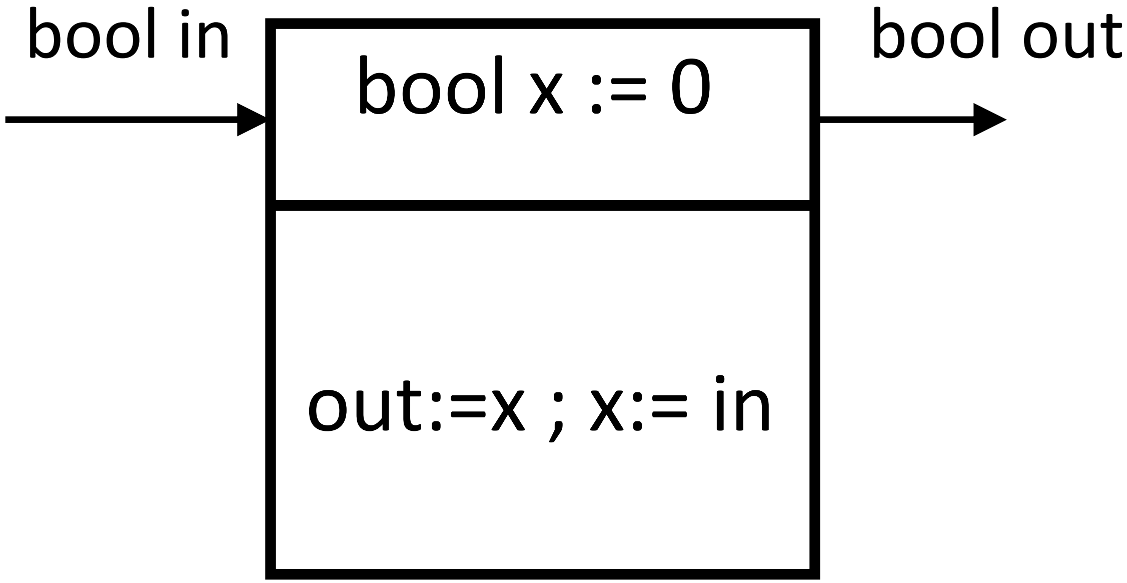
(Boolean = { 0, 1})

- Input variable: in of type Boolean
- Output variable: out of type Boolean
- State variable: x of type Boolean, initialized to 0
- In each round, component updates output from the state and state from input



Execution of “Delay”

- Initialize state to 0
- Repeatedly execute rounds
- In each round:
  - Choose value for input (provided from environment, e.g. by user)
  - Execute update code



$$0 \xrightarrow{1\,/\,0} 1 \xrightarrow{1\,/\,1} 1 \xrightarrow{0\,/\,1} 0 \xrightarrow{0\,/\,0} 0 \xrightarrow{1\,/\,0} 1 \quad \dashrightarrow$$

Synchrony hypothesis

- Time needed to execute update is negligible compared to arrival times between consecutive inputs

- Synchronous execution is a *logical abstraction*
  - *Execution time of update code is 0*
  - *Production of outputs, updates to state and arrival of inputs happen instantaneously*

- With multiple components, assume all execute synchronously and simultaneously

Let’s Formalize an SRC

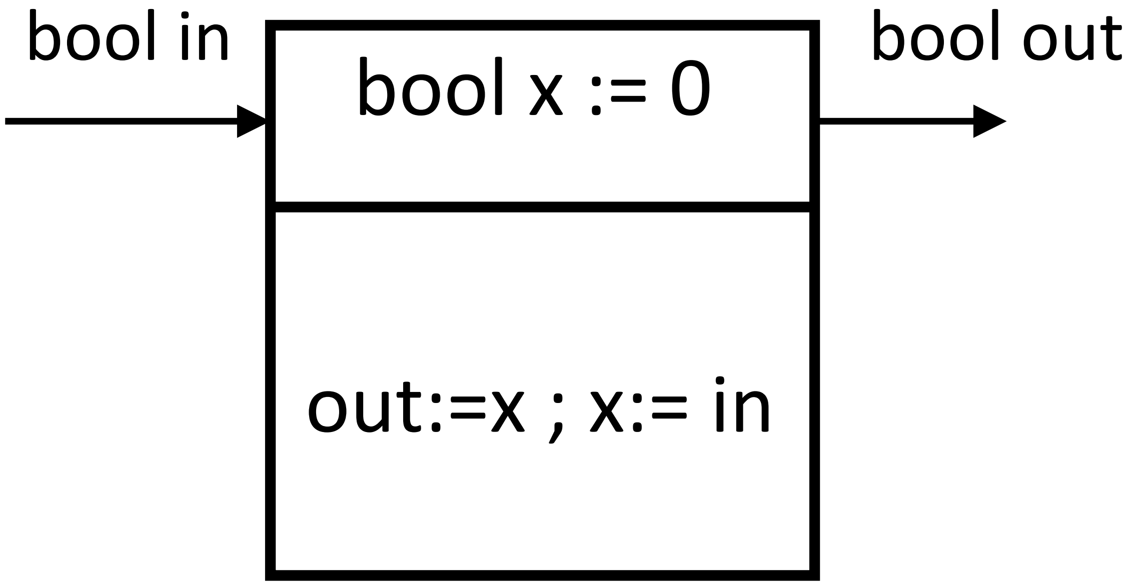
- SRC is defined as a tuple: $(Q_I, Q_x, Q_o, \llbracket init \rrbracket, \llbracket react \rrbracket)$, where:

Symbol	Designation	Examples
Q_I	Set of Inputs	$\{\text{bool } in\}$
Q_X	Set of State Variables	$\{bool\ x\ \}$
Q_o	Set of Outputs	$\{bool\ out\}$
$\llbracket init \rrbracket$	Set of initial States	$x \coloneqq 0$
$\llbracket react \rrbracket$	Set of Updates	$out \coloneqq x$ $x \coloneqq in$

Semantics of updates & initialization

- Let the set of input, output, and state values be Q_I, Q_O, Q_X
- Semantics of the initialization function:
  - At time/round 0, maps the state variables to some specified value (or values) in Q_X
- Semantics of the update function (some sequence of conditionals and assignments):
  - A set R of transitions where each transition is of the form: $q \xrightarrow{i/o} \mathrm{q}'$, where q is the old value of the state variables, q' is the new value of the state variables, i is the value of the input in that round, and o is the value of the output
  - R is a subset of $Q_X \times Q_I \times Q_O \times Q_X$

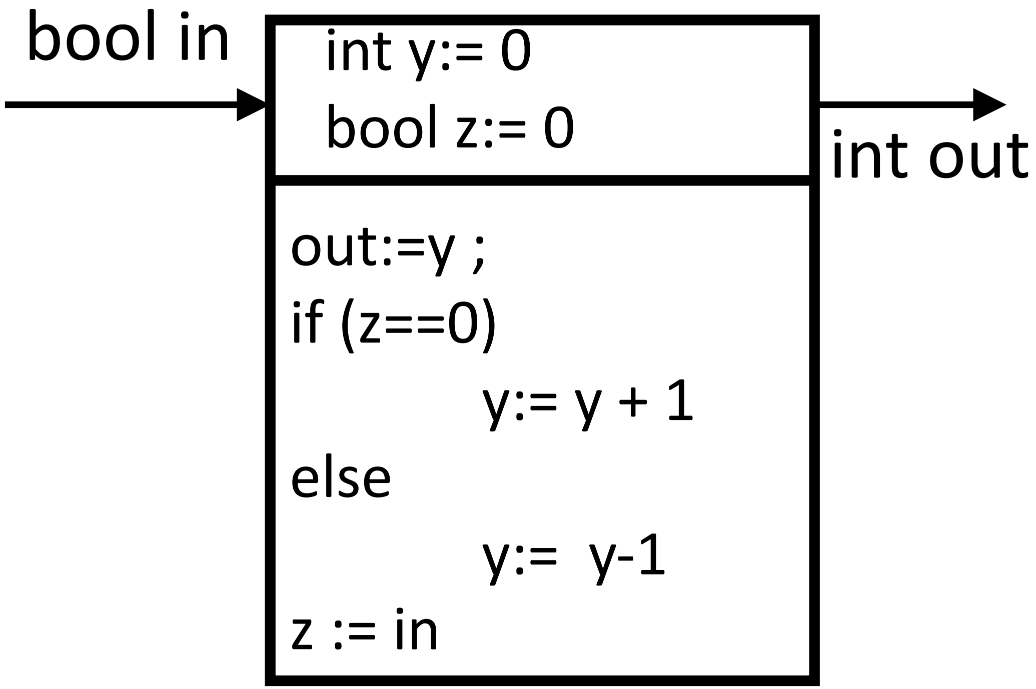
What are the Q_I, Q_O, Q_X for these SRCs?



```
bool x := 0
```

```
out:=x ; x:= in
```

$Q_I = \{0,1\}, Q_X = \{0,1\}, Q_O = \{0,1\}$

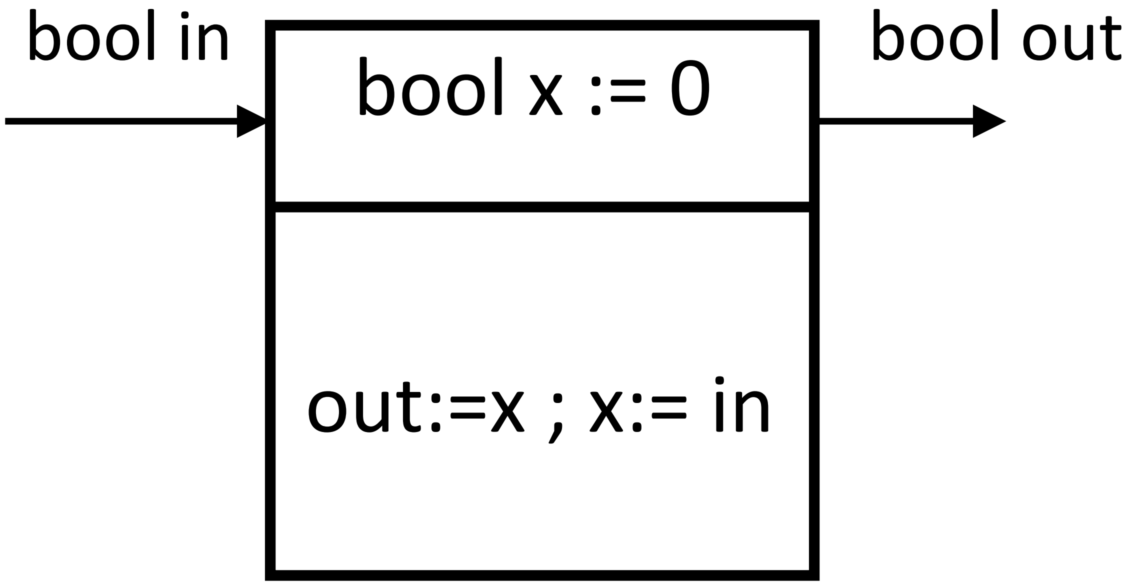


```
int y:= 0
bool z:= 0
```

```
out:=y ;
if (z==0)
        y:= y + 1
else
        y:=  y-1
z := in
```

$Q_I = \{0,1\}, Q_X = \text{int}\times\{0,1\}\,, Q_O = \text{int}$

Transitions for Delay



bool in

bool x := 0

out:=x ; x:= in

bool out

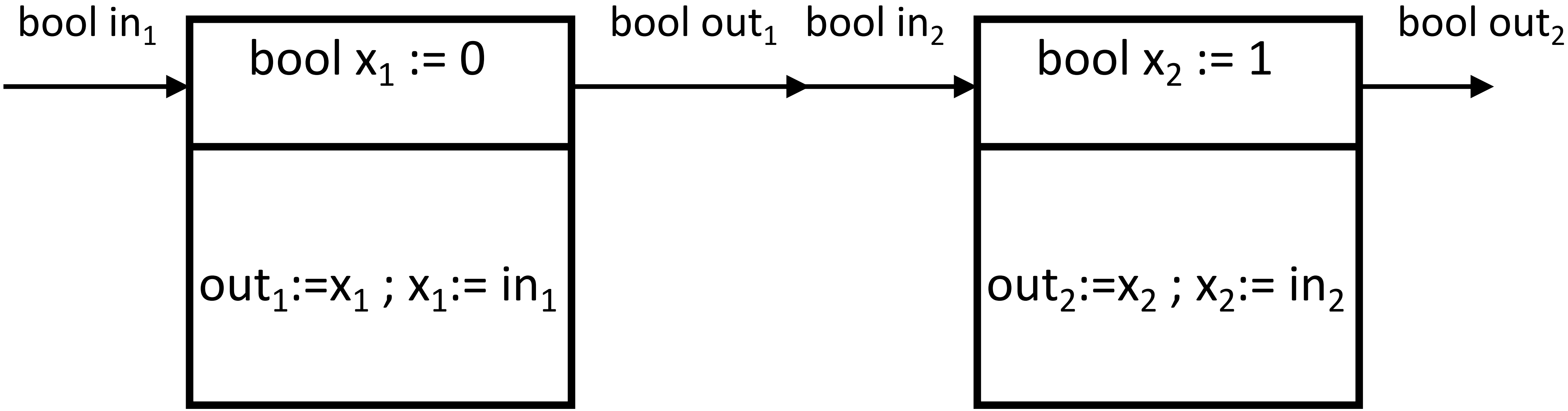
$$0 \xrightarrow{0/0} 0$$

$$0 \xrightarrow{1/0} 1$$

$$1 \xrightarrow{0/1} 0$$

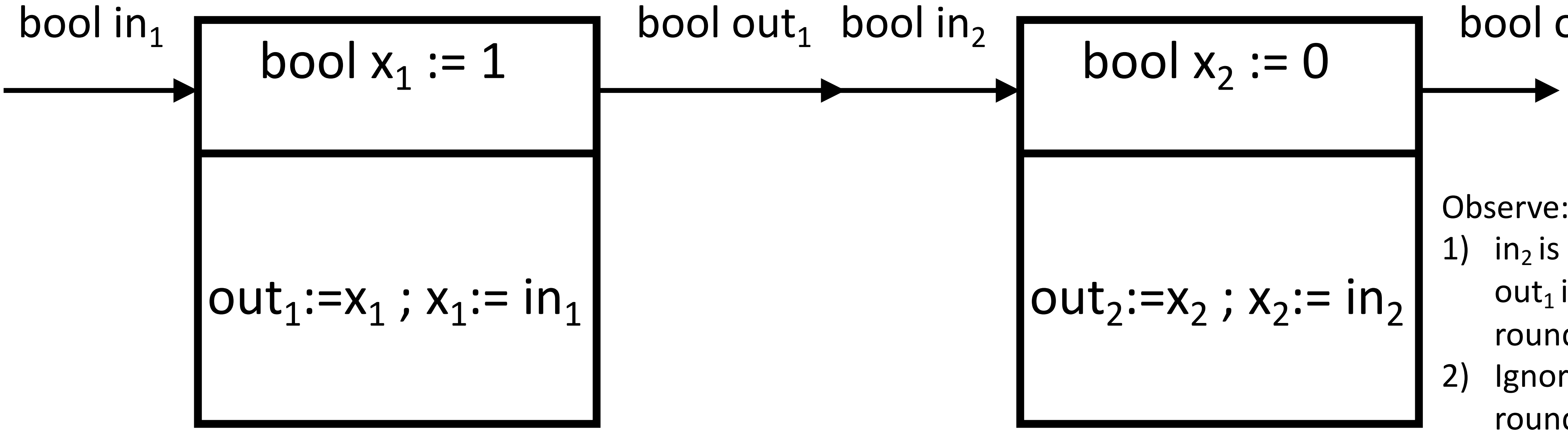
$$1 \xrightarrow{1/1} 1$$

Composition of Synchronous Components



Delay sequentially composed with Delay

Composition of Synchronous Components



bool in_1

bool x_1 := 1

out_1:=x_1 ; x_1:= in_1

bool out_1

bool in_2

bool x_2 := 0

out_2:=x_2 ; x_2:= in_2

bool out_2

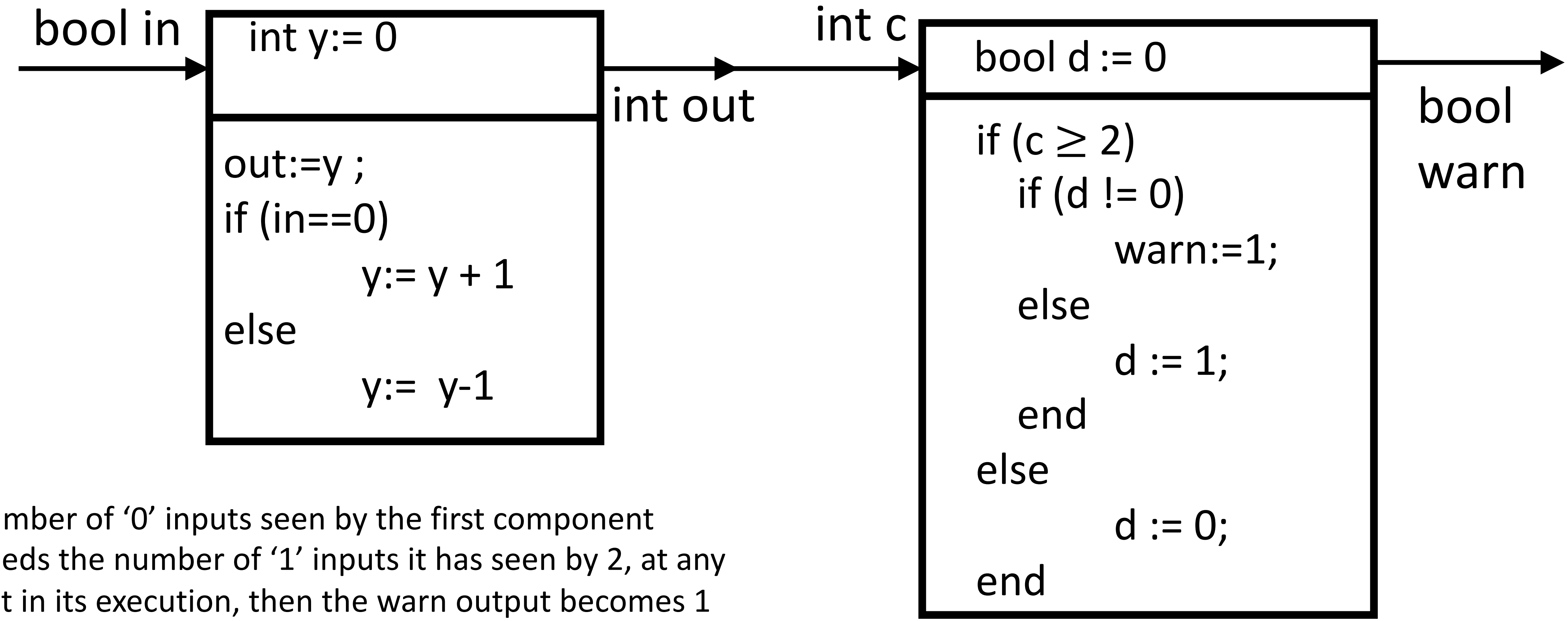
$$1 \xrightarrow{1\,/\,1} 1 \xrightarrow{0\,/\,1} 0 \xrightarrow{0\,/\,0} 0 \xrightarrow{1\,/\,0} 1 \xrightarrow{1\,/\,1} 1 \quad \dashrightarrow \quad \text{Delay 1}$$

$$0 \xrightarrow{1\,/\,0} 1 \xrightarrow{1\,/\,1} 1 \xrightarrow{0\,/\,1} 0 \xrightarrow{0\,/\,0} 0 \xrightarrow{1\,/\,0} 1 \quad \dashrightarrow \quad \text{Delay 2}$$

Observe:

1) in_2 is the same as out_1 in every round
2) Ignoring first 2 rounds, outputs of d2 are the inputs to d1 delayed by 2 rounds

What does this model achieve?



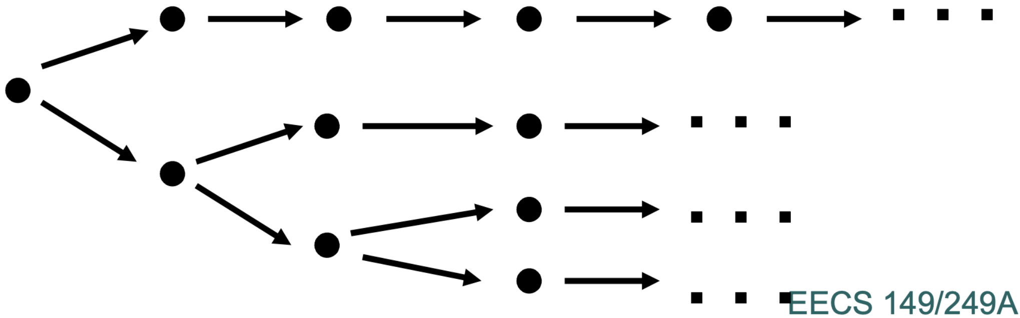
```
int y:= 0
---
out:=y ;
if (in==0)
        y:= y + 1
else
        y:=  y-1
```

```
bool d := 0
---
if (c ≥ 2)
   if (d != 0)
          warn:=1;
   else
          d := 1;
   end
else
          d := 0;
end
```

If number of ‘0’ inputs seen by the first component exceeds the number of ‘1’ inputs it has seen by 2, at any point in its execution, then the warn output becomes 1

Deterministic Component

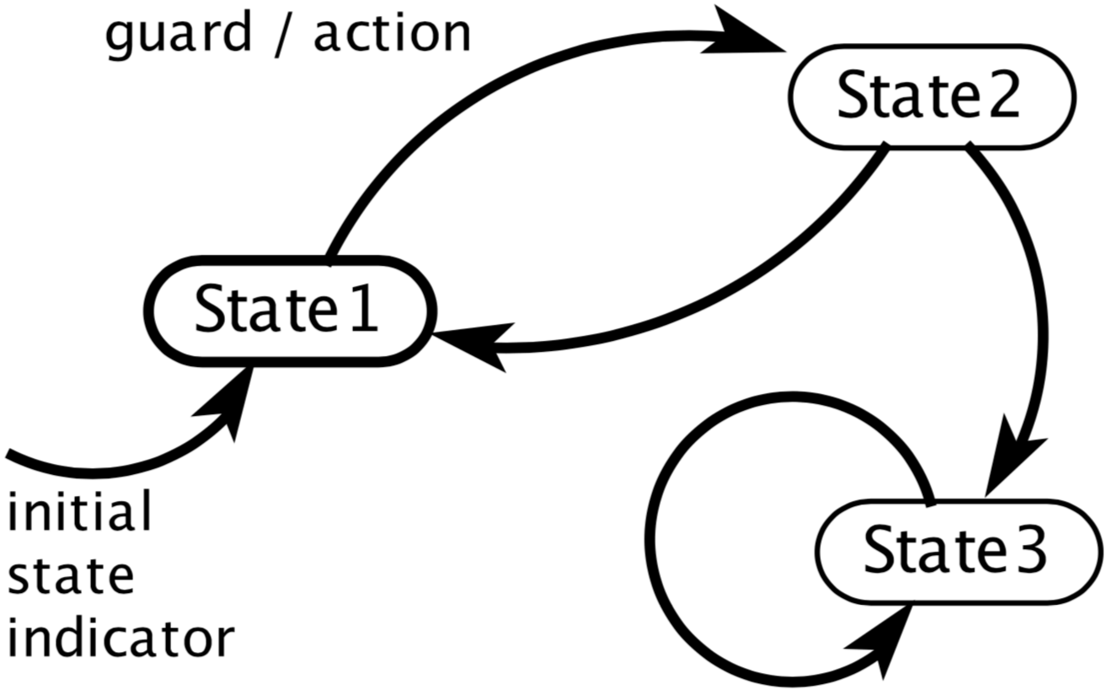
- An SRC is deterministic if:
  - It has a single initial state
  - Updates ensure that for every state q and input i, there is a unique state q' and output o such that (q, i, o, q') is a transition
- Determinism means for same input sequence, you get same state/output sequence every single time
- Note:
  - Nondeterminism is useful for modeling uncertainty/unknown and compactness



  - It is not the same as probabilistic/random choice!

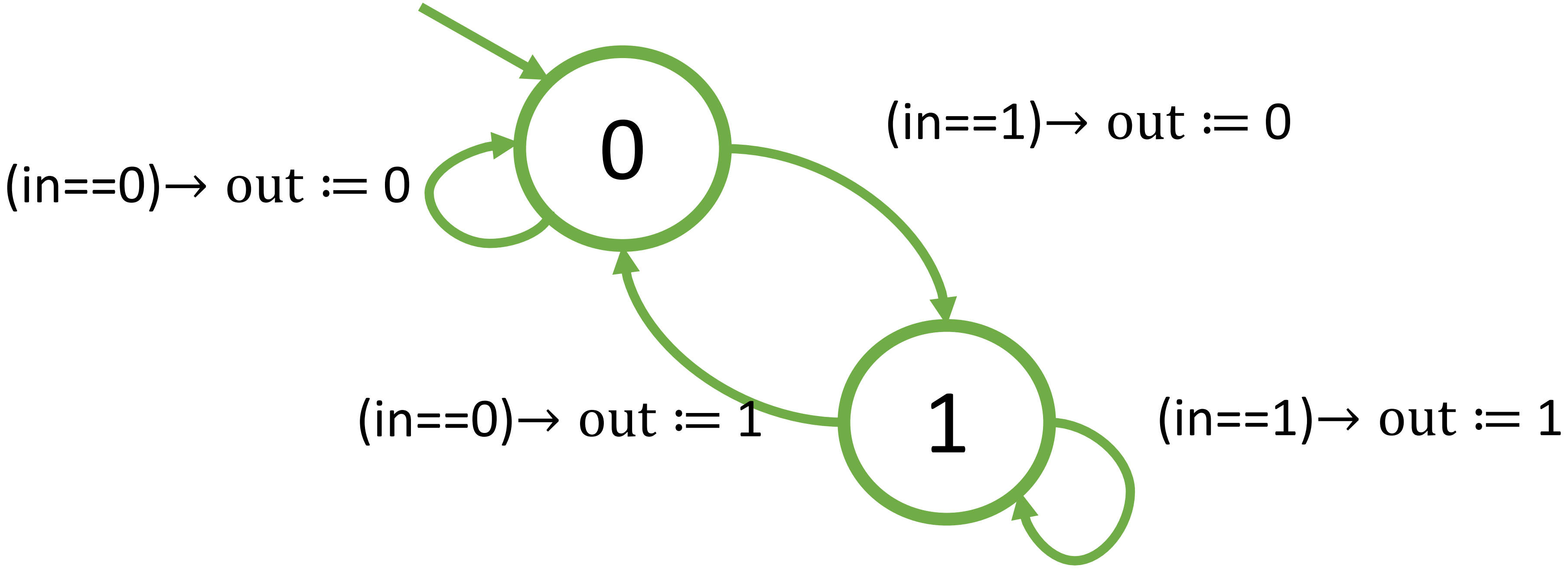
Extended State Machines

- Commonly used to describe behavior of MBD models

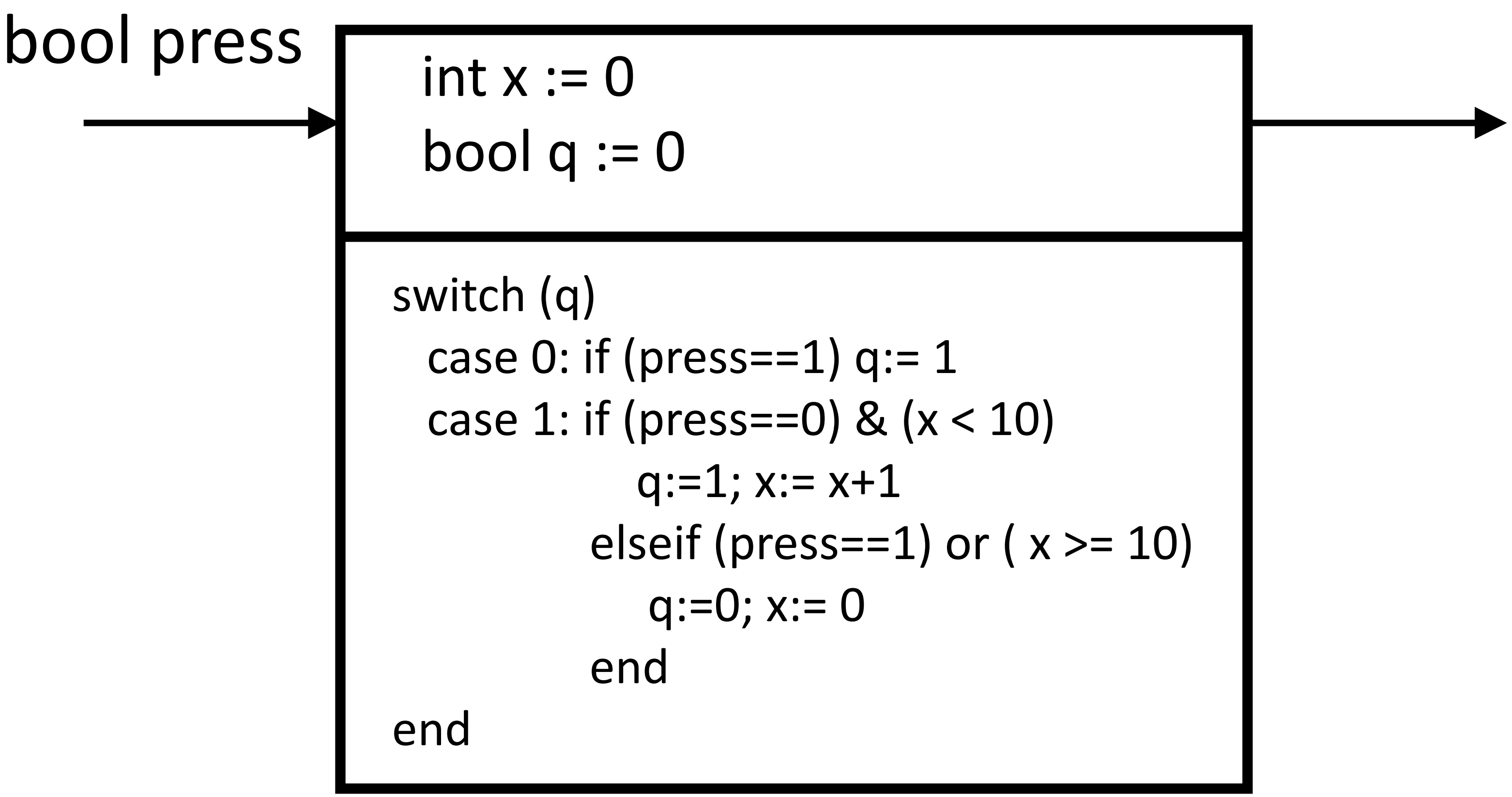


Extended State Machines

- Does this ESM remind you of something?



Component Switch: What does this do?

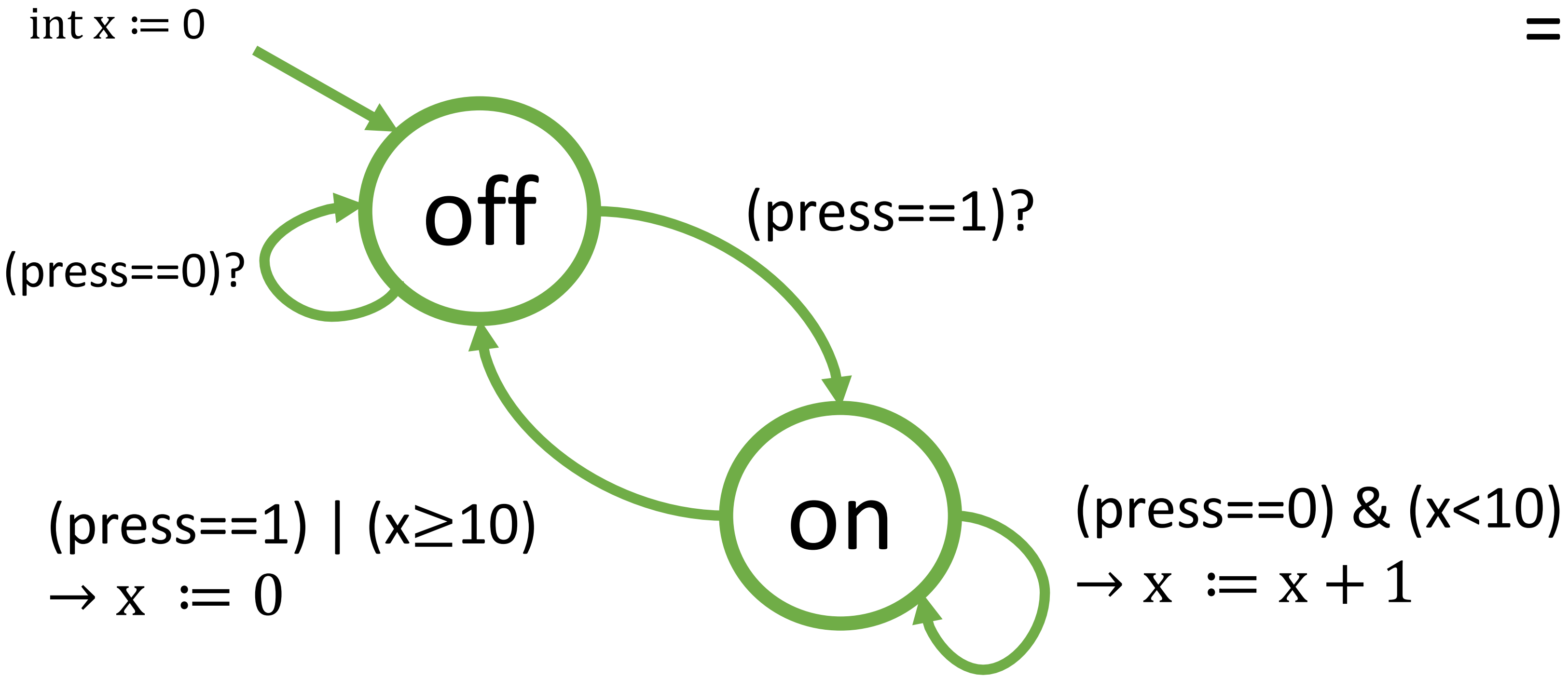


```
int x := 0
bool q := 0
```

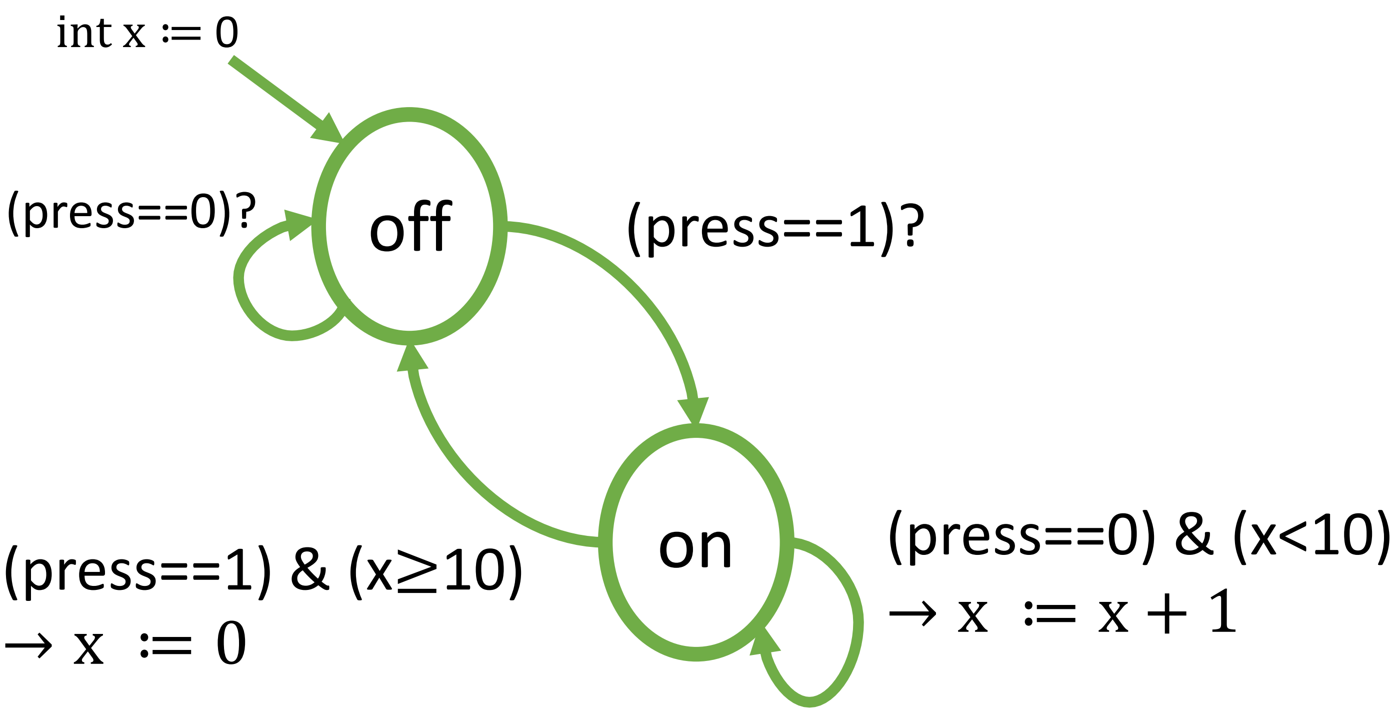
```
switch (q)
  case 0: if (press==1) q:= 1
  case 1: if (press==0) & (x < 10)
               q:=1; x:= x+1
             elseif (press==1) or ( x >= 10)
               q:=0; x:= 0
             end
end
```

ESM corresponding to Switch SRC

q = 0 : off
  = 1 : on

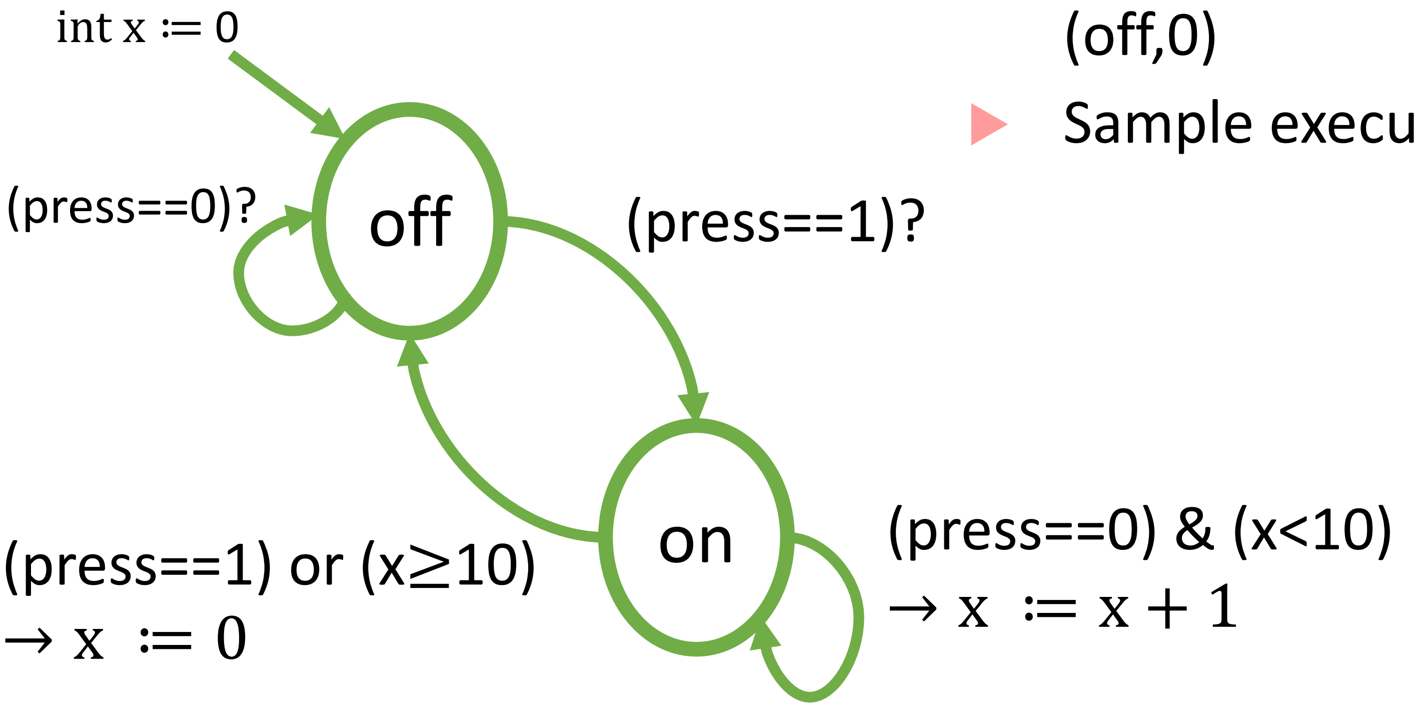


ESM notation



- Implicit variable called “mode” that is a *discrete* state variable over some finite enumeration. Here: {on, off}
- SRC transition may correspond to mode-switch
- Each mode-switch has guard/update. Example:
  - Guard: (press==0) & (x<10) and
  - Update: x:= x+1

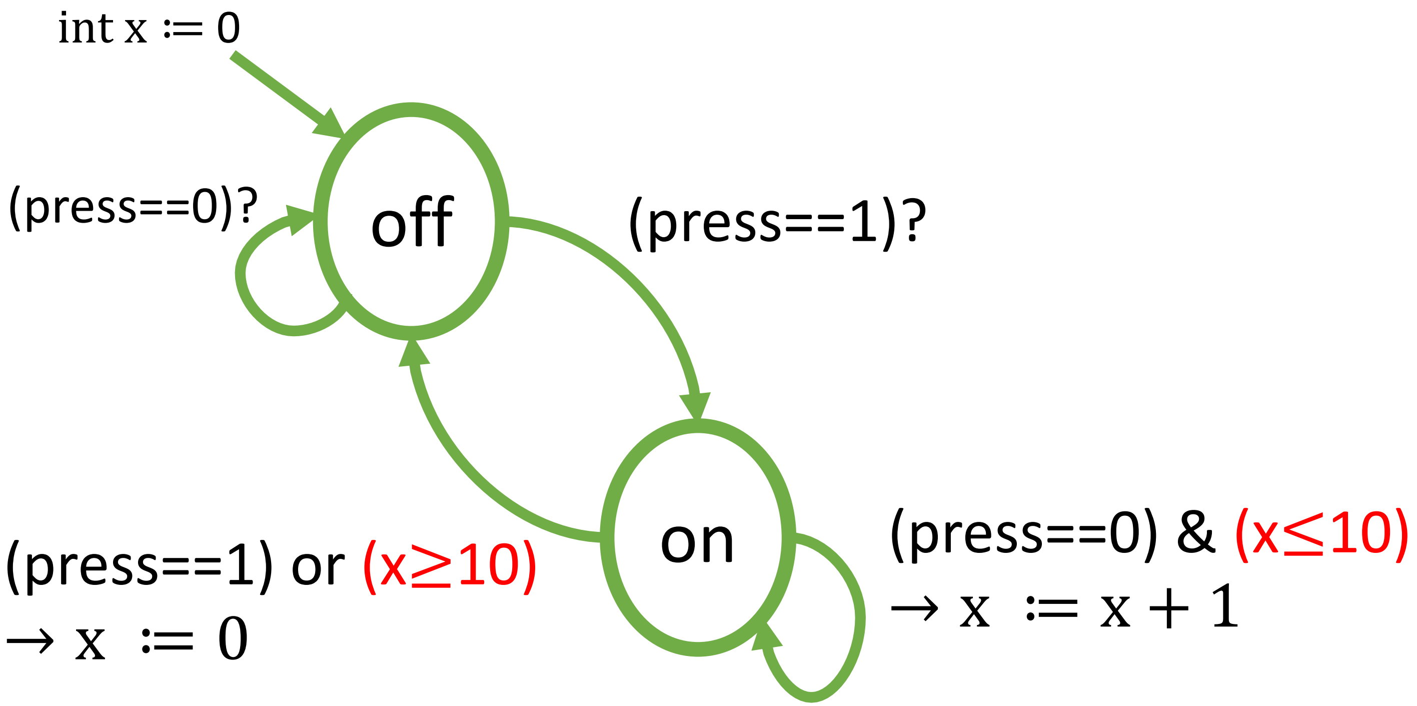
ESM execution



- Start in mode off; initial state = (off,0)
- Sample executions:

$$\begin{array}{cc} (off,0) & (off,0) \\ \downarrow 0 & \downarrow 0 \\ (off,0) & (off,0) \\ \downarrow 1 & \downarrow 1 \\ (on,0) & (on,0) \\ \downarrow 0 & \downarrow 0 \\ (on,1) & (on,1) \\ \vdots & \vdots \\ \downarrow 0 & \downarrow 0 \\ (on,10) & (on,5) \\ \downarrow 0 & \downarrow 1 \\ (off,0) & (off,0) \end{array}$$

ESM transitions could be nondeterministic!

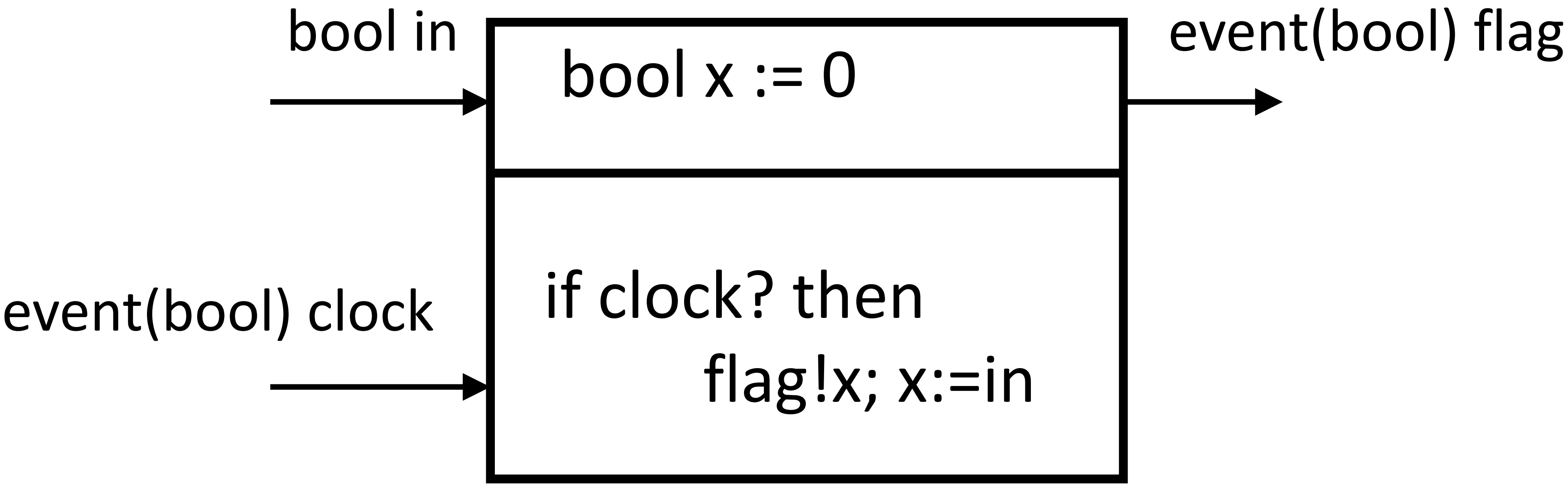


Event-triggered Components

- What to do if we want some components to *not* participate in some rounds?
- Event is a special input/output variable, which can be *absent* or *present*
- Event variable has value only if it is *present*
- Syntax:

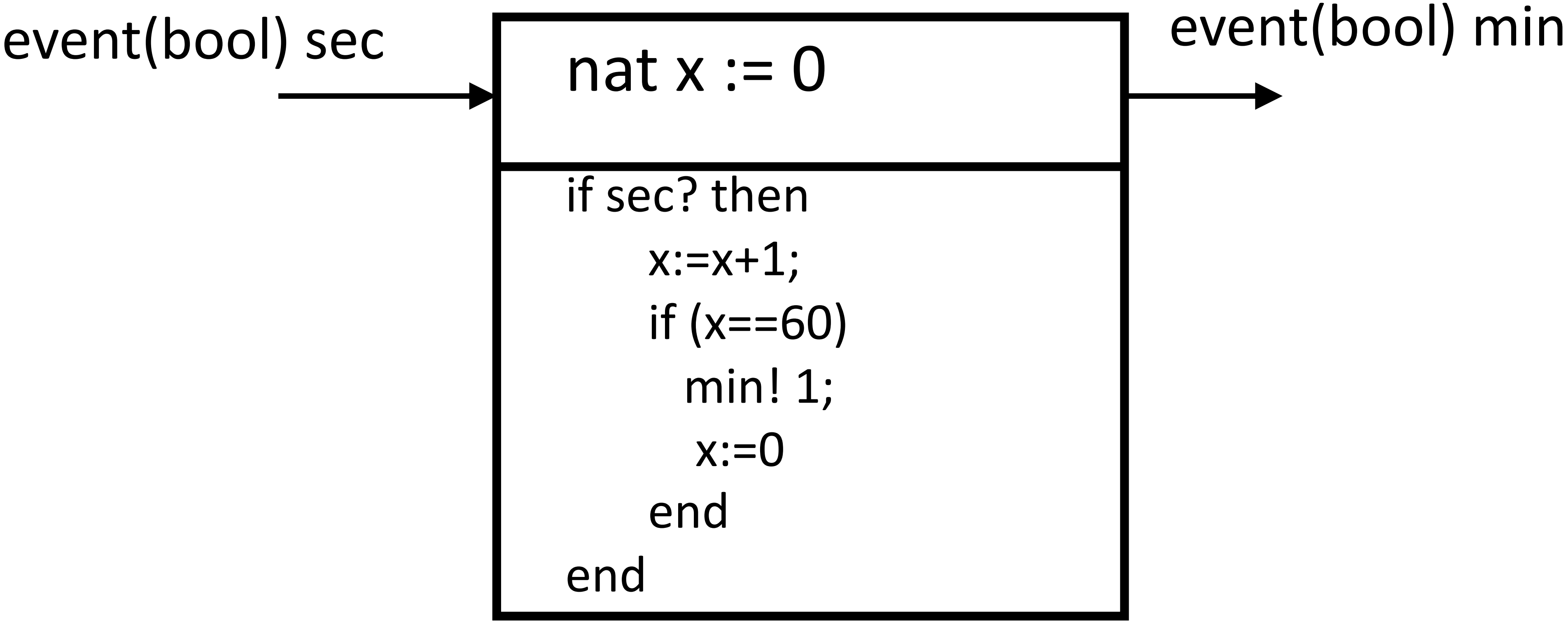
e?	True if e is present
e!a	e gets the value of the assignment a

Event-triggered Copy



Event-triggered Components

- No need to execute in a round where triggering events are absent



event(bool) sec

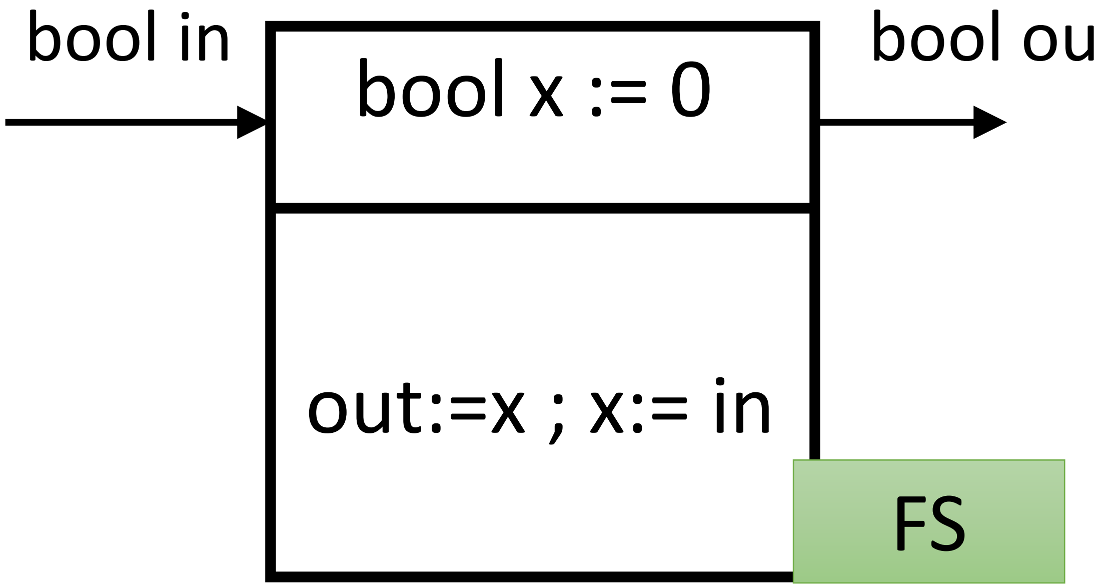
event(bool) min

```
nat x := 0
```

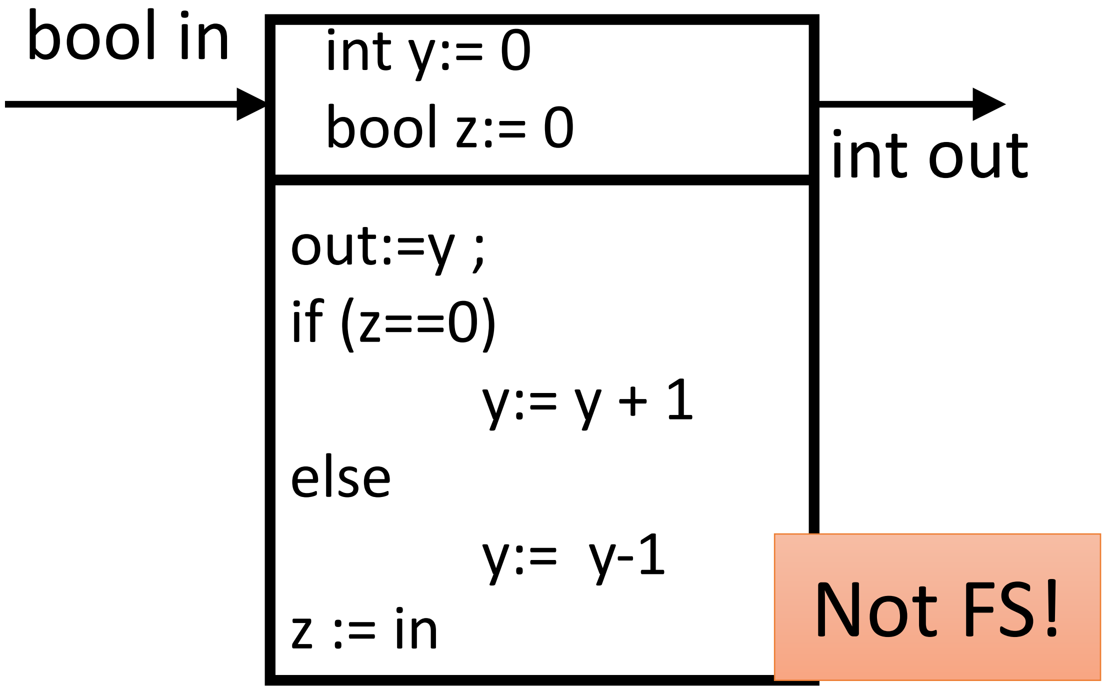
```
if sec? then
     x:=x+1;
     if (x==60)
        min! 1;
         x:=0
     end
end
```

Finite-state Components

- Component is finite state if all variables are over finite types

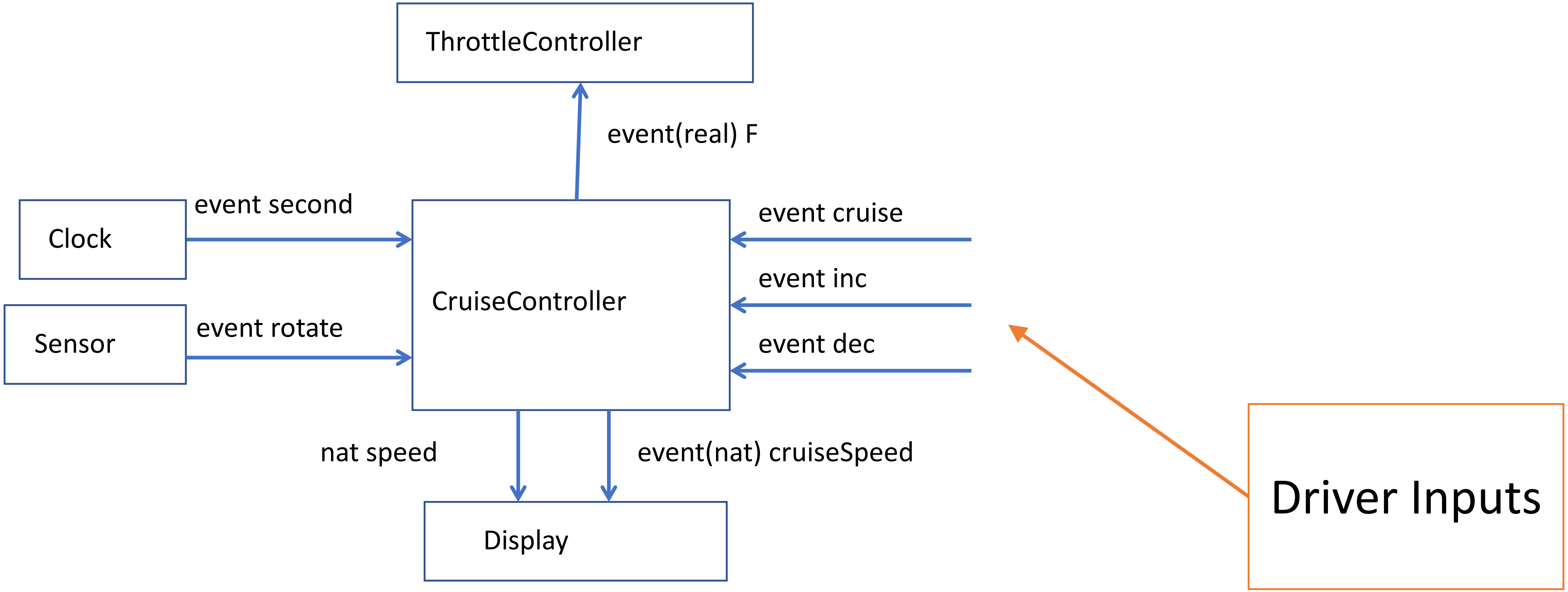


```
bool x := 0
out:=x ; x:= in
```



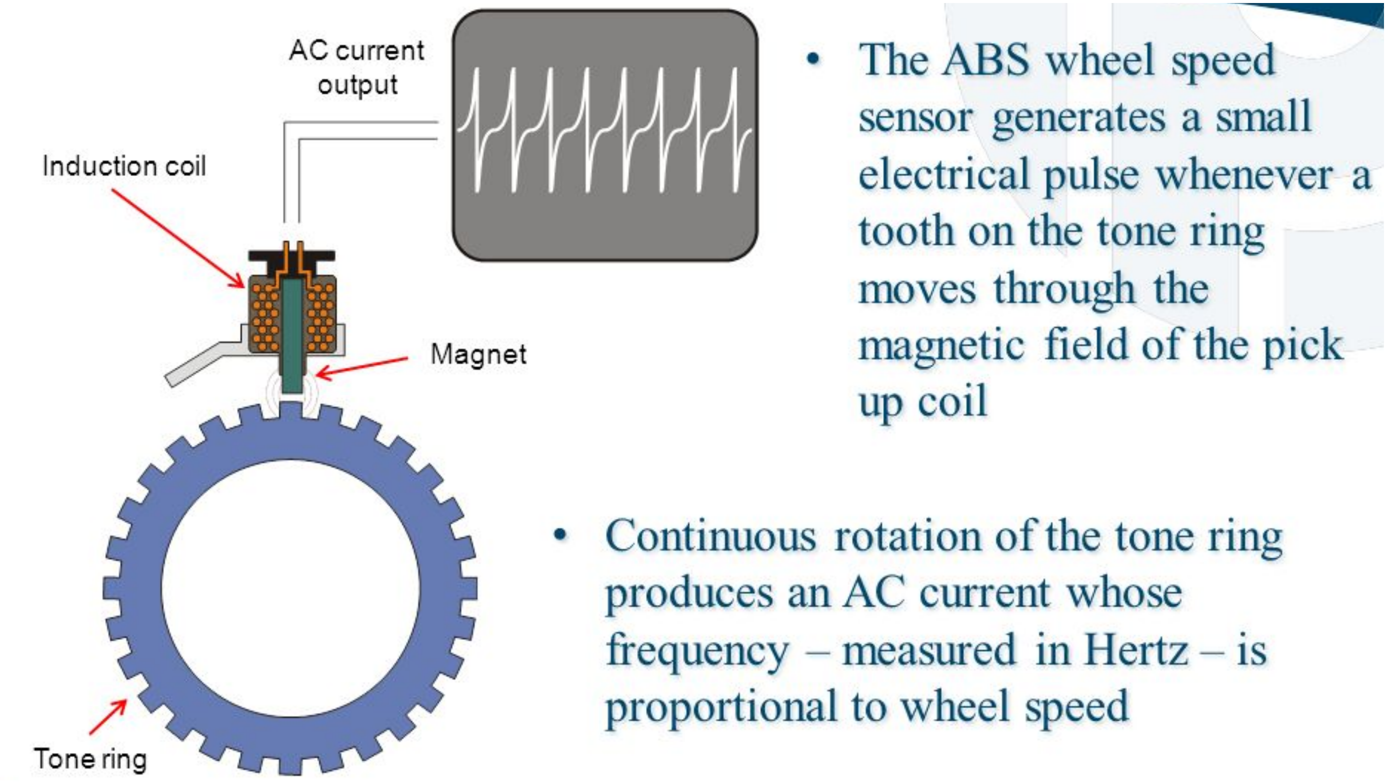
```
int y:= 0
bool z:= 0
out:=y ;
if (z==0)
        y:= y + 1
else
        y:=  y-1
z := in
```

Cruise Controller Example



Sensors

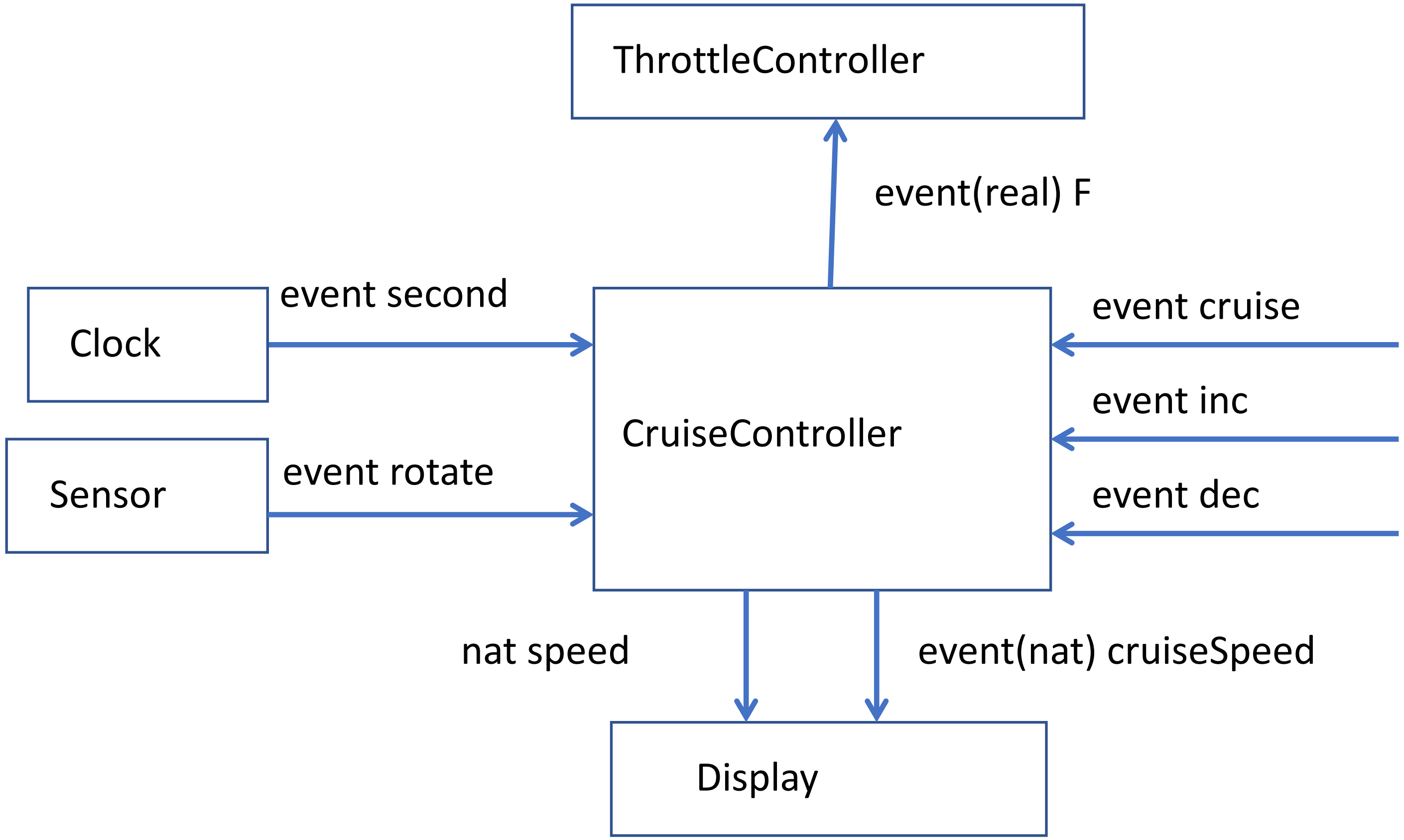
- Rotation Sensor: Wheel speed sensor or vehicle speed sensor
- Type of a tachometer
- Counts number of rotations per second and as the wheel radius is known, can compute the linear speed of the car



- The ABS wheel speed sensor generates a small electrical pulse whenever a tooth on the tone ring moves through the magnetic field of the pick up coil
- Continuous rotation of the tone ring produces an AC current whose frequency – measured in Hertz – is proportional to wheel speed

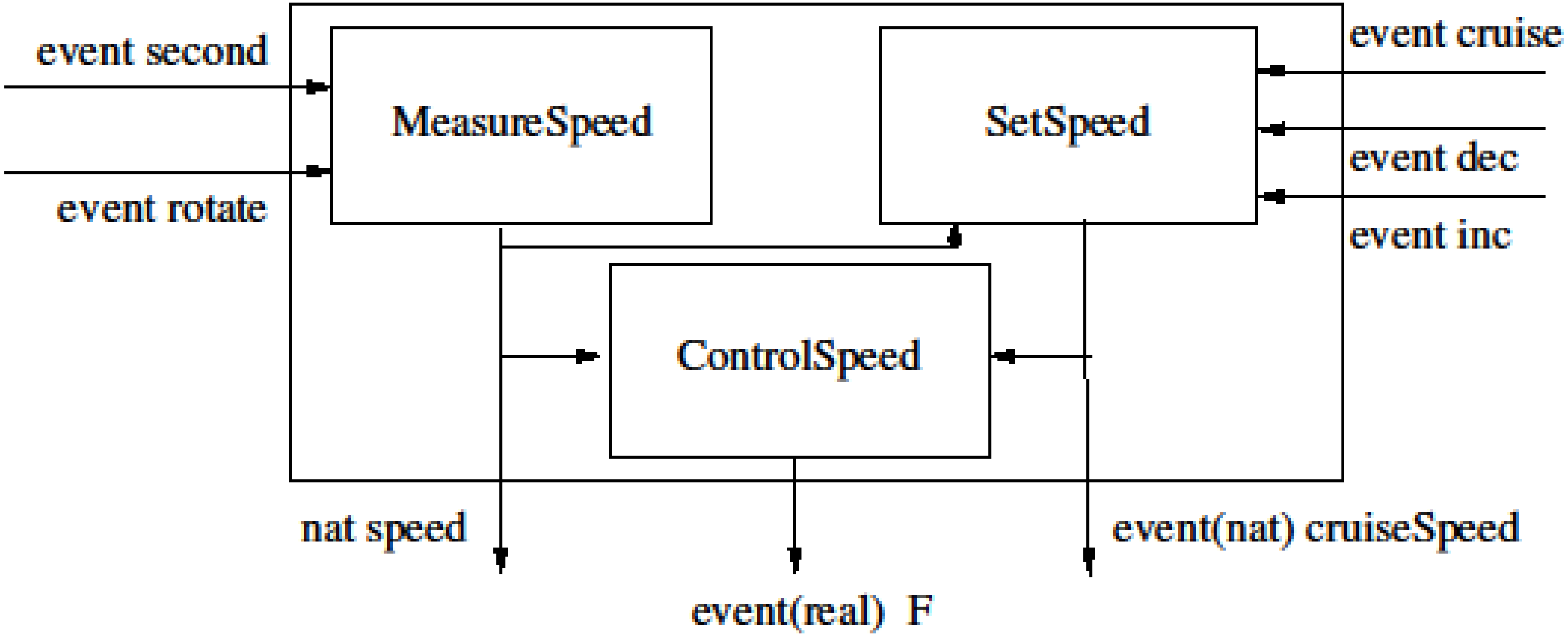
(From Porter and Chester Institute slides on Google Image Search)

Actuator

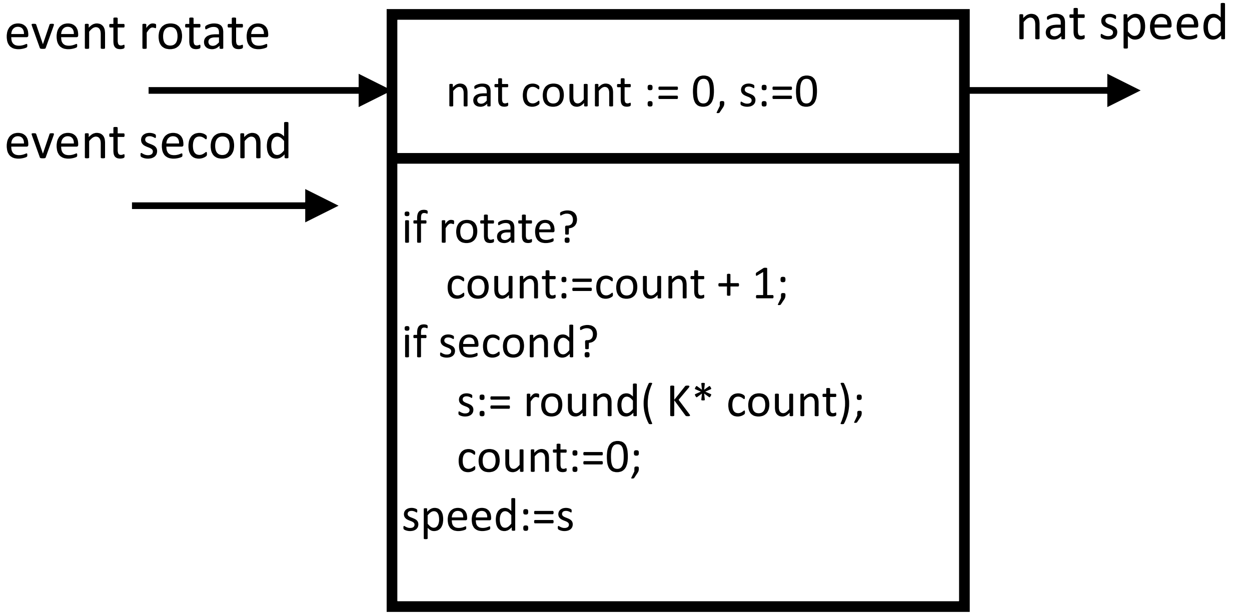


- ThrottleController is an actuator that gets a force/torque required to adjust the throttle plate which leads to tracking the desired speed

Decomposing CruiseController further



MeasureSpeed SRC



```
nat count := 0, s:=0
```

```
if rotate?
   count:=count + 1;
if second?
    s:= round( K* count);
    count:=0;
speed:=s
```

MeasureSpeed SRC

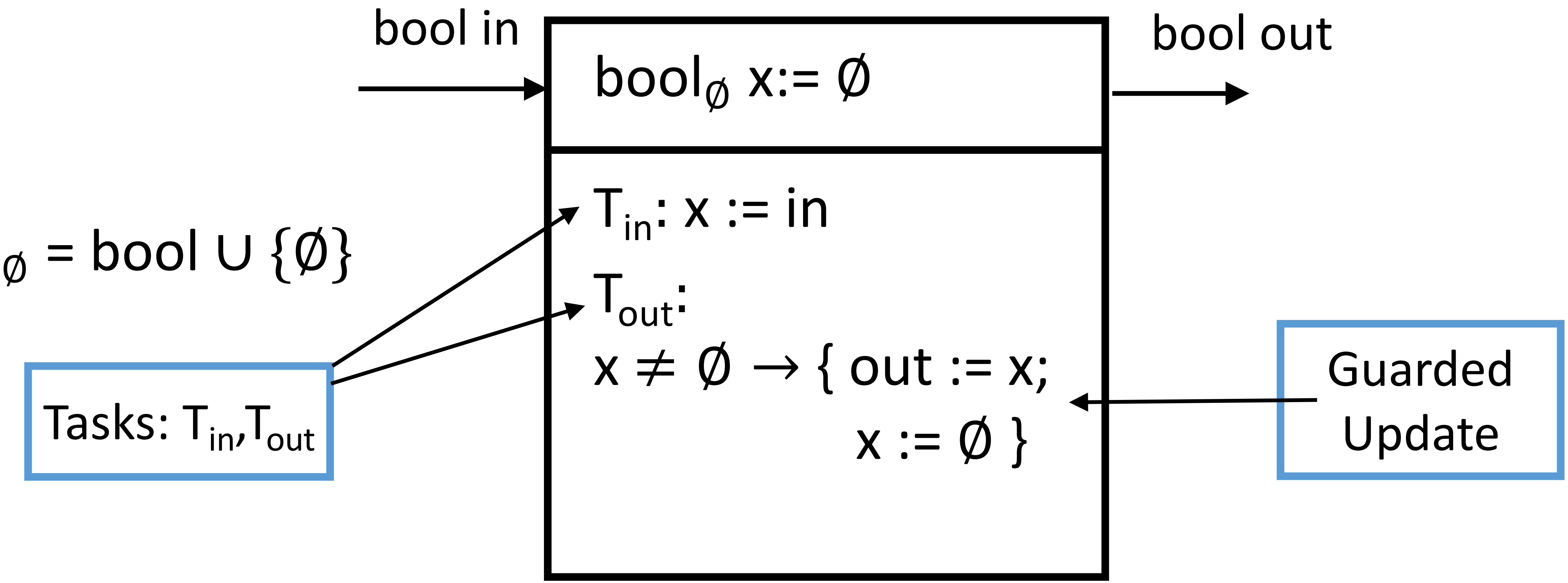
Asynchronous Components

Asynchrony

- Synchrony: All components execute in a sequence of rounds in lock-step
- Asynchrony: No lock-step computation!
- Natural model for networked, distributed communicating components executing independently and at possibly different speeds
- As there is no central, global clock, explicit coordination is required between components
- Examples:
  - Processes in distributed computation, multiple threads in any modern OS
  - Interrupt-driven processing

Asynchronous Reactive Component Example

$\text{bool}_\emptyset = \text{bool} \cup \{\emptyset\}$



bool in

$\text{bool}_\emptyset\ x := \emptyset$

bool out

T_{in}: $x := in$

T_{out}:

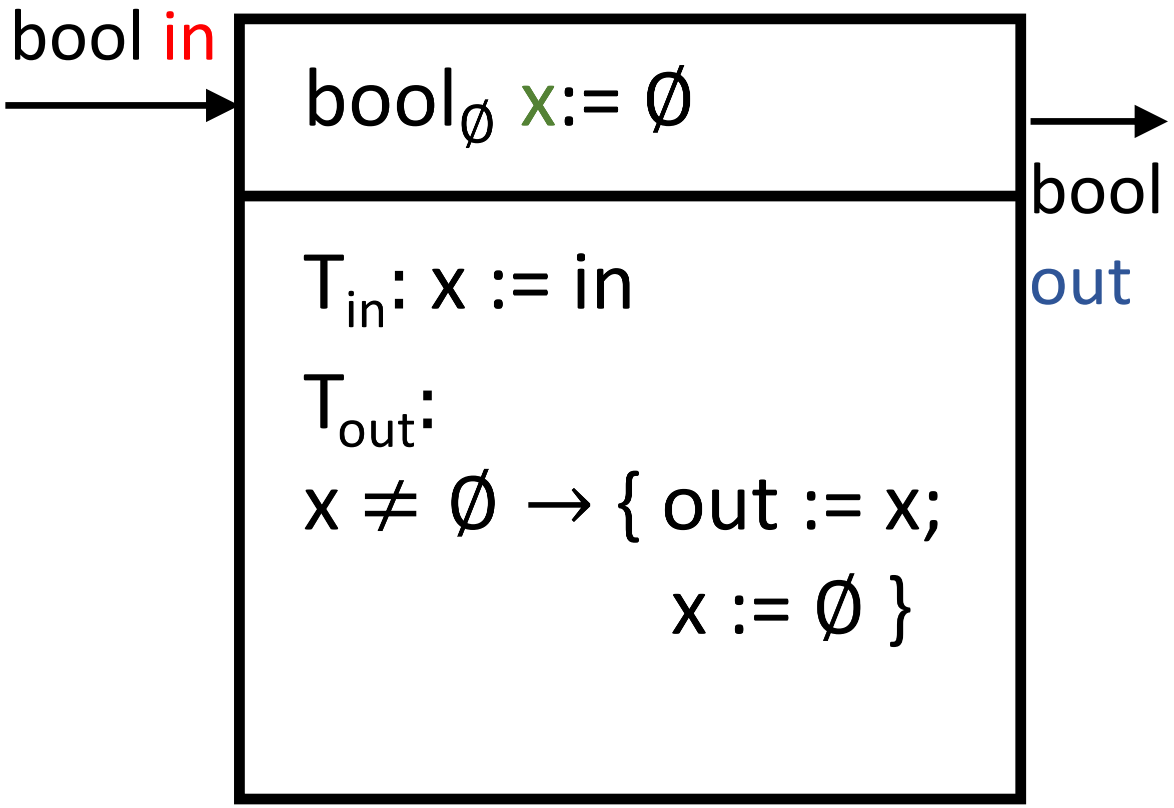
$x \neq \emptyset \rightarrow \{\ out := x;$

$x := \emptyset\ \}$

Tasks: T_{in},T_{out}

Guarded Update

Asynchronous Reactive Component



bool in

$\text{bool}_{\emptyset}$ x:= ∅

T_{in}: x := in

T_{out}:

x ≠ ∅ → { out := x;
x := ∅ }

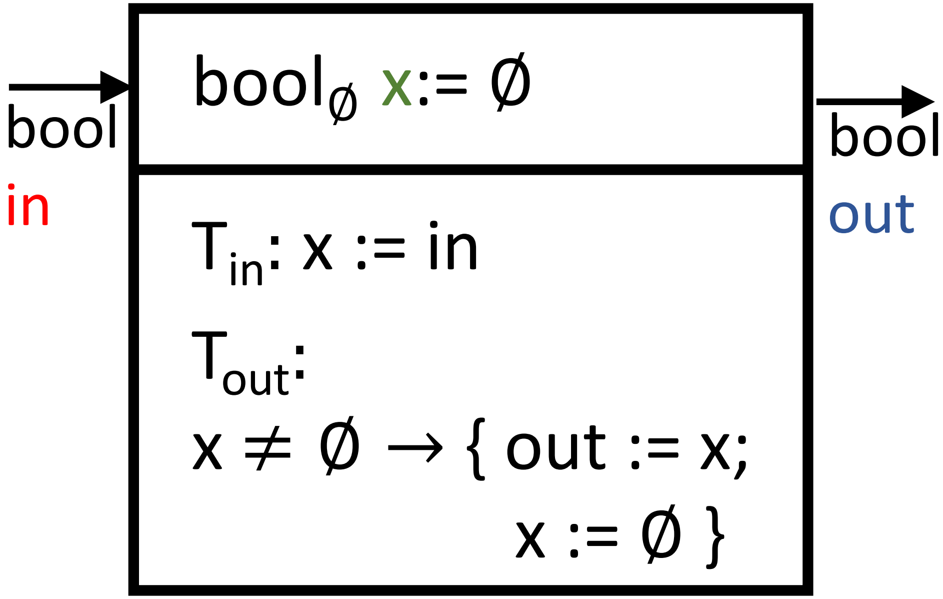
bool out

- Input channel in of type bool
- Output channel out of type bool
- State variable x of type bool+∅. The value ∅ indicates empty or null.
- x initialized to ∅
- Input task T_{in} reads input value into x
- Output task T_{out} produces output if x is not empty

Asynchronous Reactive Component Execution

- Execution Model: In each step only one task is executed
- Task can be executed only if it is enabled (i.e. if its guard condition is true)
- If multiple guard conditions are true, one task is nondeterministically executed
- Sample execution:

$$\emptyset \xrightarrow[\mathrm{T}_{\mathrm{in}}]{\mathrm{in?0}} 0 \xrightarrow[\mathrm{T}_{\mathrm{out}}]{\mathrm{out!0}} \emptyset \xrightarrow[\mathrm{T}_{\mathrm{in}}]{\mathrm{in?1}} 1 \xrightarrow[\mathrm{T}_{\mathrm{in}}]{\mathrm{in?0}} 0 \xrightarrow[\mathrm{T}_{\mathrm{out}}]{\mathrm{out!0}} \emptyset$$



bool in

$\text{bool}_{\emptyset}$ x:= Ø

T_{in}: x := in

T_{out}:

x ≠ Ø → { out := x;
x := Ø }

bool out

Buffer

Example: Asynchrony + Nondeterminism

int x:= 0, y:= 0

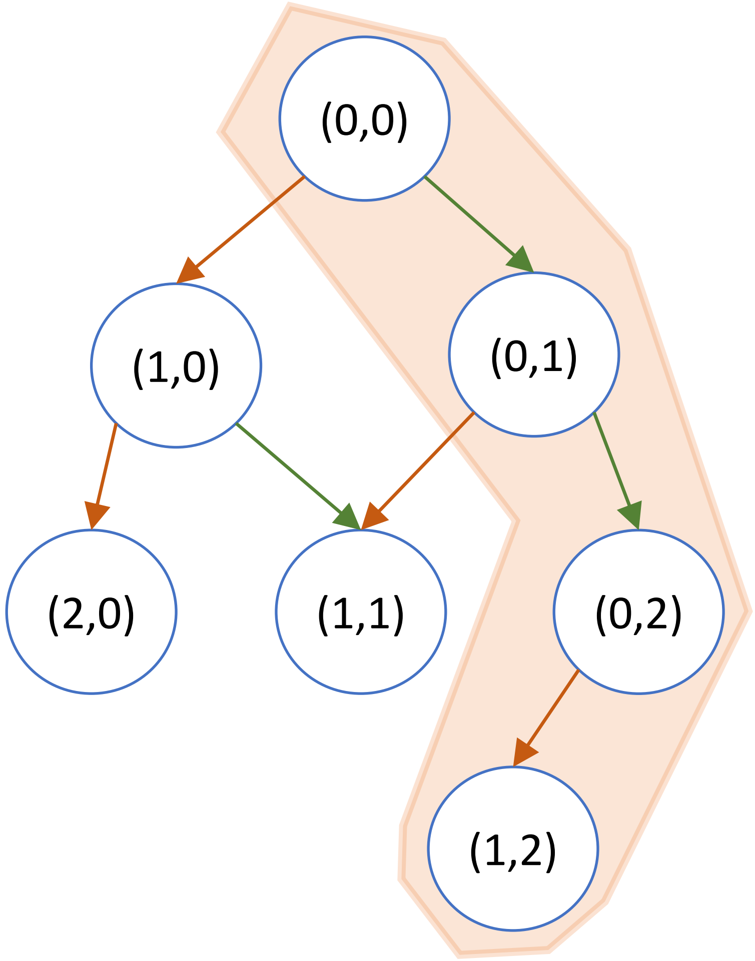
T_x: x := x+1

T_y: y:= y+1

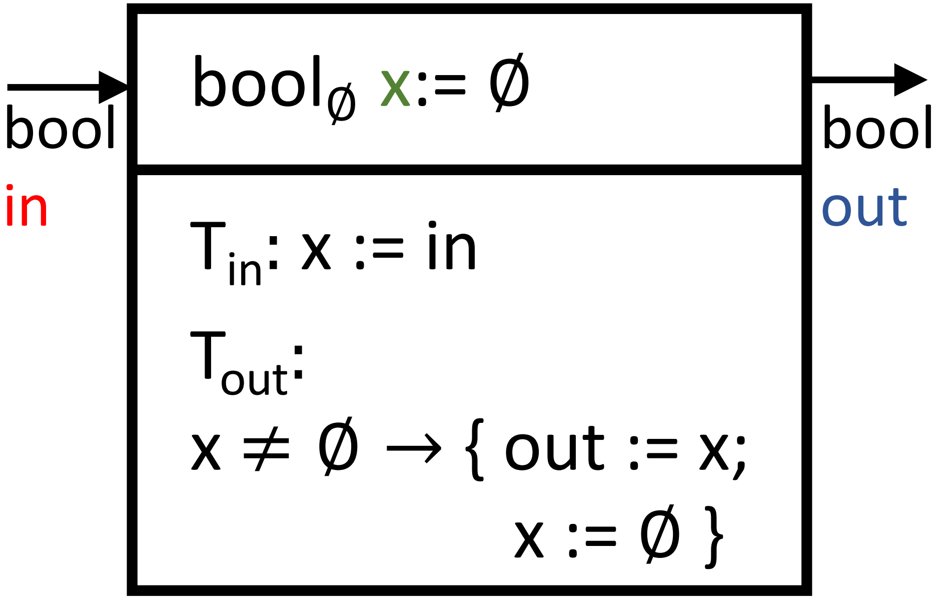
- ARC may have no inputs or outputs, just internal tasks
  - Update may have no guards
- In each step, execute T_x or T_y
- Sample execution:

$$(0,0) \underset{T_y}{\rightarrow} (0,1) \underset{T_y}{\rightarrow} (0,2) \underset{T_x}{\rightarrow} (1,2) \underset{T_y}{\rightarrow} (1,3)$$

- Interleaved model of concurrency



Asynchronous Process/Reactive Component



$\text{bool}_{\emptyset}\ x := \emptyset$

T_{in}: $x := in$

T_{out}:

$x \neq \emptyset \rightarrow \{\ out := x;$
$\qquad x := \emptyset\ \}$

- Set of input channels: I
  - ESM representation: in?v, where v is the value to be received
- Set of output channels: O
  - ESM representation: out!v, where v is the value to be written
- Set of state variables X
- Initialization Init which maps state variables to initial values

Updates are different from SRCs!

Input Task defines updates of the form: **G → x:= E(X,in)**

- Guard condition G: some expression over ***only*** state variables X; input task can be executed only if G is true
- For each in in I, we associate a read-set (X ∪ {in}): variables that can appear in E for input task associated with in (rationale: can read value on in only if that task is enabled)
- Defines a set of input actions of the form: $q \xrightarrow{\mathrm{in}?v} q'$
  - where q is value of state variables before update, and q satisfies G
  - value of state variables after update is q' = E(X↦q, in↦v)

Updates are different from SRCs!

Output Task: defines updates of the form: **G** → **out := E(X)**

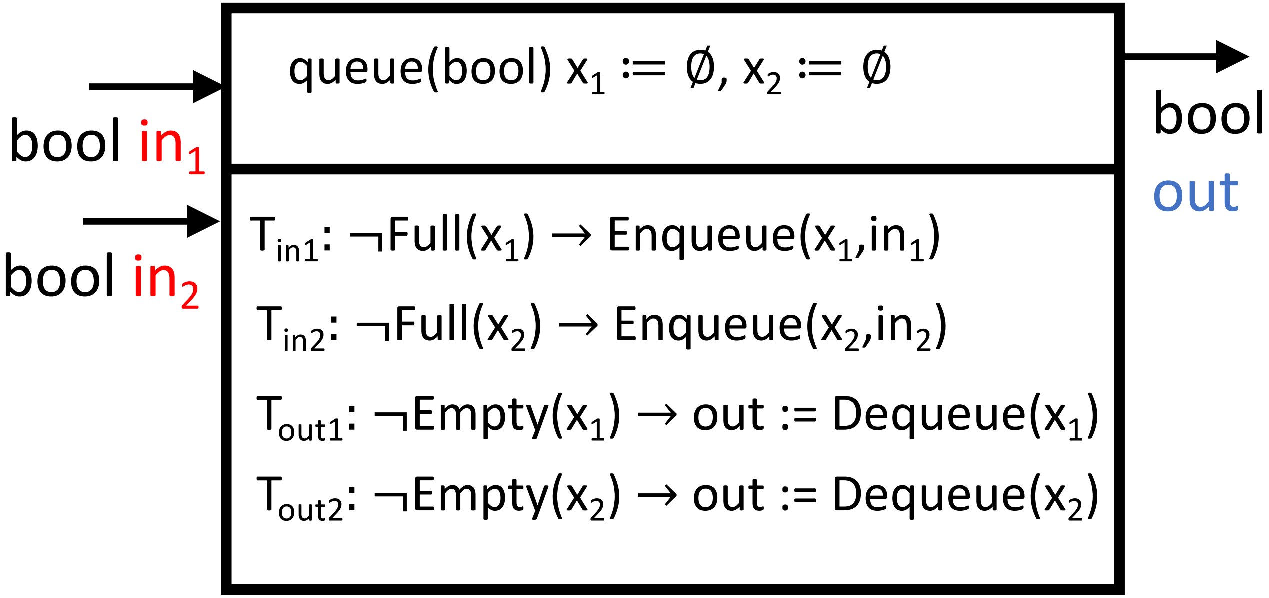
- Guard condition G: some expression over ***only*** variables in X; output task can be executed only if G is true
- Any expression containing only state variables can appear in E
- Defines an output action of the form $q \xrightarrow{out!v} q'$
  - where q is value of state variables before update, and q satisfies G
  - value of state variables after update is q'
  - value v is output on channel out

Updates are different from SRCs!

Internal Task: defines updates of the form: **G → x := E(X)**

- Guard condition G: some expression over ***only*** variables in X; internal task can be executed only if G is true
- Any expression containing only state variables can appear in E, only state variables appear on LHS
- Defines an internal action of the form $q \xrightarrow{\varepsilon} q'$
  - where q is value of state variables before update, and q satisfies G
  - value of state variables after update is q’
  - No input is read or output is produced!

Asynchronous Example



bool in_1

bool in_2

queue(bool) $x_1 \coloneqq \emptyset$, $x_2 \coloneqq \emptyset$

T_{in1}: $\neg\text{Full}(x_1) \rightarrow \text{Enqueue}(x_1, in_1)$

T_{in2}: $\neg\text{Full}(x_2) \rightarrow \text{Enqueue}(x_2, in_2)$

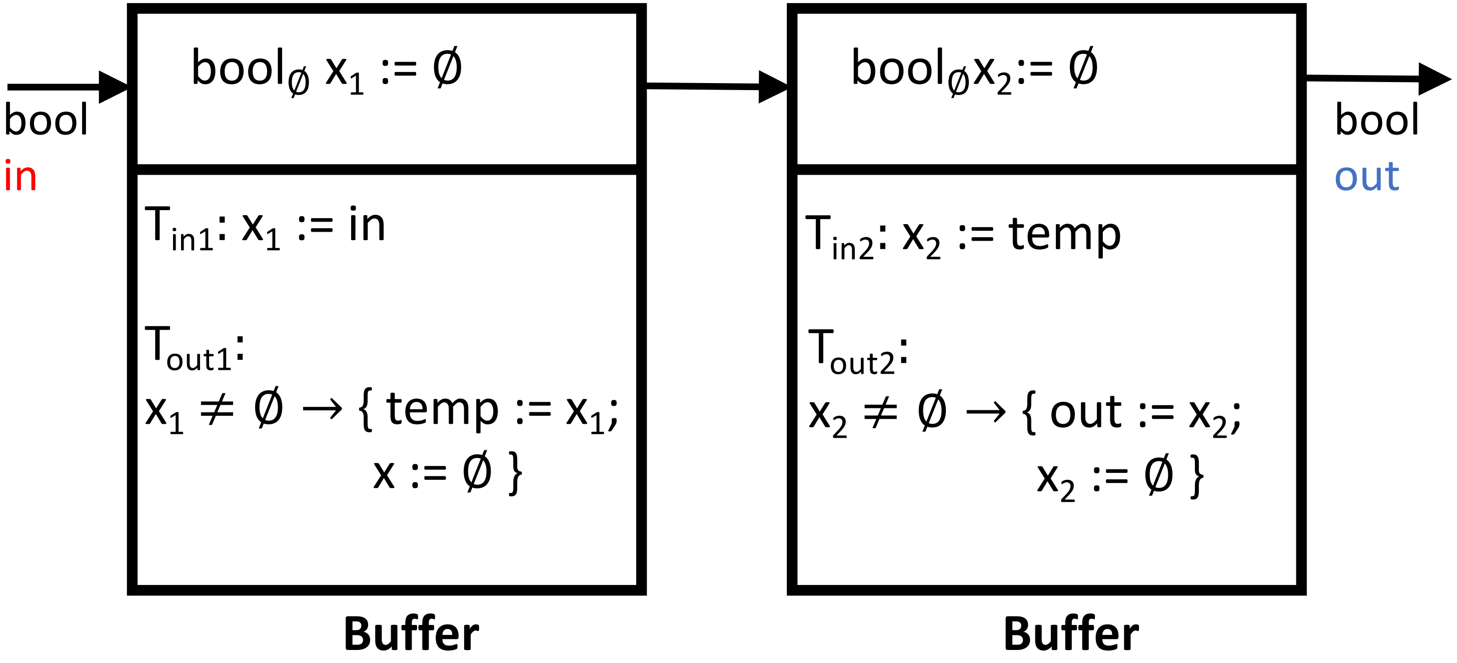
T_{out1}: $\neg\text{Empty}(x_1) \rightarrow out := \text{Dequeue}(x_1)$

T_{out2}: $\neg\text{Empty}(x_2) \rightarrow out := \text{Dequeue}(x_2)$

bool out

Asynchronous Processes can also be represented with extended state machines

Composing Asynchronous Processes



bool
in

$\text{bool}_\emptyset\ x_1 := \emptyset$

T_{in1}: $x_1 := \text{in}$

T_{out1}:
$x_1 \neq \emptyset \rightarrow \{ \text{temp} := x_1;$
$x := \emptyset \}$

Buffer

$\text{bool}_\emptyset x_2 := \emptyset$

T_{in2}: $x_2 := \text{temp}$

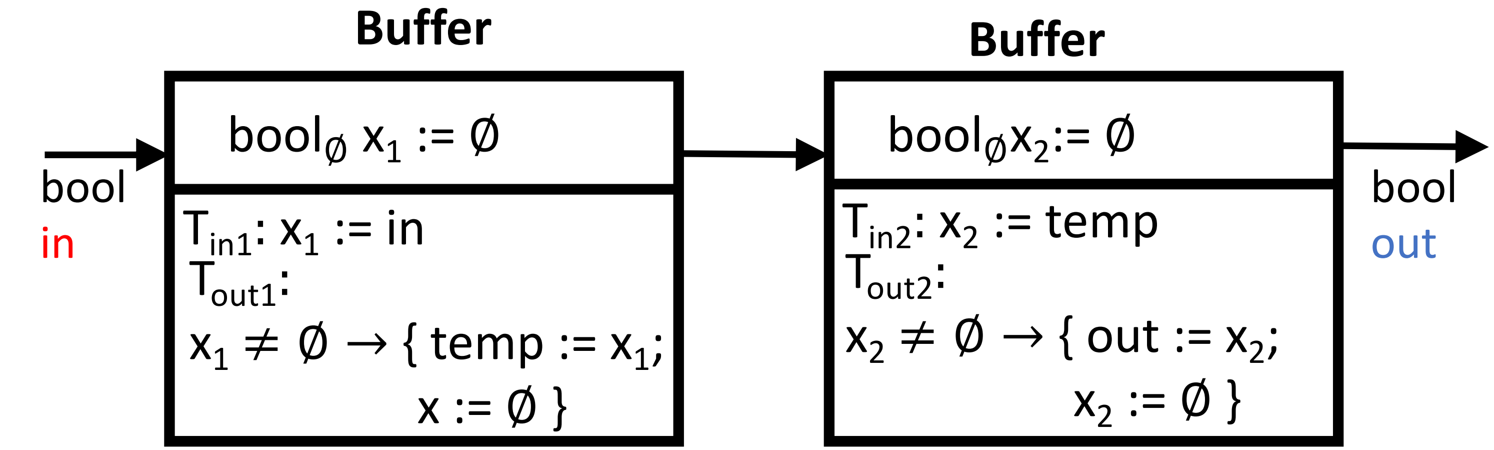
T_{out2}:
$x_2 \neq \emptyset \rightarrow \{ \text{out} := x_2;$
$x_2 := \emptyset \}$

Buffer

bool
out

- Parallel composition: Inputs, Outputs, States and Initialization similar to the synchronous case
- Input consumption needs to be synchronized with output production for the ‘temp’ variable

Composed DoubleBuffer



Buffer

bool bool in

$\text{bool}_{\emptyset}\ x_1 := \emptyset$

T_{in1}: $x_1 := \text{in}$

T_{out1}:

$x_1 \neq \emptyset \rightarrow \{\ \text{temp} := x_1;$

$x := \emptyset\ \}$

Buffer

$\text{bool}_{\emptyset} x_2 := \emptyset$

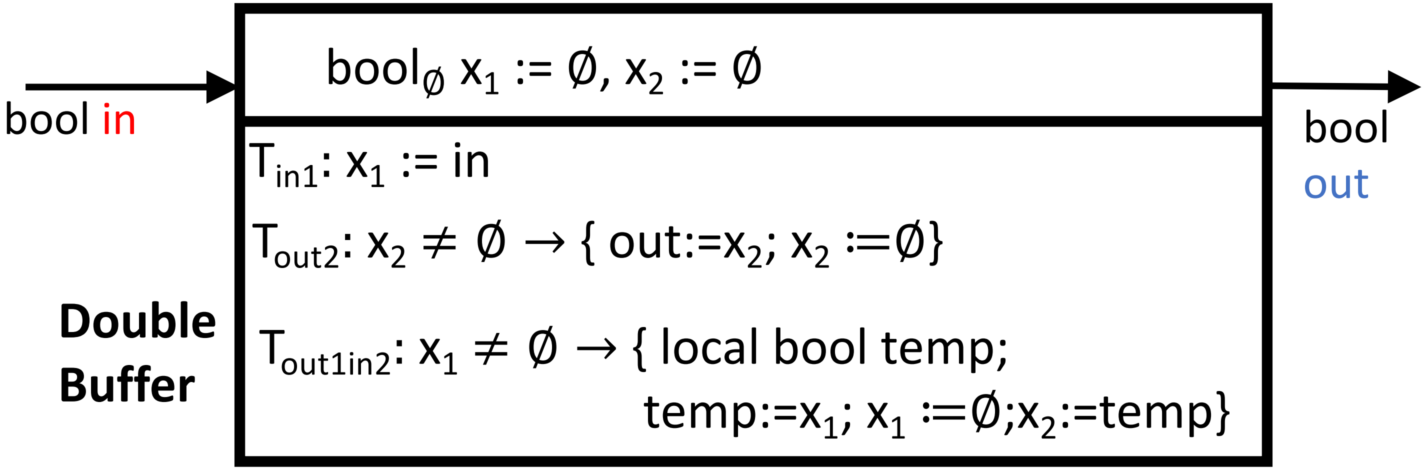
T_{in2}: $x_2 := \text{temp}$

T_{out2}:

$x_2 \neq \emptyset \rightarrow \{\ \text{out} := x_2;$

$x_2 := \emptyset\ \}$

bool out



bool in

$\text{bool}_{\emptyset}\ x_1 := \emptyset,\ x_2 := \emptyset$

T_{in1}: $x_1 := \text{in}$

T_{out2}: $x_2 \neq \emptyset \rightarrow \{\ \text{out}{:=}x_2;\ x_2 := \emptyset\}$

$T_{out1in2}$: $x_1 \neq \emptyset \rightarrow \{$ local bool temp;

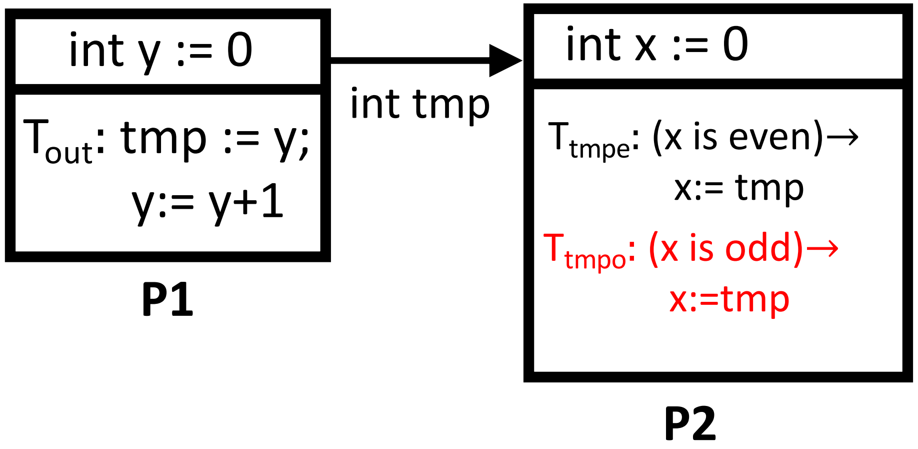
$\text{temp}{:=}x_1;\ x_1 := \emptyset; x_2{:=}\text{temp}\}$

bool out

Double Buffer

- Defining $P_1 \mid P_2$
- In each step only 1 task executes
- If y is an output channel of P_1 and input channel of P_2:
- A_1: output task for P_1 with code: $G_1 \rightarrow U_1$
- A_2: input task for P_2 with code: $G_2 \rightarrow U_2$
- Composition has output task for y with code: $G_1 \wedge G_2 \rightarrow U_1;U_2$

Blocking vs. Non-blocking Synchronization



int y := 0

T_{out}: tmp := y;
y:= y+1

P1

int tmp

int x := 0

T_{tmpe}: (x is even)→
x:= tmp

T_{tmpo}: (x is odd)→
x:=tmp

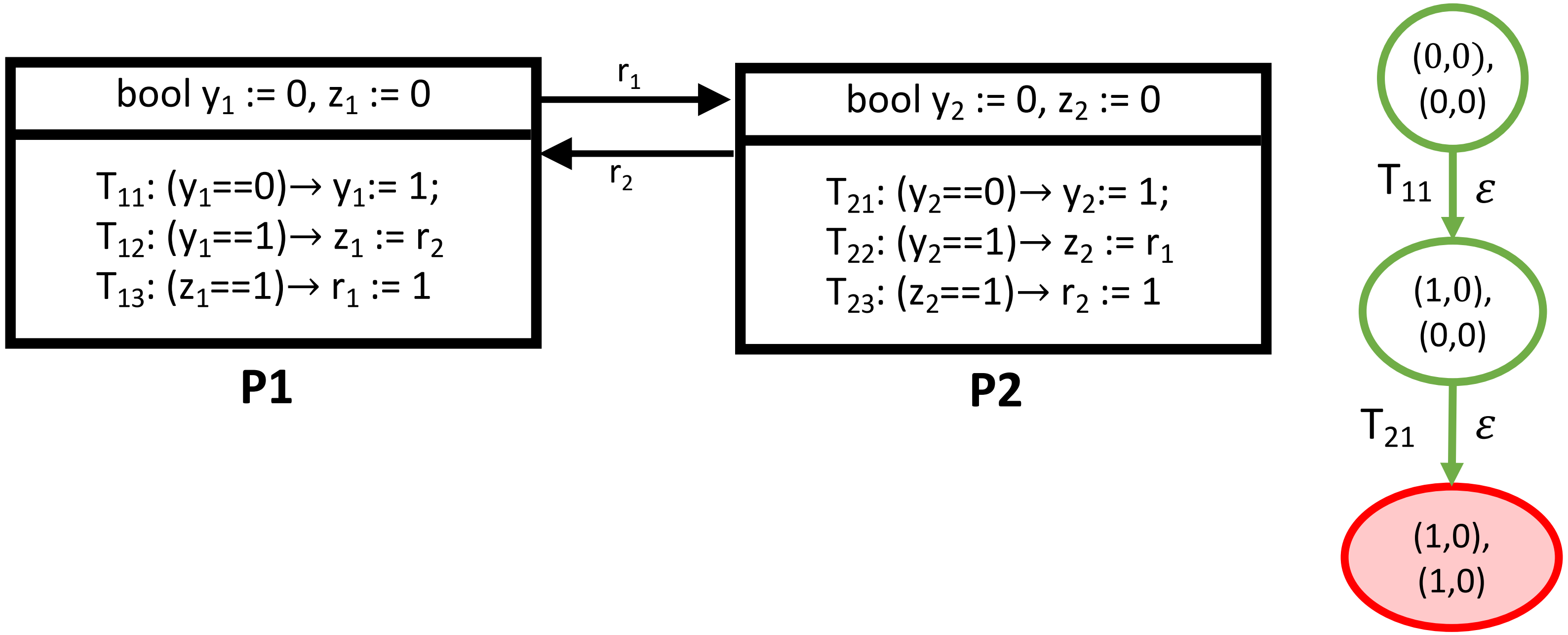
P2

How do you make P2 non-blocking?

- Task T_{out} of P1 can *produce* a value on the output only if P2 has an input task that is enabled to *consume* the value with some input task
- In this example, once x becomes odd, P2 cannot consume (no enabled input task) and it **blocks** communication
- Process is **non-blocking** on channel in if at least one guarded update corresponding to input task for in is enabled
- Process is **non-blocking** if for every input channel, the disjunction of all guards corresponding to input tasks for that channel is *valid* or the Boolean formula 1 (true).

Deadlocks

- Common error in asynchronous designs
- Caused by each process waiting for another process to execute a task, but no task is enabled



bool $y_1 := 0, z_1 := 0$

T_{11}: $(y_1{=}{=}0) \rightarrow y_1 := 1$;
T_{12}: $(y_1{=}{=}1) \rightarrow z_1 := r_2$
T_{13}: $(z_1{=}{=}1) \rightarrow r_1 := 1$

P1

bool $y_2 := 0, z_2 := 0$

T_{21}: $(y_2{=}{=}0) \rightarrow y_2 := 1$;
T_{22}: $(y_2{=}{=}1) \rightarrow z_2 := r_1$
T_{23}: $(z_2{=}{=}1) \rightarrow r_2 := 1$

P2